SYNTHESIS, ANALYSIS AND SIMULATION OF A FOUR-BAR MECHANISM USING MATLAB PROGRAMMING

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ABSTRACT

The four-bar mechanism is a class of mechanical linkage in which four links are pinned together to form a closed loop in order to perform some useful motion. This has long been, and continues to be, an effective tool for mechanical design engineers. This paper considers synthesis, analysis and simulation of the four-bar linkage analytically for three and four precision positions of the motion generation problem. Kinematic synthesis of the four-bar mechanism using the complex number method is presented. The results of the synthesis process are analyzed to determine motion characteristics of the mechanism. These motion characteristics are then used for simulation of the mechanism. Matlab programs are written for solving the equations developed in the synthesis and analysis problems. Matlab is also used to develop user-friendly Graphic User Interface windows for data input and output as well as for simulation.

INTRODUCTION

Mechanism synthesis is the process of generating the geometry of a mechanism that will perform a specific task. Inhere, geometry generation means determining the lengths of the individual links that make up the mechanism for performing a desired task. In this paper the focus will be on the planar four bar mechanism as shown in Fig. 1.

There are three broad categories that are considered in the synthesis process. These are outlined below.

Function Generation defines the relationship between the position of the output link and the position of the input link. The motion that the coupler link goes through is not of concern. Examples of function generation include the automobile accelerator, the control stick in an aircraft, and the piston in an engine mechanism.

Path Generation defines locations through which a tracer point, or a point of interest, on the coupler will pass. The orientation of the coupler link is not important, but the times at which the tracer point passes the prescribed locations may be important. In this case the synthesis problem is path generation with prescribed timing. The film advancing mechanism in a movie camera is an application of path generation.

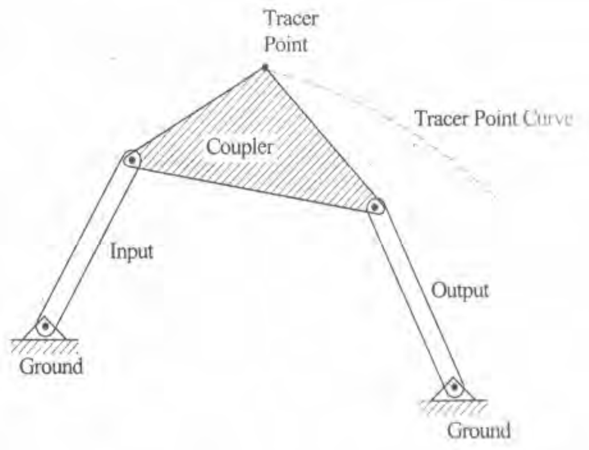


Figure 1 Four-bar mechanism

Motion Generation (Rigid-body Guidance) defines locations through which the tracer point passes and the corresponding orientation of the coupler link at those locations. Examples of motion generation include the power lift gate on a truck, the lift mechanism on a dumpster truck, and the windshield wipers on an automobile.

For all three synthesis types the prescribed conditions that the mechanism must satisfy are called the *precision positions*. For function generation the precision positions consist of the pairs of input and output angles, β_i and γ_i respectively, that the mechanism must meet. For path generation the precision positions are the pairs of coordinates (x_i, y_i) which the tracer point must pass through. In addition, for prescribed timing the angle of the input ($\square_i$) will also be specified. In the case of motion generation the precision positions are the coordinates the tracer point passes through (x_i, y_i), as well as the orientation of the coupler defined by angle $\square_i$. This paper deals with Motion Generation problem of kinematic synthesis of a four-bar mechanism. The synthesized mechanism is then analyzed for determining its motion characteristics. These results are then used for the animation of the mechanism.

Conventional mechanism design procedures using card board models, prototypes of the mechanism, making trial and error calculations, and a combination of the above, have recently been getting replaced with the use of computer software. Software packages can assist the designer in performing repetitive and programmable calculations fast and thus help in minimizing the errors that can be induced by human fatigue.

Various means of designing or synthesizing a mechanism to produce desired motion in a four-bar mechanism have been developed. Sandor and Erdman (1984) describe a method of solving the problem using graphical constructions. They also provide analytical equations to solve the same problem computationally. Nikravesh 1988; Haug 1989; Shabana 1994, proposed methods based on the well-established absolute coordinate method for kinematic analysis [3 }.

Indeed, there are a number of tools like KYNSYN, Watt, LINCAGES [1] available, which are commercially very expensive, that can be used in the analysis and synthesis of planar mechanisms. However, in this work, Matlab programming language has been used as the graphic interface, which is more user friendly and has also built in routines that ease the programming difficulties.

Synthesis of the four-bar mechanism

In dimensional synthesis, the component dimensions that comprise a chosen mechanism are specified to allow the mechanism to perform a given task [5].

Motion of the coupler can only be approximated by several discrete precision points. That is, the resulting linkage can create the motion desired precisely at these positions and approximately at other positions. The more precision points are used, the closer is the actual motion of the coupler to the ideal desired motion. But the problem becomes more difficult to solve as the number of precision positions is increased. Fortunately, many real world problems only need several critical positions to be satisfied precisely. Tolerance is usually allowed at positions other than the precision points [6]. A four bar linkage can satisfy up to five prescribed positions for the standard-form solutions of motion generation problem [5].

