SIGNIFICANCE OF THE SHAPE OF THE ASSUMED CONCRETE COMPRESSION BLOCK ON THE ANALYSIS AND DESIGN OF REINFORCED CONCRETE SECTIONS UNDER PURE FLEXURAL LOADING

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ABSTRACT

The inelastic stress-strain relationship of concrete under compressive loading is difficult to express in mathematical terms. In flexural analysis and design of reinforced concrete sections, a number of simplified shapes have been proposed to date for the concrete compression block. The parabolic-rectangular as well as the rectangular stress blocks are among the most common ones. In this study, the effect of assuming a parabolic-rectangular or rectangular shape of the concrete compression block on the analysis and design of reinforced concrete beams has been investigated. Analytical expressions are derived for the steel reinforcement ratio, the concrete compressive force coefficient and the design bending moment coefficient and the corresponding graphs are plotted in parallel. It is also shown with the aid of examples that the selected shape of the concrete compression block has little influence on the analysis and design of reinforced concrete rectangular sections.

***Key words:** Simplified stress block, Stress-strain, Parabolic-Rectangular, Flexure in RC sections*

INTRODUCTION

Ultimate limit state (ULS) design is the critical condition for strength, it is a method based on designing structural members to their maximum load carrying capacity. In designing reinforced concrete beams according to ULS, the constituent materials – both concrete and reinforcement – are supposed to be subjected to their design strengths. ULS method makes proper utilization of the full strengths of the materials; and hence it is a realistic design approach that is widely adopted in many codes of practices including the Ethiopian Building Code Standards (EBCS).

The behavior of concrete under compressive loading has been extensively studied. Direct compression test results conducted on concrete specimens display a non-linear stress strain behavior. Studies on the flexural behavior of concrete performed on specially prepared specimen subjected to pure bending exhibit non-linear stress strain patterns [1-4]. The determination of the total compressive force in the concrete mass and its location thus becomes an intricate task.

The shape of the concrete compressive stress blocks in beams subjected to flexure has to reflect the concrete stress-strain diagram. To represent the inelastic stress-strain behavior for the flexural analysis and design of reinforced concrete beams, simplified shapes are recommended for practical cases. The most widely used shapes of concrete compressive block are parabolic, parabolic-rectangular, simplified-rectangular and bi-linear (trapezoidal).

In this study, analytical investigation is performed to examine the effect of the shape of the assumed concrete compressive stress block on the analysis and design of a given reinforced concrete beam section subjected to pure bending. Furthermore, numerical investigation has been performed to assess the implication of the derived equations. Of the many possible shapes of the concrete compressive stress blocks, this study focuses on the parabolic-rectangular and the simplified-rectangular shapes as these are the two main idealizations of stress-strain diagrams for concrete recommended text books and code of practices, including the EBCS. As both are simplifications of the inelastic stress-strain relationship, the term 'simplified' would be misleading and it is not used hence-forth in this work.

The investigation is based on making use of the mechanics of singly reinforced rectangular concrete sections subjected to pure flexure. The study also tries to observe the effect of variation in material properties, i.e., changing the concrete class and the grade of steel reinforcement used.

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STRESS – STRAIN RELATIONSHIP IN REINFORCED CONCRETE

Shapes of Concrete Compression Block

Extensive research has been conducted to study the actual stress-strain relationship of concrete under direct or flexural compression. Such investigations clearly show that the concrete stress-strain distribution is inelastic and it consists of a rising curve from zero to the maximum stress and a descending curve beyond the maximum stress. It becomes difficult to express the actual stress-strain relationship in mathematical terms. To overcome this difficulty, different shapes of concrete compression blocks have been suggested and are being in use, the most common ones being parabolic, parabolic-rectangular, rectangular and trapezoidal. Searching for simplification of the concrete stress-strain relationship, and hence the concrete compression force, is a long-standing and still an active research area [4, 9, 10]. A few of the other assumed shapes of concrete compressive stress blocks that have been proposed through time are shown in Fig. 1(1, 4).

