ENERGY EFFICIENT TOPOLOGY FOR WIRELESS MESH NETWORKS

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ABSTRACT

Wireless Mesh networks employing the IEEE 802.11 a/b/g standard operate in the unlicensed frequency band. Their deployment has seen a tremendous growth since their introduction particularly for providing community based connectivity services. A number of wireless access points are deployed to provide these connectivity requirements. Due to this controlling the transmit power by the access points has become necessary as it is required to reduce interference and cost of operation and enhance performance. In this paper we study the problem of transmit power control by the access points in a wireless mesh network using computational geometry and coalition formation game theoretic framework from economics. We consider a typical wireless mesh network topology where mesh nodes are placed on the vertices of a two-dimensional lattice. We analyze the power control problem using coalition formation game theory employing utilities based on the coverage areas of the access points by associating a cost function with the utility as the payoff of the coalition members. Our work focuses on the access layer of a wireless mesh local area network. We show that by forming a coalition with fair allocation of payoffs among the members and assigning different radio ranges and coverage areas to the access points, a significant amount of transmit power can be saved while covering the required service area of the network.

Keywords: *coalitional game theory, wireless mesh network, power control, core, utility, payoff, Voronoi diagram*

INTRODUCTION

Cities, municipalities and large campus compounds are embracing WiFi and mesh networking technologies as a means for providing enhanced online services to their communities. Wireless mesh networks have emerged as the extension to the infrastructure WLAN deployments in public and private outdoor installations such as large academic and corporate campuses, municipalities, city downtowns and to some extent, multi-unit apartments and residential complexes.

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A wireless mesh network, in general, is comprised of three layers [1]. These are the access layer, the mesh layer and backhaul (wired network) layer. The access layer is used to provide wireless access to end user devices. The purpose of the mesh layer is to connect the access points of a collection of mesh nodes to the wired backhaul network layer. The wired backhaul layer provides connectivity of the mesh layer to a Point of Presence (PoP). This is usually a high speed Ethernet connection that might employ fiber optics technology.

One of the major components of a wireless mesh network design is the assignment of coverage areas and hence radio ranges to the access points. These assignments signify the transmit power level usage of the mesh nodes at the access layer. In this paper we focus on the assignment of coverage areas and radio ranges (and hence the transmit power levels) for the access layer devices in a wireless mesh network. We employ the coalition formation game theoretic frame work from economics to analyze our power control problem.

In [2] the authors have studied the power control by base stations in a cellular network in a shared media using computational geometry and competitive game theory framework. Their study suggests that Nash equilibrium can be achieved if the base station use equal radio ranges in a competitive cellular network for an efficient usage of power. It is also a common industrial practice in designing wireless mesh networks to assign equal radio ranges for the access points [1].

Game theory is used to study power control of user devices in wireless networks, notably in cellular systems as studied in [4], [5], [7] and [14]. Game theory is also used to study cooperation in wireless ad-hoc networks, for example in [6], [8] and [12].The authors of this paper have used computational geometry and linear programming in [13] to address the problem of power control in spontaneously deployed wireless networks.

In this paper we are proposing that a significant amount of transmit power can be saved if access points in a mesh network (at the access layer) employ different (rather than the same) radio ranges. We show that the saving in transmit power does not affect the coverage of the service area. We use computational geometry and coalition formation game theory to analyze the power control problem in the mesh network. We also demonstrate the efficiency of our radio range assignment scheme using simulation work.

This paper is organized in the following manner. In section II we describe the system model. In section III we outline the corresponding coalition formation power control game and solve this game for a two-dimensional lattice topology. In section IV, we present our simulation results and section V concludes our paper.

SYSTEM MODEL

We assume a wireless mesh network where the mesh nodes, hereafter called the base stations (BS-s), are grouped into two. We further assume that the two groups of base stations are operated by two operators and we call the operators O_1 and O_2. Note that, in practice, all the BS-s can be operated by a single operator but we make the two operators assumption to suit our cooperative game model. Operator $i \in \{O_1, O_2\}$ controls a set of BS-s denoted by b_i. We denote the union of all BS-s by B. There exists no BS-s that belongs to different operators located at the same location. We also assume several users equipped with WLAN access devices to access the communication network.