The analytic synthesis procedure is algebraic and is less intuitive; hence, quite suitable for computerization. In order to obtain a handle on the design variables and free choices, an analytical model of the linkage must be developed. Several mathematical techniques for modeling linkages have been utilized for planar synthesis objectives. These include algebraic methods, matrix methods, and complex numbers. For planar linkages, the complex numbers technique is the simplest, yet the most versatile method [5]. This technique uses vectors to designate each of the links in the mechanism. Fig. 2 shows the schematic representation of the four-bar mechanism shown in Fig. 1 by replacing the links by vectors [4].

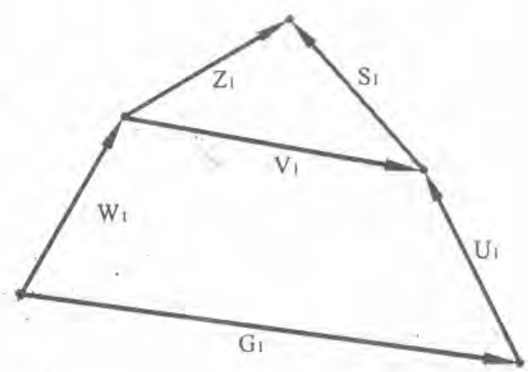


Figure 2 Schematic representation of the four-bar mechanism

Loop Equations

The equations and process for synthesizing the four-bar mechanism are developed by introducing dyads. The mechanism can be broken in to two dyads, $\boldsymbol{W_1}$ and $\boldsymbol{Z_1}$ which make up the input dyad, and $\boldsymbol{U_1}$ and $\boldsymbol{S_1}$ which make up the output dyad. Specifying these two dyads is enough to determine the entire mechanism because the remaining vectors $\boldsymbol{V_1}$ and $\boldsymbol{G_1}$ can be derived from:

$$\boldsymbol{V_1} = \boldsymbol{Z_1} - \boldsymbol{S_1}$$

$$G_1 = W_1 + V_1 - U_1 \tag{1}$$

The angles between V_1 and Z_1, Z_1 and S_1, V_1 and S_1 are fixed, as these vectors make up the rigid coupler link known as a terinary link. Fig. 3 shows the linkage in three successive positions.

The equations that are used to generate solutions for the input and output dyads are called loop equations. The first loop begins at the ground pivot and proceeds out along the second position dyad vectors, W_2 and Z_2, along the displacement vector from the first precision point, P_1, to the second precision point, P_2, and then back along the first position dyad vectors, Z_1 and W_1, to the ground pivot.

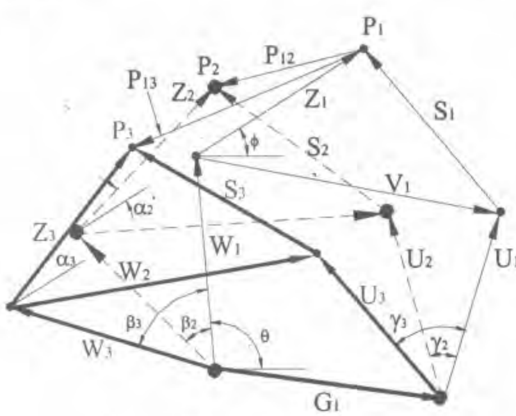


Figure 3

For the input and output dyads, the loop equations become:

$$\begin{aligned} W_2 + Z_2 - P_{12} - Z_1 - W_1 &= 0 \\ W_3 + Z_3 - P_{13} - Z_1 - W_1 &= 0 \end{aligned} \tag{2}$$

Moving the displacement vector to the right hand side and using the relationship between the second position and first position vectors

$$\begin{aligned} W_1 e^{i\beta_2} + Z_1 e^{i\alpha_2} - Z_1 - W_1 &= P_{12} \\ W_1 e^{i\beta_3} + Z_1 e^{i\alpha_3} - Z_1 - W_1 &= P_{13} \end{aligned} \tag{3}$$

And these equations can be simplified to:

$$\begin{aligned} W_1\left(e^{i\beta_2} - 1\right) + Z_1\left(e^{i\alpha_2} - 1\right) &= P_{12} \\ W_1\left(e^{i\beta_3} - 1\right) + Z_1\left(e^{i\alpha_3} - 1\right) &= P_{13} \end{aligned} \tag{4}$$

Equations (4) are commonly referred to as the *standard form loop equations*, and each of them contains two scalar equations. Resolving the vectors into real and imaginary parts, we have the real parts as:

$$\begin{aligned} &w\cos\theta(\cos\beta_2 - 1) - w\sin\theta\sin\beta_2 + \\ &\quad z\cos\phi(\cos\alpha_2 - 1) - z\sin\phi\sin\alpha_2 = p_{12}\cos\delta_2 \\ &w\cos\theta(\cos\beta_3 - 1) - w\sin\theta\sin\beta_3 + \\ &\quad z\cos\phi(\cos\alpha_3 - 1) - z\sin\phi\sin\alpha_3 = p_{13}\cos\delta_3 \end{aligned} \tag{5}$$

and the imaginary ones as:

$$\begin{aligned} &w\sin\theta(\cos\beta_2 - 1) - w\cos\theta\sin\beta_2 + z\sin\phi(\cos\alpha_2 - 1) - \\ &\quad z\cos\phi\sin\alpha_2 = p_{12}\sin\delta_2 \\ &w\sin\theta(\cos\beta_3 - 1) - w\cos\theta\sin\beta_3 + z\sin\phi(\cos\alpha_3 - 1) - \\ &\quad z\cos\phi\sin\alpha_3 = p_{13}\sin\delta_3 \end{aligned} \tag{6}$$

In these equations there are twelve variables. Namely, w, θ, β_2, β_3, z, ϕ, α_2, α_3, p_{12}, p_{13}, δ_2 and δ_3. Six of these (α_2, α_3, p_{12}, p_{13}, δ_2 and δ_3) are input variables and since we have only four equations we can solve for only four of the remaining six unknowns. The other two unknowns are taken as *free choices* [5]. Taking □□and □□as the free choices, as it is commonly done, the transcendental non-linear equations are linearized.