Basic Assumption

Apart from having a simplified concrete stress block, further simplifying assumptions are employed in the analysis and design of reinforced concrete sections subjected to flexure, such as:

- plane sections remain plane after bending,
- tensile strength of concrete is neglected,
- concrete is assumed to fail in compression
- when the strain reaches ε_{cu} which equals 0.0035 for bending and 0.002 for direct compression,
- the maximum tensile strain in the reinforcement is taken to be 0.01.

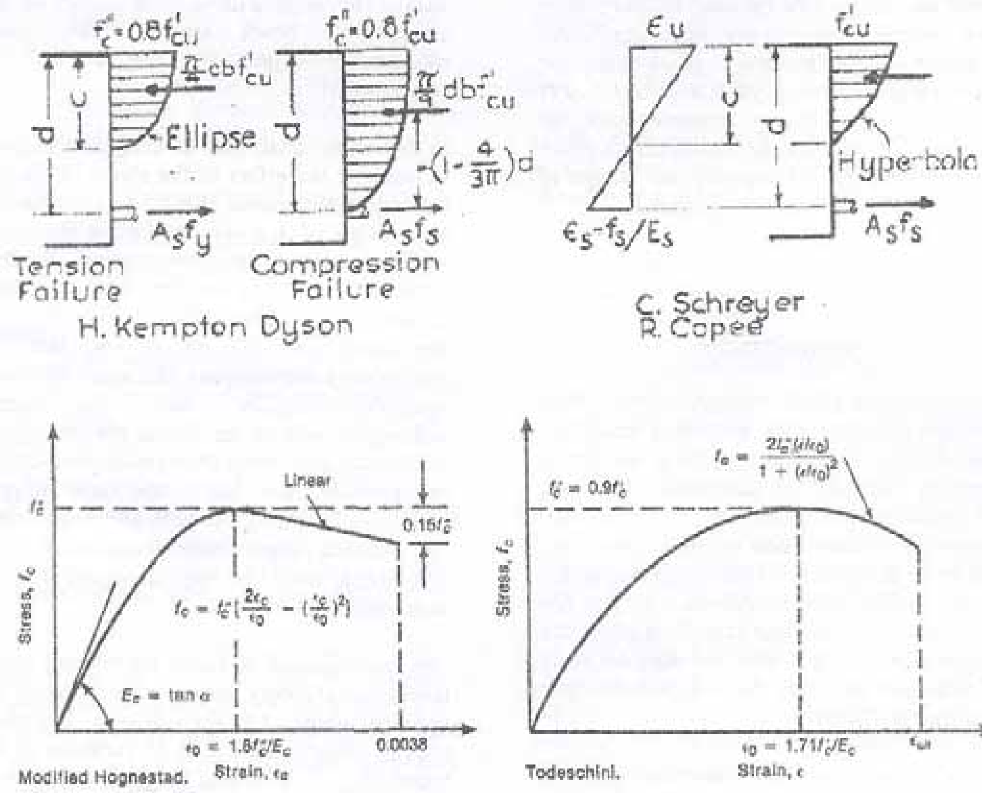

Figure 1 Different assumptions of the shape of stress-strain diagram in concrete

As a result, the strain profile over a reinforced concrete beam section is linear and the strain lines that correspond to the various failure types are depicted in Fig. 2 (5, 14).

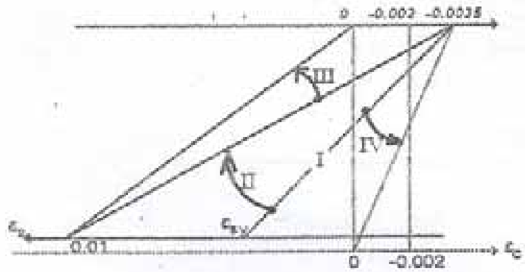

Figure 2 Possible strain diagrams in RC sections

FLEXURAL ANALYSIS OF REINFORCED CONCRETE RECTANGULAR SECTIONS

Analysis of the Various Failure Types

From the strain profile in Fig. 2 case I as shown by the straight line corresponds to a state of strain in which both the extreme concrete compressive strain and the tensile reinforcement reach their ultimate and yield values, respectively, at the same time. This is usually referred to as the *balanced failure* case. The corresponding location of the neutral axis is given by:

