INVESTIGATION OF ADAPTIVE BEAMFORMING ALGORITHMS FOR COGNITIVE RADIO TECHNOLOGY

Mulugeta Atlabachew, Institute of Technology Jimma University
Mohammed Abdo Tuko*, Department of Electrical & Computer Engineering
Addis Ababa Institute of Technology, Addis Ababa University

ABSTRACT

Frequency spectrum is one of the biggest natural resource which has a significant impact on the development of wireless communication technologies. Therefore, utilizing this natural resource in an efficient way accelerates the technological advancement. The spectrum allotment strategy that has been serving well the wireless communication family is the fixed spectrum allocation strategy. However, the increasing demand to use wireless technologies increased the competition for spectrum. As a result, there is no usable frequency spectrum left unoccupied. In spite of this spectrum scarcity, different research shows that most of the times most of the spectrum bands are not in use. The proposed solution to overcome this problem is to use the cognitive radio technology.

Cognitive radio is a wireless communication technology which adds intelligence to the existing wireless communication scenario. As every wireless communication requires antenna, in this paper the feasibility of smart antenna to this intelligence system is studied and the performance (based on computational complexity, convergence rate and radiation pattern characteristics) of different adaptive beamforming algorithms are investigated. The investigation result shows that the Sample Matrix Inversion (SMI) algorithm besides its best convergence rate, also produces radiation pattern that best suits the behavior of cognitive radio technology.

***Key words**: Cognitive Radio, smart antenna, and Adaptive beamforming algorithms.*

GENERAL BACKGROUND

Communication in general is a transmission of signal (information) from one point (source) to the other (destination). Basically, it is an inherent behavior of all living matter to communicate, in particular human beings have used this phenomenon as a tool to change this world in all dimensions. Therefore, the history of communication is totally linked to the history of living matter. Different disciplines classify types of communication differently but from communication engineering point of view, it can be broadly classified as either wired or wireless communication. This paper is confined to the latter types of communication.

Because of its most convenient features, the wireless communication is leading the market of communication technology. The ever-increasing demand of the world to use wireless technology has motivated both researchers and the business community to come up with new services and ideas. However, this motivation is being restricted by the scarcity of spectrum bands; hence all spectrum bands of wireless communication are already occupied [1]. This is so, because of the fixed spectrum allocation strategy used. To alleviate this problem cognitive radio (CR) technology is proposed [2, 3].

Cognitive radio is a wireless technology that senses the external environment, learns from experience, plans based on knowledge, and decides based on reasoning. In general, it adds intelligence to the existing wireless communication.

PROBLEM DESCRIPTION

Though cognitive radio technology has many advantages, it has limitations like complexity, interference and detection [4, 5]. This work tries to give a solution to the interference and detection problems.

Since the cognitive radio is a wireless technology it requires an antenna to establish the wireless link. Therefore, it is possible to overcome the problem associated with interference by using an appropriate antenna. For this technology we propose to use smart antenna (adaptive array antenna) because it has much more beyond the interference reduction capability like increasing spectrum utilization efficiency, increasing capacity, extended coverage area, reducing power requirement, reducing the amount of electromagnetic radiation to the globe etc [6-8]. Omnidirectional antenna is excluded due to its obvious power dissipation and its being source of interference to others.

This adaptive array antenna has got its 'smartness' from digital signal processing that is incorporated within the adaptive antenna array system. The main purpose of the digital signal processing unit is to

*E-mail: abdotuko@yahoo..com

add adaptability nature to the array antenna in such a way that the antenna dynamically produces narrow and stronger beam to the direction of the intended user and nulls to the direction of interferers by tracking the new locations of both the user and interferers. This process is known as beamforming.

SPECIFIC OBJECTIVES

This work exclusively emphasizes on the following objectives:

- To study the fundamental behaviors of smart antenna and propose it for the cognitive radio technology.
- To investigate the performances of different adaptive beamforming algorithms such as sample matrix inversion (SMI), least mean square (LMS), recursive least square (RLS), constant modulus (CM), and least square constant modulus (LS-CM) and choose the one that best suits the cognitive radio architecture.

COGNITIVE RADIO (CR)

Cognitive radio was first coined by Joseph Mitola III [3], and he defined it as follows: "The term cognitive radio identifies the point at which wireless personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio resources and related computer-to-computer communications to:

- detect user communications needs as a function of use context, and
- Provide radio resources and wireless services most appropriate to those needs.