Base stations and mobile devices operate in the same unlicensed frequency band. Each of these devices might perform power control but in our paper we focus on the power control of the transmit signal of the BS-s, i.e., BS to mobile devices.

According to the physical model of signal propagation [10], the transmit power of a base station $b_i \in B$ can be received by a user device u if its signal-to-interference-noise ratio (SINR) exceeds a reception threshold β:

$$\frac{P_i g_{iu}}{N_0 + \sum_{j \in B, j \neq i} P_j g_{ju}} \geq \beta \qquad (1)$$

Where P_i is the transmission power of BS b_i, g_{iu} is the channel gain between the BS b_i and the user device u and N_0 is the Gaussian thermal noise. We assume the channel gain depends only on the distance of the transmitter and the receiver and we normalize the effect of the antenna characteristics, thus we have $g_{iu} = d_{iu}^{-\alpha}$ between the BS b_i and user device u, where $2 \leq \alpha \leq 5$ is the path loss exponent that characterizes the radio signal propagation properties of the environment. Eq. (1) captures how the reception power depends on the most important factors, namely on the transmission power and the distance between transmitter and receiver. Note that we considered the local average of the received signal but in reality, on a small time scale, the transmit power signals have a time-varying property due to fading.

We assume that (1) holds for every point in the service area for at least one base station and that the user device u attaches to the base station b_i with the best SINR. Thus, we can write, for any other base station b_l, that:

$$\frac{P_i d_{iu}^{-\alpha}}{N_0 + \sum_{j \in B, j \neq i} P_j d_{ju}^{-\alpha}} \geq \frac{P_l d_{lu}^{-\alpha}}{N_0 + \sum_{m \in B, l \neq m} P_m d_{mu}^{-\alpha}} \qquad (2)$$

We abstract away the end users and assume that their expected position is uniformly distributed over the service area. Note that this also means a balanced load on the base stations (i.e., no users have to switch base stations due to the lack of available bandwidth).

Let us assume that the transmit signals propagate in an almost open area, meaning $\alpha = 2$. This is a valid assumption particularly when we are considering an outdoor implementation of the wireless mesh network. Then (2) defines a multiplicatively weighted power diagram [9], which determines the set of points in the service area (potential places of user devices) that are attached to a given base station. The Voronoi diagram with multiplicatively weighted distances has a complex shape and is generally difficult to derive analytical solution for the power control problem. Hence, we apply a radio range model that is simpler and widely used in the literature. An example of this approach can be found in [2].

Let us derive from (1) the radio range of the transmit signal of the BS b_i as the Euclidian distance within which the users are able to attach to this base station if there is no interference from other devices. This is usually the case when the network employs the Distributed Coordination Function (DCF).

$$r_i \propto \sqrt{\frac{P_i}{\beta N_0}} \qquad (3)$$

According to this radio range model, we can define the additively weighted power distance as:

$$\mathrm{pow}(u, b_i : w_i) = d_{iu}^2 - w_i \quad (4)$$

Where d_{iu} is the Euclidian distance between the points u and b_i and w_i is a weight assigned to point b_i.

We can now define the Voronoi region $V(b_i)$ around a base station $b_i \in B$ as the set of points u that are "closer" to point b_i than to any other point b_j (*i.e.*, $b_i \neq b_j$). Hence, we can write $V(b_i)$ as:

$$V(b_i) = \{u | \mathrm{pow}(u, b_i : w_i) \leq \mathrm{pow}(u, b_j : w_j) \text{ for } i \neq j\} \quad (5)$$

We can write the Voronoi diagram V(B) of all base stations B as:

$$V(B) = \cup V(b_i) \quad (6)$$

In this paper, we substitute $w_i = r_i^2$ and hence we obtain a Voronoi diagram in the Laguerre geometry [9]. This model corresponds to a Voronoi diagram, where the distance is defined as a tangential Euclidean distance to circles centered at the base stations' locations and radii corresponding to their radio ranges.