$$x_{bal} = \frac{700}{700 + f_{yd}} d \qquad (1)$$

To guarantee a balanced failure condition, a specific amount of reinforcement, $A_{s,bal}$, would be required. This is an ideal amount of reinforcement which would rarely be attained in practice as there are a number of other factors governing the provision of reinforcements in a section. A lower amount of reinforcement than $A_{s,bal}$ would lead to a failure mode characterized by the first yielding of the tensile reinforcement, usually classified as tension failure. Cases II and III in Fig. 2 are both *tension failure* conditions. In case II, the strain in the steel at failure is greater than the yield strain but it is less than 0.01, i.e. the maximum permitted strain in the tensile reinforcement; and the extreme concrete compressive strain attains the value 0.0035. In case III, the strain in the tensile reinforcement reaches 0.01 but that in the concrete would be less than 0.0035. For the derivation of the analytical expressions, further two special cases of case III are identified based on the extreme concrete strain (either the extreme concrete compressive strain lies between 0.002 and 0.0035 or it is below 0.002). Finally case IV is a section characterized by a higher amount of reinforcement than $A_{s,bal}$. It represents the situation where the extreme concrete compressive strain reaches ε_{cu} while the strain in the tension reinforcement is below the ε_{yd}, a case usually referred as compression failure. If code recommended guidelines for the design of ductile sections are properly employed, case IV could never be encountered in practice.

Resultant Concrete Compressive Force and Its Location

A singly reinforced rectangular section with both Par-Rec as well as Rect compressive stress blocks is shown in Fig. 3. The necessary parameters for the analysis and design of an RC section can be determined by using equilibrium condition of the forces as well as compatibility of strains on the section.

Without going into the routine derivation process, the analytical expressions for the concrete compressive force ratio – i.e the dimensionless quantity α_c obtained by dividing the resultant compressive force under the assumed compressive block by (f_{cd} bd) is provided in Table 1. Using the compressive stress block and the strain diagram, expressions are also derived in Table 2 for the neutral axis depth coefficient *(x/d)* and the distance from the extreme compression face to the resultant compressive force in the concrete ($\overline{y}$). These values are the most essential parameters for the analysis and design of reinforced concrete section subjected to pure bending [6,7,8,9,14].

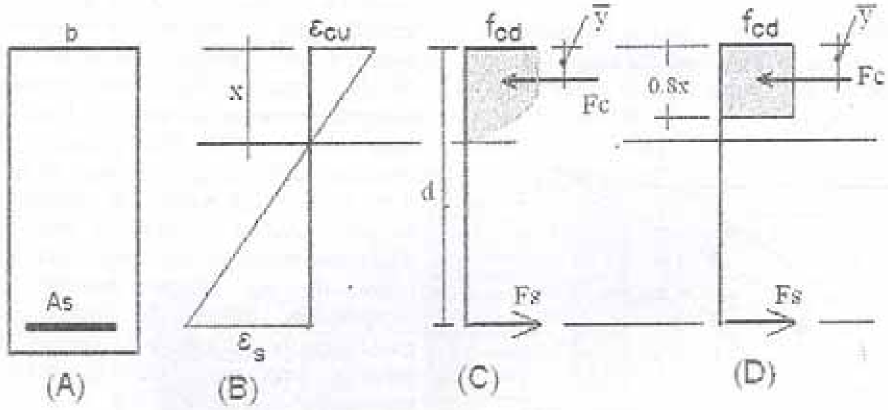

Figure 3 A reinforced concrete rectangular section (A), Strain diagram (B), Parabolic-rectangular stress block (C) and Rectangular stress block (D)

Table 1: Total concrete compressive for coefficient $\alpha_c = \frac{F_c}{f_{cd} \cdot b \cdot d}$

Case	Parabolic – Rectangular	Rectangular
I	$\alpha_c = \frac{17}{21} \cdot \frac{x}{d}$	$\alpha_c = 0.8 \cdot \frac{x}{d}$
II		
IV		
III	$\alpha_c = \frac{1}{15}\left(16\frac{x}{d} - 1\right)$ for $0.002 < \varepsilon_c < \varepsilon_{cu}$	
	$\alpha_c = \frac{x}{d}\left(\frac{5x/d}{1-x/d} - \frac{1}{3}\left(\frac{5x/d}{1-x/d}\right)^2\right)$ for $\varepsilon_c < 0.002$	$\alpha_c = \frac{x}{d}\left(\frac{8x/d}{1-x/d}\right) \cdot \left(1 - \frac{2.5x/d}{1-x/d}\right)$**
***This has been derived using parabolic stress block in concrete*		