Since then the concept has got popularity and different groups are working on it for its feasibility [2, 3, 5, 9-13], and. [20]. Those working groups have developed their own working definition but the central ideas can be summarized as follows: Cognitive radio refers to an intelligence wireless system that

- senses and is aware of its operational environment.
- does not operate in a fixed assigned band but it rather searches an appropriate band to operate without any user intervention.
- can be trained to dynamically and autonomously adjust its radio operating parameters accordingly.
- learns from experience, plans based on knowledge, and decides based on reasoning.

Therefore, CR improves spectrum utilization by making it possible for a secondary user (unlicensed user) to access a *spectrum hole* unoccupied by the primary user (licensed user) at the right location and time in request. A spectrum hole is a band of frequency assigned to a primary user, but, at a particular time and specific geographic location, the band is not being utilized by that primary user.

The spectrum freedom obtained from CR will increase the number of wireless operators which will undoubtedly increases the interference level. Therefore, to reduce interference, overcome detection problem, and increase coverage area, appropriate technologies must be chosen for this new technology.

SMART ANTENNAS

Generally speaking, all types of antennas exhibit directivity except the isotropic antenna, which does not exist in the real world. Though, the level of directivity varies from one type to the other, directive antennas have many areas of applications in wireless communication. Array antenna technology is a more practical way of producing highly directive radiation pattern than producing the required radiation by using single and large antenna. Besides, it has the following advantages [14, 15]:

- It produces narrow, electronically steerable and more directive beams.
- It tracks multiple targets
- It produces low side lobes

The above mentioned advantages have been used by deploying array antennas into the wireless communication technologies.

Adding some intelligence to the array antenna helps in tracking the dynamic wireless environment and user location. It is just because of this intelligence that the array antenna system has got the so called naming of Smart Antenna / Adaptive Array Antenna / Adaptive Beamforming. Fig. 1 shows the block diagram of an adaptive array antenna.

Smart antenna may be considered as a marriage of array antenna and digital signal processing technology to improve the performance of wireless communication technology by changing its radiation pattern dynamically to suppress noise, interference and reject multipath. In general, deploying smart antennas to the wireless technology have the following benefits [6, 8, 16-18],

i. Reduction in co-channel interference
ii. Range improvement / range extension
iii. Increase in capacity
iv. Reduction in transmitted power
v. Reduction in handoff
vi. Mitigation of multipath effects
vii. Compatibility with TDMA, FDMA, CDMA, SDMA