Equation (4) can be written in matrix form as:

$$\begin{bmatrix} e^{\beta_2} - 1 & e^{\alpha_2} - 1 \\ e^{\beta_3} - 1 & e^{\alpha_3} - 1 \end{bmatrix} \begin{Bmatrix} W_1 \\ Z_1 \end{Bmatrix} = \begin{Bmatrix} P_{12} \\ P_{13} \end{Bmatrix} \tag{7}$$

Equation (7) is solved for W_1 and Z_1, using the Cramer's rule. Taking β_2 and β_3 as free choices,

$$W_1 = \frac{\begin{vmatrix} P_{12} & e^{\alpha_2} - 1 \\ P_{13} & e^{\alpha_3} - 1 \end{vmatrix}}{\begin{vmatrix} e^{\beta_2} - 1 & e^{\alpha_2} - 1 \\ e^{\beta_3} - 1 & e^{\alpha_3} - 1 \end{vmatrix}} \tag{8}$$

$$Z_1 = \frac{\begin{vmatrix} e^{\beta_2}-1 & P_{12} \\ e^{\beta_3}-1 & P_{13} \end{vmatrix}}{\begin{vmatrix} e^{\beta_2}-1 & e^{\alpha_2}-1 \\ e^{\beta_3}-1 & e^{\alpha_3}-1 \end{vmatrix}} \tag{9}$$

Similarly, from the right hand dyad, U_1 and S_1 are obtained from:

$$\begin{bmatrix} e^{\gamma_2}-1 & e^{\alpha_2}-1 \\ e^{\gamma_3}-1 & e^{\alpha_3}-1 \end{bmatrix} \begin{Bmatrix} U_1 \\ S_1 \end{Bmatrix} = \begin{Bmatrix} P_{12} \\ P_{13} \end{Bmatrix} \tag{10}$$

Equation (10) can be solved for U_1 and S_1, taking γ_2 and γ_3 as free choices.

$$U_1 = \frac{\begin{vmatrix} P_{12} & e^{\alpha_2}-1 \\ P_{13} & e^{\alpha_3}-1 \end{vmatrix}}{\begin{vmatrix} e^{\gamma_2}-1 & e^{\alpha_2}-1 \\ e^{\gamma_3}-1 & e^{\alpha_3}-1 \end{vmatrix}} \tag{11}$$

and

$$S_1 = \frac{\begin{vmatrix} e^{\gamma_2}-1 & P_{12} \\ e^{\gamma_3}-1 & P_{13} \end{vmatrix}}{\begin{vmatrix} e^{\gamma_2}-1 & e^{\alpha_2}-1 \\ e^{\gamma_3}-1 & e^{\alpha_3}-1 \end{vmatrix}} \tag{12}$$

W_1, Z_1, U_1 and S_1 are complex numbers representing the links of the four bar linkage. The remaining vectors V_1 and G_1 are obtained from Eq. (1).

The lengths of the links are expressed by the magnitudes of the corresponding vectors. Designating the lengths of the ground link by r_1, the input crank link by r_2, the coupler link by r_3 and output link by r_4, we have,

$$r_1 = |G_1|,\ r_2 = |W_1|,\ r_3 = |V_1|, r_4 = |Z_1| \tag{13}$$

Synthesis of Four-bar mechanisms for Four positions

In a manner similar to the synthesis of four-bar mechanisms for three positions, the following equations can be developed considering four positions for the left hand dyad.

$$\begin{bmatrix} e^{i\beta_2}-1 & e^{i\alpha_2}-1 \\ e^{i\beta_3}-1 & e^{i\alpha_3}-1 \\ e^{i\beta_4}-1 & e^{i\alpha_4}-1 \end{bmatrix} \begin{Bmatrix} \mathrm{W}_1 \\ \mathrm{Z}_1 \end{Bmatrix} = \begin{Bmatrix} \mathrm{P}_{12} \\ \mathrm{P}_{13} \\ \mathrm{P}_{14} \end{Bmatrix} \tag{14}$$

In this case, the resulting six scalar equations have seven unknowns and can not be solved unless one of the unknowns is taken as a free choice. Although one of the unknowns is made a free choice, the system of equations, unlike to that for three positions, is still non-linear. Rearranging the equations yields,

$$\begin{bmatrix} e^{i\beta_2}-1 & e^{i\alpha_2}-1 & \mathrm{P}_{12} \\ e^{i\beta_3}-1 & e^{i\alpha_3}-1 & \mathrm{P}_{13} \\ e^{i\beta_4}-1 & e^{i\alpha_4}-1 & \mathrm{P}_{14} \end{bmatrix} \begin{Bmatrix} \mathrm{W}_1 \\ \mathrm{Z}_1 \\ -1 \end{Bmatrix} = 0 \tag{15}$$