Reinforcement Ratio and Design Moment Capacity

An expression for the reinforcement ratio ρ required for force equilibrium in the section at ULS can be obtained from equilibrium of forces on the section, as shown in Eq. (2)

$$\rho = \alpha_c \frac{f_{cd}}{f_s} \text{ where } \begin{cases} f_s = f_{yd} & \text{for Cases } I, II, III \\ f_s = 700\left(\frac{1-x/d}{x/d}\right) & \text{for Cases } IV \end{cases} \quad (2)$$

The bending moment ratio μ, a dimensionless quantity obtained by dividing the design moment capacity of the section by the value $(f_{cd}\, b\, d^2)$, is given by Eq. (3)

$$\mu = \frac{M_d}{f_{cd} \cdot b \cdot d^2} = \frac{Fc \cdot (d - \bar{y})}{f_{cd} \cdot b \cdot d^2} = \alpha_c \frac{(d - \bar{y})}{d} = \alpha_c \left(1 - \frac{\bar{y}}{d}\right) \quad (3)$$

Relationships among x/d vs ρ, α_c and μ

The significance of the derived analytical expressions is made apparent if relationships between the x/d ratio and the values of the ρ, α_c, and μ are explored. With the aid of a specially prepared Excel table Table 2, comparison of the above three parameters for all the possible values of x/d at ULS are computed for C25 concrete and S300 steel. As recommended in EBCS2 for a ductile failure of the section, an upper limit for x/d of 0.448 has been accounted for in the derivation process [5]. For other combinations of concrete and steel, only the concrete class and the steel f_{yk} values in part 1 of Table 2 need to be replaced with appropriate value; other parts of the table are automatically updated. The corresponding results of x/d vs. ρ and x/d vs. μ are subsequently plotted in Fig. 4 and 5 (7).

Table 2: Relationships between x/d vs α_c, μ and ρ for Par-Rec as well as Rect stress blocks for C25 and S300

I

Concrete C	25	*Mpa*
Steel f_{yk}	300	*Mpa*

ϵ_{cu}	0.0035	
γ_c	1.5	
γ_s	1.15	
E_s	200000	*Mpa*

II

f_{ck}	20	*Mpa*
f_{cd}	11.33	*Mpa*
f_{yd}	261	*Mpa*

ϵ_{yd}	0.001
ρ_{bal}	0.0253
x/d_{bal}	0.7285

III

x/d	ρ		α_c		μ	
	Par-Rec	Rect	Par-Rec	Rect	Par-Rec	Rect
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.05	0.00052	0.00174	0.00850	0.01740	0.01680	0.01700
0.10	0.00197	0.00348	0.03500	0.05780	0.05580	0.05550
0.15	0.00406	0.00521	0.08040	0.10060	0.10030	0.09460
0.20	0.00637	0.00695	0.14670	0.16000	0.15340	0.14720
0.25	0.00869	0.00869	0.20000	0.20000	0.18620	0.18000
0.30	0.01101	0.01043	0.25330	0.24000	0.21460	0.21120
0.35	0.01231	0.01216	0.28330	0.28000	0.23920	0.24080
0.40	0.01407	0.01390	0.32380	0.32000	0.26680	0.26880
0.45	0.01583	0.01564	0.36430	0.36000	0.29260	0.29520