In vector form

$$y(t) = \overline{W}^{H}\,\overline{X}(t) \tag{2}$$

Where: $\overline{W} = [w_1, w_2, \cdots, w_N]^T$

$\overline{X}(t) = [x_1(t), x_2(2), \cdots, x_N(t)]^T$, and

$(.)^H$ Signifies Hermitian transpose

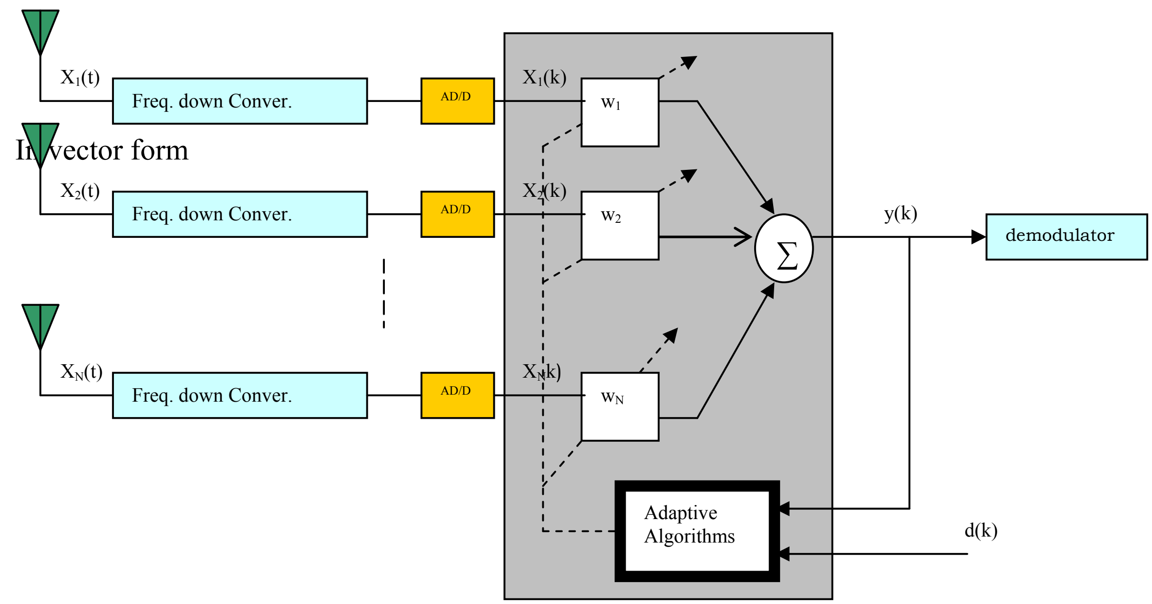


Figure 1 Block diagram of an adaptive array antenna.

Because of the above mentioned benefits, smart antennas are proposed to be used in cognitive radio technology to combat mainly interference and reduce false alarm detection but in conjunction with this, the other benefits could also be enjoyed [5, 19].

The output of any beamformer is given by the following relation [7, 17],

$$y(t) = \sum_{n=1}^{N} w^{*}_{n} x_n(t) \tag{1}$$

Where

w_n is a complex weight applied to the n^{th} element.

$x_n(t)$ is the signal received by the n^{th} element at time t

(.)* signifies complex conjugate.

For digital beamformer (adaptive array) the inputs to the beamformer are fed in digital form as shown in Fig.1. Therefore, the output of the beamformer at the k^{th} sample is given by [7, 8]:

$$y(k) = \sum_{n=1}^{N} w^{*}_{n} x_n(k) \tag{3}$$

$$y(k) = \overline{W}^{H}\,\overline{X}(k) \tag{4}$$

The objective of the adaptive element in the smart antenna system is to find weight vector $\overline{W}$ in such a way that the formed radiation pattern from the antenna array would acquire the following characteristics [7]

- Producing very strong beam to the direction of intended user.
- Formation of nulls to the direction of unintended users/interferers.

Consider M number of users with signals impinging upon the array and let $\overline{X}_k(t)$ denote received signal vector corresponding to the kth user.

For LOS communication $\overline{X}_k(t)$ may be expressed as [8]

$$\overline{X}_k(t) = \gamma_k a(\theta_k) s_k(t) \tag{5}$$

Where γ_k is the scalar complex path amplitude, $a(\theta_k)$ is the array response vector in the direction of arrival θ_k of the kth user, and $s_k(t)$ is the complex baseband signal impinging upon the array from user k.

Then the total received signal vector by the array becomes

$$\overline{X}(t) = \sum_{k=1}^{M} \overline{X}_k(t) + n(t) \tag{6}$$

Where $n(t)$ accounts for receiver noise as well as background channel noise which can be taken as Additive White Gaussian Noise (AWGN). In matrix form Eq. 6 could be rewritten as

$$\overline{X}(t) = A\overline{S}(t) + n(t) \tag{7}$$

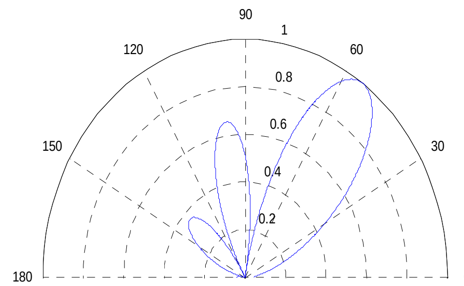
Where:

$$A = [a(\theta_1) \quad a(\theta_2) \quad \cdots \quad a(\theta_M)]$$

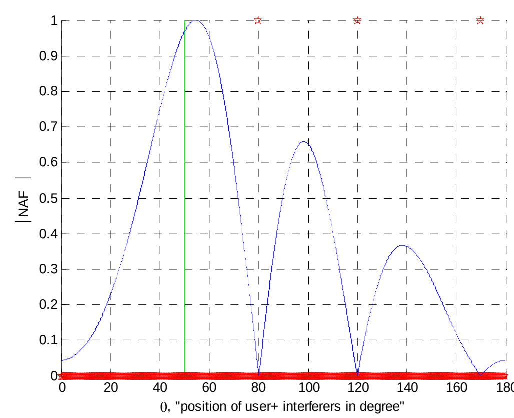
$$\overline{S}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_M(t) \end{bmatrix}$$

Information about the direction (location) of the users is contained in the steering matrix, A.