From Eq. (15) it can be noted that, because of the -*1* term, the vector can never be equal to zero. Therefore, the matrix must be singular or non-invertible. Hence, the determinant of that matrix equals zero. i.e.,

$$\begin{vmatrix} e^{i\beta_2}-1 & e^{i\alpha_2}-1 & P_{12} \\ e^{i\beta_3}-1 & e^{i\alpha_3}-1 & P_{13} \\ e^{i\beta_4}-1 & e^{i\alpha_4}-1 & P_{14} \end{vmatrix} = 0 \tag{16}$$

Expanding the determinant yields

$$\mathbf{a}e^{i\beta_2} + \mathbf{b}e^{i\beta_3} + \mathbf{c}e^{i\beta_4} - (\mathbf{a}+\mathbf{b}+\mathbf{c}) = 0 \tag{17}$$

where,

$$\boldsymbol{a} = (e^{i\alpha_3}-1)P_{14} - (e^{i\alpha_4}-1)P_{13}$$
$$\mathbf{b} = (e^{i\alpha_4}-1)\mathrm{P}_{12} - (e^{i\alpha_2}-1)\mathrm{P}_{14}$$
$$\boldsymbol{c} = (e^{i\alpha_2}-1)P_{13} - (e^{i\alpha_3}-1)P_{12}$$

Taking one of the three unknowns in Eq. (17) as the free choice, it is possible to solve for the other two unknowns using the Symbolic Math Toolbox of Matlab. If we make β_2 the free choice we can solve for β_3 and β_4. Now, the problem is reduced to the linear system of equations given by Eq. (18) which can be solved for the remaining unknowns W_1 and Z_1 using Cramer's rule.

$$\begin{bmatrix} e^{i\beta_2}-1 & e^{i\alpha_2}-1 \\ (e^{i\beta_3}+e^{i\beta_4}-2) & e^{i\alpha_3}+e^{i\alpha_4}-2 \end{bmatrix} \begin{Bmatrix} \mathrm{W}_1 \\ \mathrm{Z}_1 \end{Bmatrix} = \begin{Bmatrix} \mathrm{P}_{12} \\ \mathrm{P}_{13}+\mathrm{P}_{14} \end{Bmatrix} \tag{18}$$

Analysis of the four-bar mechanism

To carry out the analysis of the four-bar mechanism, the equations that define the relative movement of the links are derived using the physical dimensions obtained from the synthesis. The configuration of the linkage depends on a free parameter, which is typically the rotation angle of one link, known as the driving link.

In this paper we are concerned with those mechanisms that generate closed coupler curves for a complete crank rotation. Such mechanisms are called *Grashof Mechanisms*.

The Grashof's Criterion

Grashof's criterion states that *the sum of the shortest and longest links of a planar four-bar linkage can not be greater than the sum of the remaining two links, if there is to be continuous relative rotation between two links* [5].

If this rule is satisfied, in other words, if

$$l + s < p + q,$$

where l and s are the longest and shortest links of the four-bar mechanism, we may have a crank-rocker, a double-crank, or a double-rocker depending on the position and configuration of the links.

Position Analysis

The position problem basically consists of determining the position of all the links in the system given the positions of the fixed and the *input* links which can also be called *guided* or *driving* elements. Mathematically, the initial position problem reduces to determining the vector of dependent coordinates from the known coordinates corresponding to the input elements that satisfy the nonlinear system of *constraint equations*.

The position problem is always based on solving the constraint equations, which make up the following set of non-linear equations:

$$\Phi(\mathbf{q}) = 0 \qquad (19)$$

where $\boldsymbol{q}$ is the vector of the system dependent coordinates. It is assumed that there are at least as many equations as there are unknown variables or coordinates. To solve the system of nonlinear equations given by Eq. (19), it is customary to resort to the well-known Newton-Raphson method which has quadratic convergence in the neighborhood of the solution.

The Newton-Raphson method is based on linearizing the system of Eqs. (19) in which the system of equations is replaced by the first two terms of its Taylor series expansion around a certain approximation $\boldsymbol{q}^{(0)}$ of the desired solution. Once the substitution has been made, the system becomes

$$\boldsymbol{\Phi}(q,t) \cong \boldsymbol{\Phi}(q^{(0)}) + \boldsymbol{\Phi}_q(q^{(0)})(q^{(1)} - q^{(0)}) \qquad (20)$$

where matrix $\boldsymbol{\Phi}_q$ is the Jacobian matrix for the constraint equations. $\boldsymbol{\Phi}_q$ is the matrix of partial derivatives of the constraint equations with respect to the dependent coordinates where m is the number of constraint equations and n is the number of dependent coordinates.

Equation (20) represents a system of linear equations constituting an approximation to the non-linear system of Eq. (19). The vector $\boldsymbol{q}^{(1)}$, obtained from the solution of Eq. (20), will be an approximation of the solution of Eq. (19). Going through this process repeatedly, the following recursive formula is developed.

$$\boldsymbol{\Phi}(q^{(i)},t) + \boldsymbol{\Phi}_q(q^{(i)},t)(q^{(i+1)} - q^{(i)}) = 0 \qquad (21)$$

Equation (20) is solved iteratively until the error, i.e. the difference between the results of two successive iterations, is smaller than the pre-specified tolerance.