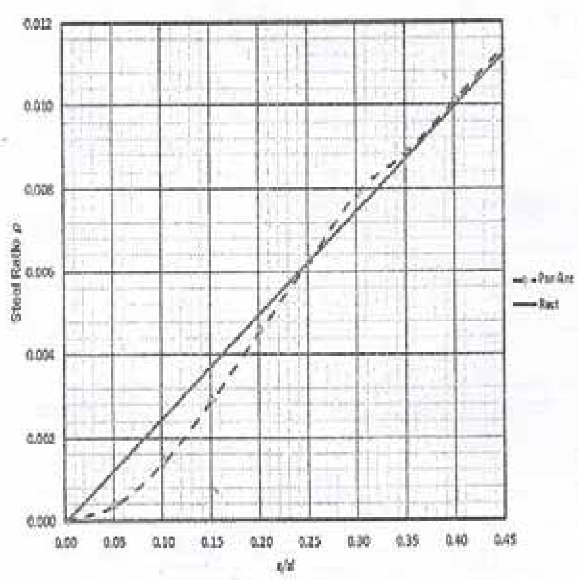

Figure 4 Reinforcement ratio $\rho = \frac{A_s}{b \cdot d}$

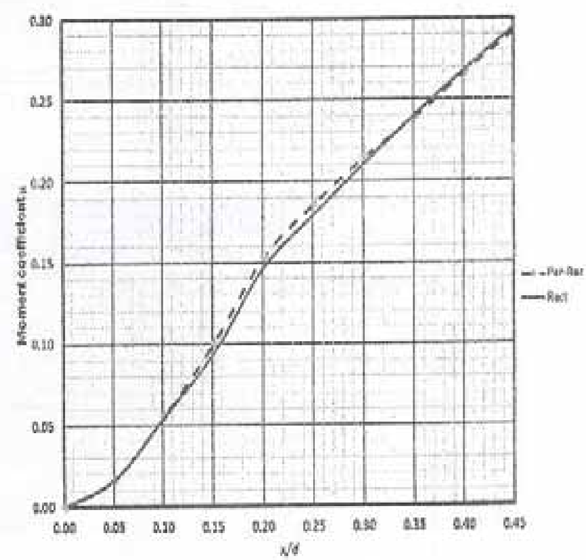

Figure 5 Bending moment coefficient $\mu = \frac{M_d}{f_{cd} \cdot b \cdot d^2}$

Table 3: Neutral axis factor x/d and position of compressive force $\bar{y}$ for parabolic-rectangular and rectangular stress blocks

Case	*Steel*	*Concrete*	*Parabolic – Rectangular*	*Rectangular*
I	$\varepsilon_s = \varepsilon_{sy}$	$\varepsilon_c = \varepsilon_{cu}$	$\frac{x}{d} = \frac{700}{700 + f_{yd}}$	
II	$\varepsilon_{sy} < \varepsilon_s < 0.01$	$\varepsilon_c = \varepsilon_{cu}$	$\frac{x}{d} = \frac{21}{17}\rho\frac{f_{yd}}{f_{cd}}$ and $\bar{y} = \frac{99}{238}x$	$\frac{x}{d} = 1.25\rho\frac{f_{yd}}{f_{cd}}$
III	$\varepsilon_s = 0.01$	$0.002 < \varepsilon_c < \varepsilon_{cu}$ $\rightarrow \frac{1}{6} < \frac{x}{d} < \frac{7}{23}$	$\frac{x}{d} = \frac{1}{16}\left(15\rho\frac{f_{yd}}{f_{cd}} + 1\right)$ and $\bar{y} = \left(\frac{x}{d} - \frac{0.628\left(\frac{x}{d}\right)^2 - 0.016\frac{x}{d} - 0.016}{\frac{16x}{15d} - 1}\right)d$	
		$\varepsilon_c < 0.002$ $\rightarrow \frac{x}{d} < \frac{1}{6}$	$\frac{x}{d}\left\{\left(\frac{5\frac{x}{d}}{1-\frac{x}{d}}\right) - \frac{1}{3}\left(\frac{5\frac{x}{d}}{1-\frac{x}{d}}\right)^2\right\} = \rho\frac{f_{yd}}{f_{cd}}$ and $\bar{y} = \left(1 - \frac{6.25\left(\frac{\frac{x}{d}}{1-\frac{x}{d}}\right)^2 - \frac{10}{3}\left(\frac{\frac{x}{d}}{1-\frac{x}{d}}\right)}{-25\left(\frac{\frac{x}{d}}{1-\frac{x}{d}}\right)^2 + 5\left(\frac{\frac{x}{d}}{1-\frac{x}{d}}\right)}\right)x$	
IV	$\varepsilon_s < \varepsilon_{sy}$	$\varepsilon_c = \varepsilon_{cu}$	$\frac{x}{d} = \frac{-7350\rho}{17f_{cd}} + \sqrt{\left(\frac{7350\rho}{17f_{cd}}\right)^2 + \frac{14700\rho}{17f_{cd}}}$ and $\bar{y} = \frac{99}{238}x$	$\frac{x}{d} = \frac{-437.5\rho}{f_{cd}} + \sqrt{\left(\frac{437.5\rho}{f_{cd}}\right)^2 + \frac{875\rho}{f_{cd}}}$