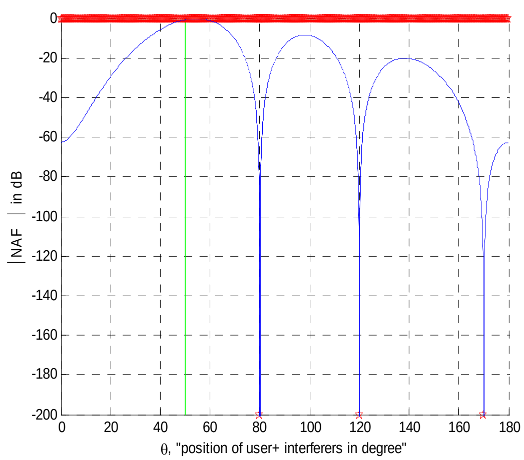
To elaborate the idea discussed above, simulations were carried and Fig. 2 shows the radiation pattern simulated for a user located at 50^0 and interferers located at 80^0, 120^0, and 170^0. In the rectangular plot of the simulation results, the asterisks (*) correspond to the location of interferers and the solid line at 50^0 corresponds to the location of the desired user. This simulation is carried out to simply show the capability of digital beamforming (adaptive beamforming) in producing strong main beam to the direction of the intended user and placing nulls to the directions of the undesired interferers' locations effectively. In this way the interference problems associated with cognitive radio technology could be alleviated by the use of smart antenna technology.



a) Polar plot of radiation pattern.



b) Rectangular plot of normalized array factor



c) Plot of array factor in dB

Figure 2 Simulation of radiation pattern for an array having 4 elements and 4 numbers of users.

To exploit the advantages of smart antenna fully for time-varying environment and extend its use to the new emerging CR technology, we have to examine the beamforming techniques used by the existing wireless communication and select the one that best suits the CR technology. To do this, there must be an adaptive algorithm that can track the change and update the system with the necessary information required to form strong beam to the directions of intended users, nulls to the directions of interferers and generate equal side lobes (with minimum detectable energy) to overcome the hidden node problem and ease detection. The next section is therefore devoted to performance study of different beamforming algorithms.

ADAPTIVE BEAMFORMING ALGORITHMS

There are two types of beamforming: *conventional* and *adaptive beamforming*. The conventional beamforming includes the entire beam shaping techniques used in conventional array. Whereas adaptive beamforming is a type of beamforming which dynamically changes array weights based on the dynamically changing environments so as to make optimum beam to the direction of the intended user and put nulls to the direction of the interferers/noise. This phenomenon is accomplished by using adaptive beamforming algorithms. The theme of this section is therefore to make theoretical investigation and compare the different adaptive beamforming algorithms.

Basically, there are two major classes of adaptive beamforming algorithms based on their requirements for training signal sequence: *Non-Blind and Blind Adaptive Algorithms* [7, 17, 19].

Non-Blind Adaptive Algorithms requires statistical knowledge of the transmitted signal in order to optimize the array weights. In other words, to extract the desired user(s) from the surrounding environment (received signals) a training signal sequences which are known both at the receiver and transmitter are transmitted. Then based on the information obtained from the received signal about the channel the array weights are optimized (adjusted) to reduce the error between the received signals sequences and the known transmitted signal sequences at the receiver.

Unlike non-blind adaptive algorithms, blind algorithms do not require training signal sequences rather they try to estimate information from the received signal.

In this section, the performance of the non-blind and blind adaptive beamforming algorithms, in particular: Sample Matrix Inversion (SMI), Least Mean Square (LMS) and Recursive Least Square (RLS) from the non-blind category; and Constant Modulus (CM) and Least Square Constant Modulus (LSCM) from the blind are dealt with based on different weight optimization criterions.