It should be noted that the Newton-Raphson iteration will not always converge to the desired solution. If the initial approximation is not close enough to the desired solution, the algorithm may diverge, or may converge to an undesired solution. There is still another source of difficulties. If the values of the input variables do not correspond to a possible physical solution, the mathematical algorithm will fail irrespective of how the initial approximation has been chosen.

Velocity Analysis

The equations that permit to solve the velocity problem originate from differentiating the constraint equations including the driving constraint equations with respect to time. Differentiating Eq. (19) with respect to time gives:

$$\Phi_q(q,t)\dot{q} = b \qquad (22)$$

Where Φ_q is the Jacobian matrix defined in Eq. (20) and vector $\dot{q}$ is the vector of dependent velocities (derivatives with respect time of the vector of dependent position variables). Vector b is partial derivative of the constraint equations with respect to time.

$$b = -\Phi_t$$

If there are no time dependent constraints, this vector will be zero. Once the position of the mechanism is known, Eq. (22) allows us to determine the velocities by starting from the velocities of the input elements. The difference between the position and velocity problems is that the position problem is non-linear where as the equations governing the velocity problem are linear. Consequently, there is only one solution to a properly posed velocity problem.

Acceleration Analysis

The solution of the acceleration problem is obtained by differentiating Eq. (22) with respect to time. In doing so, we have:

$$\Phi_q(q,t)\ddot{q} = c \qquad (23)$$

where $\ddot{q}$ is the dependent acceleration vector.

$$c = -\dot{\Phi}_{tt} - 2\dot{\Phi}_{qt}\dot{q} - (\Phi_q\dot{q})_q\dot{q}$$

If the position vector q and the velocity vector $\dot{q}$ are known, one can find the dependent acceleration vector $\ddot{q}$ by solving the system of linear Eq. (23).

Solutions to the position, velocity and acceleration Problems

Solution to the position problem:

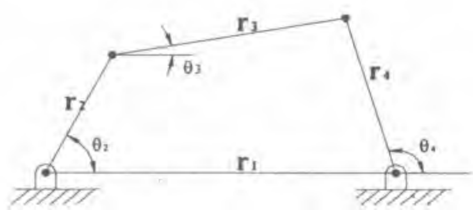


Figure 4

The vector loop equation for a four bar linkage as shown in Fig. 4 can be written as:

$$r_1 + r_2 + r_3 + r_4 = 0 \qquad (24)$$

Equation (24) can be written as,

$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} - r_1 - r_4 e^{i\theta_4} = 0 \qquad (25)$$

The position problem can be stated as: Given the value of θ_2, find those values of θ_3 and θ_4 for which the above equations are satisfied. Since both of the equations are non-linear and transcendental, Newton-Raphson iterative method is used to solve the method iteratively.

Newton-Raphson Algorithm

Let the position problem be expressed as:

$$\Phi(q) = 0$$

where q is the vector unknowns to be found.

a) Estimate the solution $q^{(0)}$

b) Take a small correction factor which is the difference between the values estimated and the solution.

$$q^{(1)} = q^{(0)} + \Delta q$$

c) Using the Taylor series expansion,

$$\{\Phi^{(0)}\} + \left[\frac{\partial \Phi}{\partial \mathbf{q}}\right]\{\Delta \mathbf{q}^{(1)}\} = 0$$

$$\{\Delta \mathbf{q}^{(1)}\} = \{\mathbf{q}^{(1)} - \mathbf{q}^{(0)}\}$$

Which can be solved for the correction factors $\{\Delta q\}$,

$$\{\Delta q^{(1)}\} = -\left[\frac{\partial \Phi}{\partial q}\right]^{-1}\{\Phi^{(0)}\}$$

Forming a recursive formula:

$$\{\Delta q^{(i+1)}\} = -\left[\frac{\partial \Phi}{\partial q}\right]^{-1}\{\Phi^{(i)}\}. \quad (26)$$

d) Set a condition for convergence $\varepsilon > 0$

e) Find $Norm_n = \Phi(q^{(i)})$

If $Norm_n < \varepsilon$, then ,

$$q = q^{(i)}$$

Stop.

Else,

$$\{\Delta q^{(i+1)}\} = -\left[\frac{\partial \Phi}{\partial q}\right]^{-1}\{\Phi^{(i)}\}$$

$$q^{(i)} = q^{(i)} + \Delta q^{(i+1)}$$

and go to (e)

Many mechanisms have multiple solutions for the position problem. The solution that will be provided by the Newton-Raphson scheme is, in general, dependent on the initial guess.