We assume that the BS-s are placed on the vertices of a two-dimensional lattice in an alternating way such that any BS that belongs to operator O_1 has four neighboring BS-s that belong to operator O_2. Let us call the minimum Euclidean distance between the BS-s d.

In the analyses of the power control problem the following assumptions are used in order to simplify our model.

- No place should remain uncovered in the service area.
- Each BS belonging to the same operator has the same radio range.
- There exists a limitation on the maximum transmission power of any base station which is defined by the standards of the wireless LAN technology.
- The base stations and the user devices have Omni-directional antennae.

COALITION FORMATION POWER CONTROL GAME

We model the power control problem as a coalition formation game between two players. The strategy of the coalition is to assign coverage areas and radio ranges to the BS-s in the coalition so that the total transmit power in the network is minimized. The coalition should also divide the value of the coalition fairly and ensure that the total service area is covered. It can achieve this by assigning the same or different coverage areas for its members.

Assume that the coalition choose to assign different radio ranges and hence different coverage areas to its members. Let us assign operator O_1's BS-s a higher radio range r_H and operator O_2's BS-s a lower range r_L. Note that the assignment can be the opposite due to the symmetric situation.

Since the placement of the BS-s is symmetric and the players apply the same radio range to all of their BS-s, we can analyze the game considering two neighboring BS-s, as shown in Fig. 1

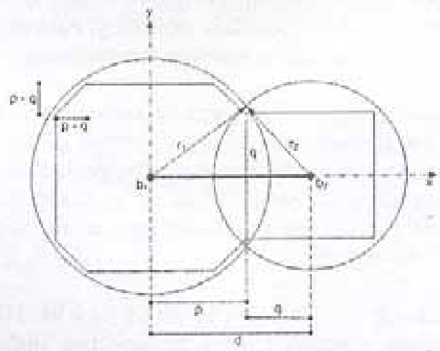

Figure1. Coverage areas of the two base stations

The coverage area of the BS with the higher radio range (A_H) and that of the BS with the lower radio range (A_L) is given by

$$A_H = \frac{d^4 + 2d^2(r_H^2 - r_L^2) - (r_H^2 - r_L^2)^2}{d^2} \quad (7)$$

$$A_L = \frac{[d^2 - (r_H^2 - r_L^2)]^2}{d^2} \quad (8)$$

Observe that $A_L = 4q^2$ *and* $A_H = 2d^2 - 4q^2$ *where* $q = \frac{d}{2} - \frac{r_H^2 - r_L^2}{2d}$ *and* $p + q = d$.

Note that the total area covered by the two base stations is equal to $2d^2$. We also require that base stations cover a non empty area. This requirement leads to the inequality $(q > 0)$

$$r_H^2 - r_L^2 < d^2 \quad (9)$$

From the geometry of figure 1, we note that

$$r_H^2 - r_L^2 = d^2 - \sqrt{2} d r_L \quad 10)$$

We model the power control problem as a coalition formation game between the two players and investigate if a stable and fair coalition can form that minimizes the total transmit power. We particularly use the coalition game with transferable utility (TU) theory from micro economics.

Definition 1: - A coalitional game with TU, (N, v) consists of

- A finite set N (The set of players)
- A function v that associates with every non empty subset B of N (a coalition) a real number *v(B)* – the worth of B.

For each coalition B the number *v(B)* is the payoff that is available for division among the members of B. Due to the TU assumption *v(B)* is divided among the members of the coalition arbitrarily. Fairness in allocation is required to maintain the coalition.

The core of a coalition game is analogous to the Nash equilibrium of a non-cooperative game: an outcome is stable if no deviation is profitable. In the case of the core, an outcome is stable if no coalition can deviate and obtain an outcome better for all its members.

Definition 2: - A coalition game (N, v) with TU is said to be superadditive if for any two disjoint coalitions $B_1, B_2 \subset N$

$$v(B_1 \cup B_2) \geq v(B_1) + v(B_2) \tag{11}$$

If the superadditivity property holds for all the coalitions, then we say that the Grand coalition, the coalition that contains all the players, can form.