SAMPLE MATRIX INVERSION (SMI) ALGORITHM

SMI algorithm is an algorithm which uses Minimum Mean Squared Error (MMSE) criterion to obtain the optimal array weight vector. Since we do not have the true auto-correlation matrix and cross-correlation vector, this algorithm replaces both of them by their corresponding estimations (time averaging) to obtain the Wiener-Hopf solution. The estimations are given by [7]:

$$\hat{R}_{xx} = \frac{1}{N}\sum_{n=1}^{N} \bar{X}(n)\,\bar{X}^{*}(n) \tag{8}$$

$$\hat{r}_{xd} = \frac{1}{N}\sum_{n=1}^{N} \bar{X}(n) d^{*}(n) \tag{9}$$

Where: *N is the block size.*

$R_{xx} = E\{\bar{X}^{H}\bar{X}\}$ is the auto-correlation matrix

$\bar{r}_{xd} = E\{\bar{X}d^*\}$ is the cross-correlation vector

$d(.)$ is the reference signal

In terms of the above estimations the Wiener-Hopf solution becomes [7]

$$\hat{\bar{W}} = \hat{R}_{xx}^{-1}\hat{\bar{r}}_{xd} \tag{10}$$

The estimation error which is also known as the residual error is given by

$$e_{est} = \hat{R}_{xx}\bar{W} - \hat{\bar{r}}_{xd} \tag{11}$$

As the block size increases the time average values better approximate the ensemble, resulting in a minimized estimation error in such a way that a much more closer solution to the Wiener-Hopf is obtained. Because of the time varying nature of the wireless channel, the block adaptation is made periodically**.**

This algorithm is very suitable for applications which have bursty nature (for discontinuous transmission) because its adaptation is made in block form. The stability of the SMI algorithm depends on the ability to invert the large covariance matrix. In order to avoid a singularity of the auto-correlation matrix, a zero- mean white Gaussian noise is added to the array response vector. It creates a strong additive component to the diagonal of the matrix. In the absence of noise in the system, a singularity occurs when the number of signals to be resolved is less than the number of elements in the array. The main limitation of SMI algorithm is its computational complexity since it uses direct matrix inversion

Least Mean Square (LMS) Algorithm

This is the second type of beamforming algorithm which uses the MMSE criterion; that searches for the optimal weight that would make the array output either equal or as close as possible to the reference signal or minimizes the Mean Square Error (MSE). Unlike SMI, the LMS is very suitable for continuous type of transmission since its optimization is based on the instantaneous received data. The optimization for LMS is done by employing the Steepest Descent Method which is a recursive way of optimizing the array weights.

The Steepest Descent Method is recursive in the sense that its formulation is represented by a feedback system whereby the computation of the filter takes place iteratively in step by step manner. When the method is applied to the Wiener filter, it provides us with an algorithmic solution that allows for the tracking of time variations in the signal's statistics without having to solve the Wiener-Hopf equations each time the statistics change.
The Steepest Descent Method is given by [20]

$$\bar{W}(k+1) = \bar{W}(k) - \frac{1}{2}\mu\nabla(MSE) \tag{12}$$

Where μ is the step size parameter (commonly it is positive constant) and controls the convergence characteristics of the algorithm.

The difference between the reference signal d(k) and the array output signal y(k) is universally taken as error of the adaptive system at that sample and it is defined as: $e(k) = d(k) - y(k)$

The mean squared error (MSE) is given by [21]:

$$MSE = E\left\{|e(k)|^2\right\}$$

and

$$\nabla(MSE) \overset{\Delta}{=} 2\frac{\partial MSE}{\partial W^*} = 2E\{X^H X\}W - 2E\{Xd^*\}$$

By substituting the gradient of the cost function i.e. the mean squared error (MSE) into Eq.12 we come up with

$$\bar{W}(k+1) = \bar{W}(k) - \mu(R_{xx}\bar{W}_{opt} - \bar{r}_{xd}) \tag{13}$$

The computation of matrix associated with the Steepest Descent Method is another problem of this method. To overcome this difficulty, the LMS algorithm replaces the auto-correlation and cross-correlation by their instantaneous values instead of their actual values. Therefore, Eq.13 can be rewritten as [20]:

$$\bar{W}(k+1) = \bar{W}(k) - \mu(\bar{X}(k)\bar{X}^H(k)\bar{W}_{opt} - \bar{X}(k)d^*(k))$$

$$\bar{W}(k+1) = \bar{W}(k) + \mu\bar{X}(k)e^*(k) \tag{14}$$

Recursive Least Square (RLS) Algorithm

RLS is a type of non-blind adaptive beamforming algorithm that uses the LS method as optimization criterion. To make the estimation problem "well-posed" as well as to track time-varying systems, the cost function is defined as [7, 17, 20].