NUMERICAL EXAMPLES

To clearly see the implication of the various expressions derived in this study, two numerical examples are provided here for the two compressive stress-blocks in comparison. The first example presents the difference in the case of analysis problems; the second considers a design problem.

Design Moment Capacity of an RC Section

Determination the design moment capacity of a section is a typical analysis problem in RC. In this section, it is required to determine the design moment capacity of a given RC section with C25 concrete, S300, b=250mm, d=450mm, and As=3Φ24. The result is tabulated for both sections in Table 4.

Table 4 Comparison of Analysis Result

	Parabolic-Rectangular	Rectangular
Steel Ratio ρ	0.0121	0.0121
x/d	0.343	0.347
μ	0.2352	0.2391

The design moment capacity of the section is thus 134.91kN-m in the case of Par-Rec and 137.14kN-m for the Rect stress block, showing a deviation of 1.6%.

Design of the Amount of Reinforcement

Given a rectangular RC section with b=300mm, d=550mm, C25 concrete and S300 steel, determine the required reinforcement if it is subjected to a design moment of 300kN-m.

The results, as obtained from Fig. 4 and 5 are given in the Table 5.

Table 5 Comparison of Design Results

	Par-Rec	Rect
μ	0.2431	0.2431
x/d	0.357	0.354
Steel Ratio ρ	0.01256	0.0123

The required amounts of reinforcement are 2072mm^2 and 2030mm^2, for Par-Rec as well as Rect stress blocks, respectively. Though this shows a 2% deviation, the actual amount of steel provided would most probably be identical, such as providing 3Φ30.

CONCLUSION

Analytical expressions for parabolic-rectangular as well as rectangular shapes of concrete compression blocks have been developed. This has been performed considering all the possible stress-strain ranges that an RC section could attain at failure. The analytical expressions for the parabolic-rectangular block are complicated while the counterparts for the rectangular stress-block are simple. Similar expressions can be derived for doubly reinforced, T- and L-sections. But, further complications exist when analyzing a non-rectangular cross-section for the case of the parabolic-rectangular stress block.

Graphs have also been prepared showing how the reinforcement ratio ρ and design moment coefficient μ vary for the two compressive stress blocks with varying *x/d*. The MS Excel table in Table 3 provided a practical platform which eases the investigation of the effect of the shape of the assumed concrete compression block as well as the type of material used. By updating the concrete class and grade of steel, all computed values and graphs are automatically updated.

Because of its simplicity, the rectangular stress block is favored in text books [11,12,13] and equally recommended in a number of major national codes including ACI 318, BS 8110, Eurocode 2, Indian standards IS 456, etc [14,15]. Many software have implemented the rectangular stress block option in the design of RC sections to ULS [16,17]. The only weakness of the rectangular stress block is that it does not provide a clear picture of the stress profile in the concrete when the section has a small amount of tensile reinforcement that leads to case III failure types, see Table 1[7].

It was also shown in this study that the shape of the concrete compression block has little impact on the analysis and design of reinforced concrete rectangular sections. Because of its ease and convenience in actual computation, its suitability to implement in computer programs, and its seamless adoptability for non-rectangular cross sections, the author recommends the rectangular stress block to be used in all practical cases and especially in training prospective engineers.

Finally, it should be noted that the results are applicable to normal strength concrete (for concrete classes of up to C50). In the case of high strength concrete, proper adjustments are required on the concrete compression block and the maximum concrete compressive strain as recommended in Eurocode 2[11,12,15].

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