Velocity problem

Differentiating Eq. (24) with respect to time gives

$$ir_2\dot{\theta}_2 e^{i\theta_2} + ir_3\dot{\theta}_3 e^{i\theta_3} - ir_4\dot{\theta}_4 e^{i\theta_4} = 0 \quad (27)$$

where $\dot{r}_1 = \dot{r}_2 = \dot{r}_3 = \dot{r}_4 = \dot{\theta}_1 = 0$

Resolving Eq. (27) into real and imaginary parts and solving it for the unknowns $\dot{\theta}_3$ and $\dot{\theta}_4$ we get

$$\begin{Bmatrix}\dot{\theta}_3\\ \dot{\theta}_4\end{Bmatrix} = \begin{bmatrix}-r_3\sin\theta_3 & r_4\sin\theta_4\\ r_3\cos\theta_3 & -r_4\cos\theta_4\end{bmatrix}^{-1}\begin{Bmatrix}r_2\dot{\theta}_2\sin\theta_2\\ -r_2\dot{\theta}_2\cos\theta_2\end{Bmatrix} \quad (28)$$

Acceleration Analysis

Differentiating Eq. (24) twice with respect to time yields

$$\begin{aligned}&ir_2\ddot{\theta}_2 e^{i\theta_2} - r_2\dot{\theta}_2^2 e^{i\theta_2} + ir_3\ddot{\theta}_3 e^{i\theta_3} - r_3\dot{\theta}_3^2 e^{i\theta_3}\\ &- ir_4\ddot{\theta}_4 e^{i\theta_4} + r_4\dot{\theta}_4^2 e^{i\theta_4} = 0\end{aligned} \quad (29)$$

Resolving Eq. (29) into real and imaginary parts, and solving for $\ddot{\theta}_3$ and $\ddot{\theta}_4$ yields

$$\begin{Bmatrix}\ddot{\theta}_3\\ \ddot{\theta}_4\end{Bmatrix} = \begin{bmatrix}-r_3\sin\theta_3 & r_4\sin\theta_4\\ r_3\cos\theta_3 & -r_4\cos\theta_4\end{bmatrix}^{-1}$$
$$\begin{Bmatrix}r_2\ddot{\theta}_2\sin\theta_2 + r_2\dot{\theta}_2^2\cos\theta_2 + r_3\dot{\theta}_3^2\cos\theta_3 - r_4\dot{\theta}_4^2\cos\theta_4\\ -r_2\ddot{\theta}_2\cos\theta_2 + r_2\dot{\theta}_2^2\sin\theta_2 + r_3\dot{\theta}_3^2\sin\theta_3 - r_4\dot{\theta}_4^2\sin\theta_4\end{Bmatrix} \quad (30)$$

Numerical Example

A numerical example is worked for the three-position motion generation problem to demonstrate the synthesis, analysis and simulation presented in the paper. Synthesis results obtained from the developed computer program are compared with results obtained from graphical synthesis method.

Graphical Method of Synthesis

1. The angular orientation and precision positions of the coupler link in three positions are drawn. In this case the precision points through which the end effector (the point of interest) must pass are

$P_1 = (0,0)$, $P_2 = (-6, 11)$ and $P_3 = (-17, 13)$

The angles of deviation of the Coupler link, link AB in the second and third positions from the first position are α_2=22° *and* α_3=68°.

Choosing points A and B means taking the four free choices (x and y coordinates of both points) that are allowed.

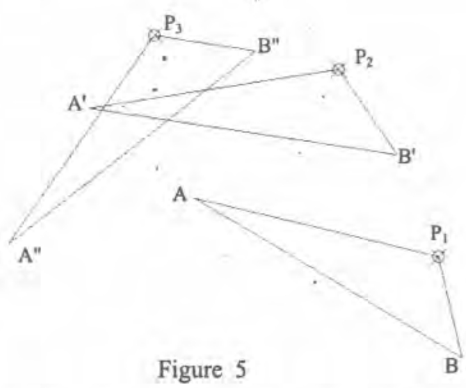


Figure 5

2. Draw construction lines from point A to A' and from point A' to A''.

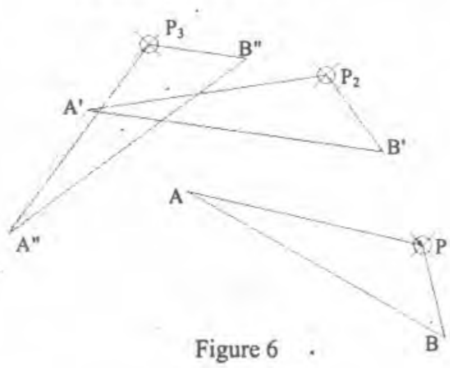


Figure 6

3. Bisect AA' and $A'A''$ and extend the perpendicular bisectors until they intersect. The point of intersection gives one of the pivot points O_2.

4. Repeat steps 2 and 3 for BB' and $B'B''$. This results in the other pivot point O_4.

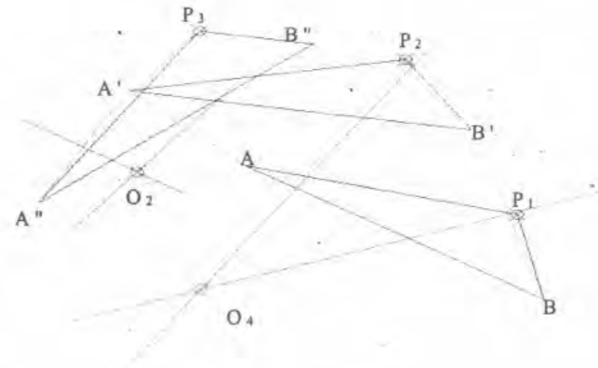
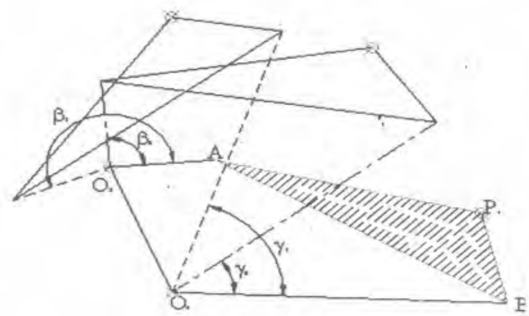