$$\varepsilon(k)=\underbrace{\sum_{i=1}^{k}\lambda^{k-i}\left|e(i)\right|^2}_{\text{Sum of Weighted Error Squares}}+\underbrace{\delta\lambda^k\left\|\overline{W}(k)\right\|^2}_{\text{Regularization term}} \tag{15}$$

Where: k is variable length of the observable data

$e(i)$ is the error function

δ is a positive real number and it is called the regularization parameter

λ is called the forgetting factor, which is a positive constant close to, but less than one. It emphasizes past data in a non-stationary environment so that the statistical variations of the data can be tracked and not "forgotten". In a stationary environment, λ = 1 corresponds to infinite memory.

The RLS algorithm can be summarized as follows [20]:

First initialize the algorithm by setting

$$\overline{W}(0)=0, \quad P(0)=\delta^{-1}I \tag{16}$$

$$\delta=\begin{cases}\text{small positive constant for high SNR}\\ \text{large positive constant for low SNR}\end{cases}$$

I is NxN identity matrix.

$$\overline{T}(k)=\frac{\lambda^{-1}P(k-1)\overline{X}(k)}{1+\lambda^{-1}\overline{X}^H(k)P(k-1)\overline{X}(k)} \tag{17}$$

$$e(k)=d(k)-\overline{W}^H(k-1)\overline{X}(k) \tag{18}$$

$$\overline{W}(k)=\overline{W}(k-1)+\overline{T}(k)e^*(k) \tag{19}$$

$$P(k)=\lambda^{-1}P(k-1)-\lambda^{-1}\overline{T}(k)\overline{X}^H(k)P(k-1) \tag{20}$$

Where: $\overline{T}(k)$ is Nx1 vector and it is called gain vector

$P(k)$ is NxN matrix and it is called inverse correlation matrix

Constant Modulus Algorithm (CMA*)*

CMA is from the blind adaptive beamforming family which requires no training signal sequence to make an optimum beam to the intended direction; it would rather try to restore important property of the transmitted signal [8, 17, 22].

In most of the communication scenario it is common to use modulation techniques with constant envelope or amplitude such as FM, FSK, PSK, MSK and the like. But in transmitting base band signals by using these modulation techniques, the transmitted signal encounters channel fading which may result both in amplitude and phase distortions. The constant envelope/modulus property of the above mentioned modulation techniques opens a window to the adaptive beamforming algorithm in order to use this property in the beamforming technology. The receiver restores the envelope of the transmitted signal by equating the received signal to some constant value that corresponds to the envelope of the transmitted signal. This is made possible by continuously updating the weight of the beamformer until the output of the array has the same modulus as that of the original transmitted signal. The class of adaptive beamforming algorithm that uses this phenomenon is known as the Constant Modulus Algorithm (CMA).

The cost function used for CMA is given by[8, 17]:

$$J(k)=E\left[\left|\left|y(k)\right|^p-\left|\alpha\right|\right|^q\right] \tag{21}$$

Where: p=1, 2 or q=1, 2 and α is the desired signal amplitude at the output of the array

Assuming that $|\alpha|$=1, then Eqn. 15 becomes:

$$J(k)=E\left[\left|\left|y(k)\right|^p-1\right|^q\right] \tag{22}$$

This so-called CMA (p, q) cost function is simply a positive measure of the average amount that the beamformer output *y (k)* deviates from the unit modulus condition. The objective is then choosing weight vector recursively in order to minimize *J* and consequently it makes y(k) as close to a constant modulus signal as possible.