Definition 3: - A payoff vector $y=[y_1,\ldots,y_N]$ is said to be group rational or efficient if $\sum_{i=1}^{N} y_i = v(N)$. A payoff vector y is said to be individually rational if the player can obtain the benefit no less than acting alone, i.e., $y_i \geq v(\{i\})$. A group rational or efficient payoff vector is also referred to as a feasible payoff profile. An imputation is a payoff vector satisfying the above two conditions.

The set *S* of stable imputations is called the Core, i.e.,

$$S = \{y: \sum_{i \in N} y_i = v(N) \text{ and } \sum_{i \in B} y_i \geq v(B)\ \forall B \subset N\} \tag{12}$$

A non-empty core means that the players have an incentive to form the Grand coalition.

In this paper we use coalition formation games rather than canonical coalition games [11]. In coalition formation games, unlike canonical games, network structure and cost for cooperation play a major role. An important characteristic which classifies a game as a coalition formation game is the presence of a cost for forming coalitions. In this paper, we propose a coalition formation game with TU and a payoff for each base station defined as a function of the coverage area, A_i, as

$$y_i = h(A_i) - C(A_i), \qquad i = H, L \tag{13}$$

Where $h(A_i)$ is the sigmoid function and is plotted in Fig. 2. The sigmoid function used in this paper is defined as

$$h(A_i) = \frac{1}{1 + e^{-a(A_i - b)}} \tag{14}$$

Which has been widely used in the study of neural networks. Clearly *h(b)=1/2*, so we call b the center of h(A). The parameter *a* defines the steepness of h(A). The derivative of h(A) satisfies

$$h'(A) = ah(A)[1 - h(A)] \tag{15}$$

The sigmoid function has a convex and concave part and it captures the allocation of the coverage areas quite naturally.

$C(A_i)$ is the cost associated with covering an area of A_i. There are at least two requirements for the cost function: *C(0) = 0* and that *C(A)* increases in coverage area. In this paper we will use a linear function defined as:

$$C(A_i) = \frac{cA_i}{d^2} \tag{16}$$

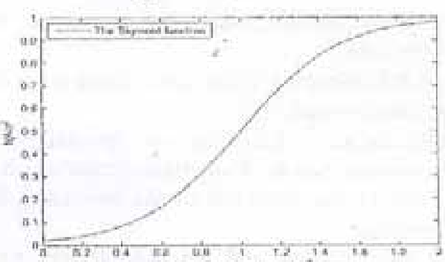

Figure 2 – The Sigmoid Function

Where *c* is the price coefficient and d is the distance between the two BS-s.

In this paper we use the value $\frac{a}{d^2}$ instead for the steepness parameter of the sigmoid function. If both base stations are to cover equal areas then they will cover an area of d^2 each. This has also be shown in [2] for base stations in a cellular network. Hence we will use the value of $b = d^2$ for the center parameter of the sigmoid function.

The values of a and c are constants and are set when the coalition is formed. The constants are strictly non-zero positive real numbers.

The goal of forming the coalition is to minimize the total transmit power by the base stations while covering the entire service area. In order to achieve its goal, the coalition might assign the higher players coverage area in the concave part of the sigmoid function while that of the lower player will be assigned a coverage area in the convex part. In doing so the coalition should assign a payoff for both base stations efficiently. That is the allocation of the coalition value should be fair.

Taking the derivative of the payoff with respect to the coverage area and setting it to zero yields

$$h(A_i) = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{c}{a}} \qquad (17)$$

One necessary condition is that $a \geq 4c$. Based on their coverage areas, we assign the higher value for the higher player while the lower value will be assigned to the lower value. Note that the higher value of Eq. (18) is in the concave part of the sigmoid function while the lower value is in the convex part. Hence, we assign the payoffs for both base stations as:

$$h(A_H) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{c}{a}} = K \qquad (18)$$

$$h(A_L) = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{c}{a}} = 1 - K \qquad (19)$$

Note that $h(A_H) + h(A_L) = 1$. Also note that $½ \leq K \leq 1$ and $K \rightarrow 1$ as $a \rightarrow \infty$. Using any of the two Eq. (18) or Eq. (19) brings the same result. Hence let us take Eq. (18) of the higher player. We can easily show that

$$A_H = d^2 + \frac{d^2}{a} \ln\left(\frac{K}{1-K}\right) = d^2 + d^2 z \qquad (20)$$