Figure 7



5. Connect O_2 with A and O_4 with B and they give links 2 and 4 respectively. O_2 and O_4 form the ground link-link 1.

Figure 8

From measurement, link 2 and link 4, given by O_2A and O_4B, respectively are found to be

$$W = 5.7550 + 0.4809i$$

$$S = 18.3746 - 0.6611i$$

The ground link, O_2O_4 is,

$$G = 3.4118 - 8.2796i$$

The other links are automatically known from the free choices.

$$V = 16.0313 - 9.4215i$$
$$Z = 14.6106 - 3.4698i$$
$$U = -1.4207 + 5.9518i$$

The angles β_2, β_3, γ_2 and γ_3 are measured to be,

$\beta_2 = 90^0$, $\beta_3 = 198^0$, $\gamma_2 = 40^0$ and $\gamma_3 = 73^0$

Analytical Synthesis

The analytical synthesis is carried out by using a computer program developed to solve the synthesis problem. Input data used are,

$P_1 = (0,0)$, $P_2 = (-6, 11)$ and $P_3 = (-17, 13)$
$\alpha_2 = 22^0$ $\alpha_3 = 68^0$

The free choices that are taken are

$\beta_2 = 90^0$, $\beta_3 = 198^0$, $\gamma_2 = 40^0$ and $\gamma_3 = 73^0$

found from the graphical method of solution so that it may be possible to compare results from both methods. Table 1 below gives comparison of the results obtained from the analytical method with those obtained by graphical method.

As can be noted from the table, using the values of the free-choices obtained from the graphical synthesis, the computer program yields the same lengths as obtained from the graphical synthesis.

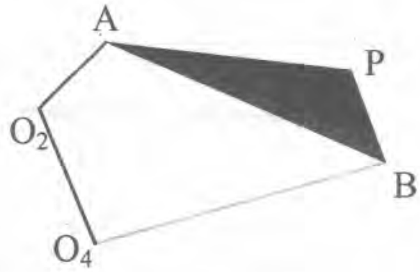


Figure 9 Synthesized four-bar mechanism

Table 1: Comparison of results

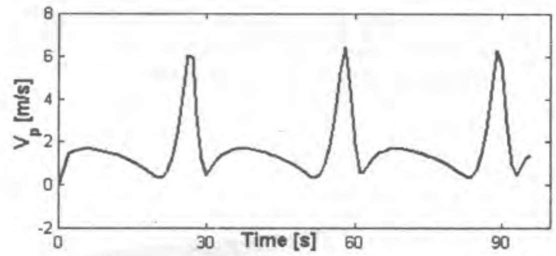
	Graphical		Computer program	
	Vector	Magnitude	Vector	Magnitude
G	3.4118 - 8.2796i	8.9550	3.4118 - 8.2796i	8.9550
W	5.7550 + 0.4809i	5.7751	5.7550 + 0.4809i	5.7751
V	16.0313 - 9.4215i	18.5948	16.0313 - 9.4215i	18.5948
S	18.3746 - 0.6611i	18.3864	18.3746 - 0.6611i	18.3864
Z	14.6106 - 3.4698i	15.0169	14.6106 - 3.4698i	15.0169
U	-1.4207 + 5.9518i	6.1190	-1.4207 + 5.9518i	6.1190

Motion characteristics of the mechanism

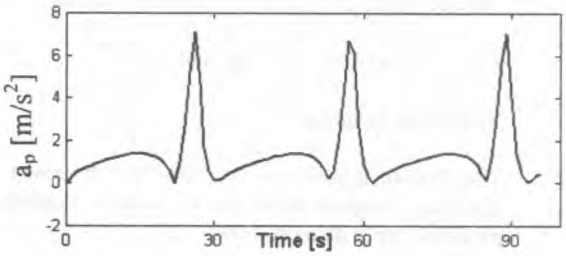
Using the results of the synthesis, the mechanism is analyzed for the motion characteristics. Plots of the simulations showing velocity and acceleration of the point of interest for 90 s simulation time,for constant velocity of the input link □□ = 0.2 rad/s are shown in Fig. 10.

User friendly windows for synthesis, analysis and simulation

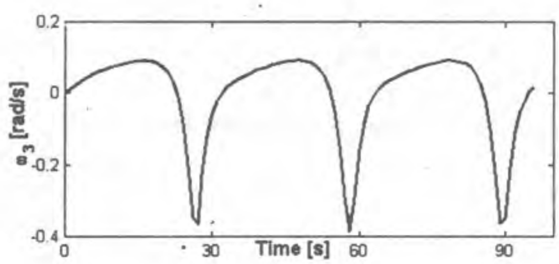
Fig. 11(a) enables selecting the number of links of the mechanism, four in our case, and the number of precision points. The inputs to the synthesis of the four-bar mechanism are introduced as shown in Fig.11 (b). The results obtained are then analyzed as shown in Fig. 11(c). The plots shown in Fig.10 (a) to Fig.10 (f) above are obtained from Fig.11(c).