Where

$$z = \frac{\ln\left(\frac{K}{1-K}\right)}{a}. \qquad (21)$$

Note that $0 \leq z \leq 1$ and $z \rightarrow 1$ as $a \rightarrow \infty$. Since z is a function of c and a, it is also a coalition parameter. Solving for $r_H^2 - r_L^2$ and considering inequality (9), we find that

$$r_H^2 - r_L^2 = d^2 - d^2\sqrt{1-z} \qquad (22)$$

Comparing Eq. (22) and Eq. (10) we find that

$$r_L^2 = \frac{d^2}{2}(1-z) \qquad (23)$$

And

$$r_H^2 = \frac{d^2}{2}\left(1 + (1 - \sqrt{1-z}\,)^2\right) \qquad (24)$$

In accordance with our system model, transmit power is proportional to the square of the radio range. Hence the totals transmit power by the two base stations in the network can be expressed as (normalizing the effects of antenna gains and frequency parameters):

$$P_T = r_H^2 + r_L^2 = 2d^2 - d^2[z + \sqrt{1-z}]$$

$$= 2d^2 - d^2 f(z) \qquad (25)$$

Where $f(z) = [z + \sqrt{1-z}]$. If initially both base stations play a radio range of d, as it is required for them to know the existence of each other, the initial total transmit power is $2d^2$. Hence we can call $f(z)$ the power reduction function and is plotted in figure 3 along with the total power as a function of z. Note that the total power in the network consisting of N BS-s with half using r_H and the other half using r_L is just N/2 time Eq. (25)

The Interference area of a given BS, b_i, is equal to $\pi r_i^2 - A_i$. The total interference area in the network is given by

$$I = \frac{N}{2}\{\pi(r_H^2 + r_L^2) - 2d^2\} \qquad (26)$$

Note that the value of z that minimizes (25) is the same value that minimizes (26). However, the value of z can be chosen to meet a design requirement for the percentage of the Interference area with respect to the coverage area.

The value of the cooperation parameter z that minimizes the total transmit power can be found by maximizing $f(z)$. It is easy to show that this value of z is found to be ¾. Hence

$$z_{opt} = \frac{3}{4} \text{ and } P_{T,min} = \frac{3d^2}{4} \quad (27)$$

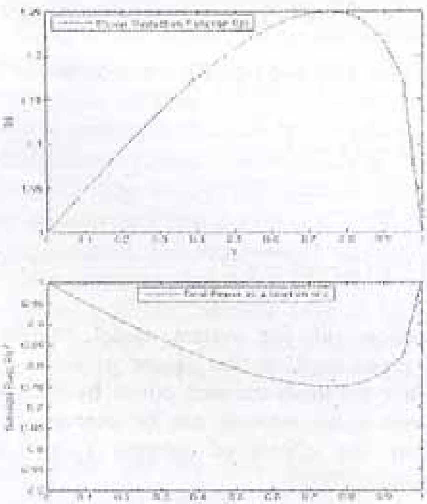

Figure 3 Power Reduction functions and normalized total Power

The values of the radio ranges and the corresponding coverage areas for the optimum value of z are given as:

$$r_H = \frac{d\sqrt{5}}{2\sqrt{2}}, r_L = \frac{d}{2\sqrt{2}}, A_H = \frac{7d^2}{4}, A_L = \frac{d^2}{4} \quad (28)$$

Note that the total transmit power is now significantly reduced while the total service area is covered $(A_H+A_L=2d^2)$. In fact when we compare our power assignment strategy with the initial power assignment, we observe that a 62.5% power saving is achieved. And when we compare our strategy with the one that is usually used in wireless mesh network design, which assigns equal coverage areas and radio ranges that is equal to , we can achieve an efficiency of 25%. The latter assignment is also found to be a Nash Equilibrium for cooperation in cellular networks in a shared medium as is shown in [2].

With the values of the radio ranges for the minimizing coalition parameter z, p and q in Fig. 1 take the values 3d/4 and d/4 respectively. The coverage area of the higher radio range player will be an eight sided polygon with the vertical and horizontal sides having lengths equal to $d/2$ $(=2q)$ and the diagonals having lengths equal to $d/\sqrt{2} = (\sqrt{2q})$. The lower range player's coverage area will be a square with sides of length $d/2$ $(=2q)$. The value of z can be adjusted (z=0.454) so that the coverage area of the higher player is an octagon while that of the lower player is still a square. But this will come at the expense of increased total transmit power ($0.807d^2$). The new total power minimizing topology of the wireless mesh network is shown in Fig. 4

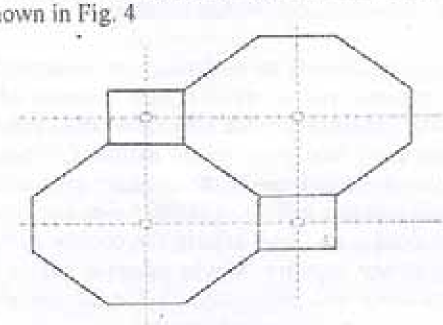

Figure 4 Power efficient topology

Fairness Considerations

In order for our assignment of transmit power and coverage area for the base stations to be a core of our coalition formation game, the value of the coalition value should be allocated in a fair manner. Hence the payoff of the two base stations should be equal for the optimum value of z. Solving for the values of a and c that keeps the value of z optimum while making the two payoff values equal requires solving an equation that is a combination of exponential and rational functions. We have found that the following values can be used to make the coalition fair.

These values will assign a payoff of 0.38 for each player, which is equal to the value they get if they were to act alone with a coverage area of d^2. With these values of the coalition parameters, our payoff vector will be group as well as individually rational.

SIMULATION RESULTS

We used the popular NS2 to simulate our work. In the simulation we used a wireless mesh network consisting of 4 BS-s with d=200m and a total of 20 mobile nodes deployed randomly in the service area. The mobile nodes transmit FTP traffic constantly to an FTP server through the base stations they are attached to. Each base station is supplied with an initial energy of 0.5 Joules. The simulation runs for an hour and the total energy consumed by the base stations recorded and used to compare three radio range assignment schemes. The three scenarios are:

- BS-s are assigned the same radio ranges equal to d.
- BS-s are assigned the same radio ranges equal to $/\sqrt{2}$.
- Our radio range assignment scheme where two base stations are assigned the radio range $r_H = \frac{d\sqrt{5}}{2\sqrt{2}}$ and the other two are assigned $r_L = \frac{d}{2\sqrt{2}}$. The topology in Fig. 4 is used for this case.

The simulation results for the three scenarios are shown in Fig. 5. The simulation results go with our finding from the mathematical analysis.

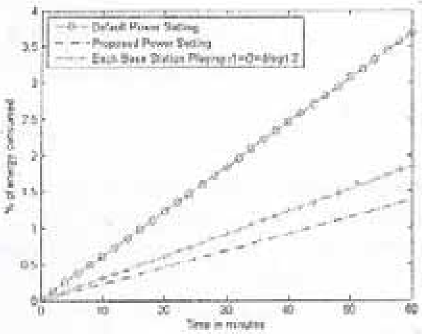

Figure 5 Simulation results for the three scenarios.

CONCLUSION

In this paper we studied the power control problem in wireless mesh networks using computational geometry and coalition formation game theory. Our results show that by assigning different radio ranges for BS-s rather than equal radio ranges, we can save a substantial amount of transmit power. Our result shows that we can achieve up to 62.5% saving in total transmit power in the network. The interference area is also decreased for the optimal value of the coalition parameter z. This helps in enhancing the performance of the wireless mesh network. But the value of z can be chosen to meet a required interference area specification at the expense of increased total transmit power. Our findings lead to the conclusions that optimum network topologies in terms of power savings can be constructed that differ from the traditional topologies where squares or hexagons are used as coverage areas of BS-s.

Our work only considers BS-s placed on the vertices of a two dimensional lattice but we are working on general topologies using the same technique. But generally it is shown that [2], [3] the problem of power control in wireless networks for general topologies is NP-hard.

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