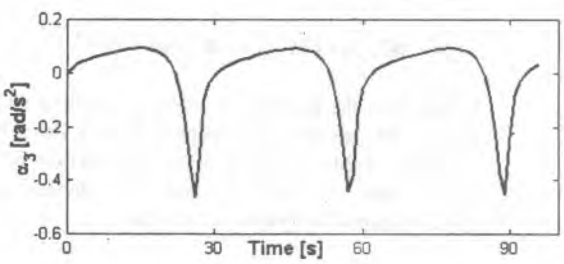
(a) Velocity of the point of interest P



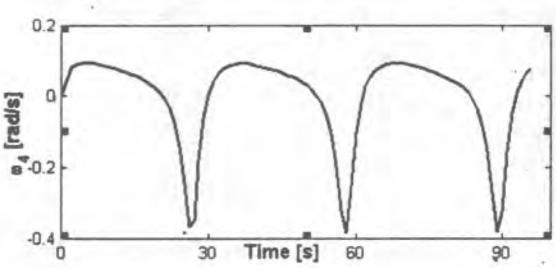
(b) Acceleration of the point of interest P



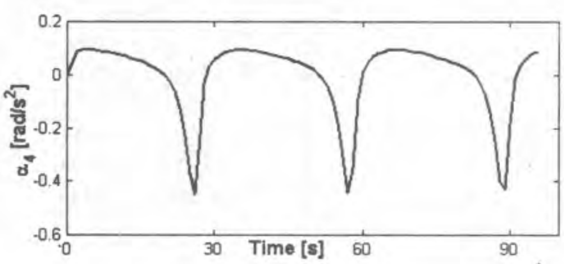
(c) Angular velocity of the coupler link



(d) Angular acceleration the coupler link



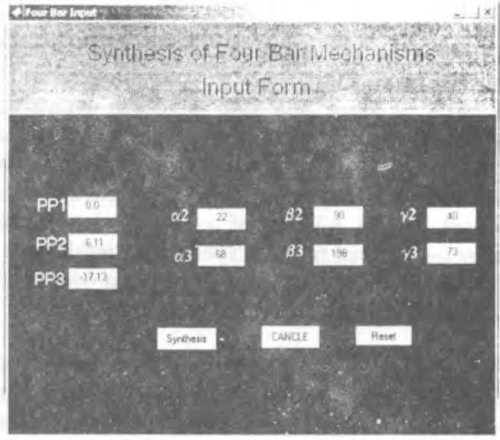
(e) Angular velocity of the output link



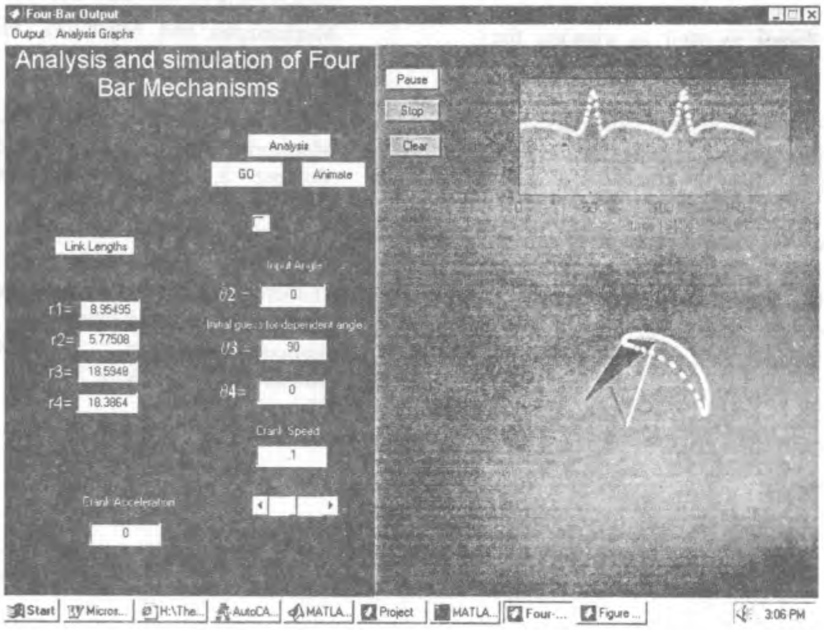
(f) Angular acceleration the output link

Figure 10 Motion characteristics of the four-bar mechanism

(a) Starting window



(b) Input window



(c) Window for Output, Analysis and Animation

Figure 11 User-friendly windows

CONCLUSION

A method for kinematic synthesis and analysis of four-bar mechanisms for the motion-generation problem of three and four precision points is presented in this paper. The dimensional synthesis is based on the complex number method approach while analysis of the motion characteristics is carried out by solution of a set of vector loop equations derived from the synthesized mechanism. Newton-Raphson iteration algorithm has been used to solve the non-linear system of equations.

A Matlab computer program is written to implement the solutions to problems described in this paper. A numerical example is worked out to compare the validity of the solutions resulting from the program against results generated from the graphical method. The computer program consists of Graphic User Interface, which helps the user with data input, output as well as viewing the simulation.

The results obtained from the computer program match exactly the results obtained from the traditional graphical method. Hence, the user-friendly program developed in this work can be used for design, analysis and animation of various four-bar mechanisms for three or four accuracy points. It can also be helpful for pedagogical purposes. It is also envisaged that this work can also be useful as a benchmark for further research in this area.

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