It is not possible to get the closed form of solution for the above cost function; rather it is simple to use an iterative method to obtain the optimal weight vector or the minimum *J* like in LSM i.e. by using Steepest Descent Method. Then, in terms of the above cost function, Eq. 12 becomes [8]:

$$\overline{W}(k+1)=\overline{W}(k)-\frac{1}{2}\mu\nabla(J) \tag{23}$$

Where: $\nabla(J)$ is the gradient of the cost function of the CMA

The $\nabla(J)$ for p=1, q=2 becomes

$$\nabla(J(k))=2\frac{\partial J(k)}{\partial\overline{W}^{*}(k)}=2E\left[(|y(k)|-1)\frac{\partial|y(k)|}{\partial\overline{W}^{*}(k)}\right]$$

Further simplification of the above equation results:

$$\nabla(J(k))=2E\left[\overline{X}(k)(y(k)-\frac{y(k)}{|y(k)|})^{*}\right] \tag{24}$$

As done for the LMS, we replace the statistical expectation with the instantaneous value so that Eq. 24 becomes [8]:

$$\nabla J(k)=2\left(\overline{X}(k)(y(k)-\frac{y(k)}{|y(k)|})^{*}\right) \tag{25}$$

Substituting Eq. 25 into Eq. 23 results in the following weight updating equation

$$\overline{W}(k+1)=\overline{W}(k)-\mu\overline{X}(k)(y(k)-\frac{y(k)}{|y(k)|})^{*}$$

$$\overline{W}(k+1)=\overline{W}(k)-\mu\overline{X}(k)e^{*}(k) \tag{26}$$

Where: μ has similar function as to LMS but we choose $\mu << 1$ to get better stability.

Like in the LMS algorithm, the convergence rate can be controlled by varying μ. However, to get much better convergence behavior, non-linear least square method need to be used.

Least Square Constant Modulus Algorithm (LS-CMA)

The constant modulus algorithm was first used by Gooch [23] in the beamforming problem. After that, many CMA-type algorithms have been proposed for use in adaptive arrays. Among them B. G. Agee [24] developed the LS-CMA by using the extension of the method of nonlinear least-squares (Gauss's method). The extension of Gauss's method states that if a cost function can be expressed in the form:

$$F(\overline{W})=\sum_{k=1}^{K}\left|g_k(\overline{W})\right|^2=\left\|g(\overline{W})\right\|_2^2 \tag{27}$$

Where: $g(\overline{W})=[g_1(\overline{W}),g_2(\overline{W}),...,g_K(\overline{W})]^T$

then the cost function has a partial Taylor-series expansion with sum-of- squares form [8]

$$F(\overline{W}+\overline{d})\approx\left\|g(\overline{W})+D^H(\overline{W})\overline{d}\right\|_2^2 \tag{28}$$

Where:

$\overline{d}$ is an offset vector, and

$$D(\overline{W})=\begin{bmatrix}\nabla(g_1(\overline{W})),\nabla(g_2(\overline{W})),..., \\ \nabla(g_K(\overline{W}))\end{bmatrix} \tag{29}$$

It can be shown that the gradient of $F(\overline{W}+\overline{d})$ with respect to $\overline{d}$ is given by [8]

$$\nabla_{\overline{d}}(F(\overline{W}+\overline{d}))=2\frac{\partial F(\overline{W}+\overline{d})}{\partial\overline{d}^{*}}$$
$$=2\left\{D(\overline{W})g(\overline{W})+D(\overline{W})D^H(\overline{W})\overline{d}\right\} \tag{30}$$

Setting $\nabla_{\overline{d}}(F(\overline{W}+\overline{d}))$ equal to zero, the offset that minimizes the cost function $F(\overline{W}+\overline{d})$ will be

$$\overline{d}=-\left[D(\overline{W})D^H(\overline{W})\right]^{-1}D(\overline{W})g(\overline{W}) \tag{31}$$

Adding $\overline{d}$ to $\overline{W}$ results in a new weight vector that minimizes the cost function. Therefore the weight update equation becomes [8, 24]:

$$\overline{W}(l+1)=\overline{W}(l)-\left[D(\overline{W}(l))D^H(\overline{W}(l))\right]^{-1}D(\overline{W}(l))g(\overline{W}(l)) \tag{32}$$

Where: l denotes the iteration number. LS-CMA is derived by applying Eq.32 to the constant modulus function

$$F(W)=\sum_{k=1}^{K}\left||y(k)|-1\right|^2=\sum_{k=1}^{K}\left||\overline{W}^H\overline{X}(k)|-1\right|^2 \tag{33}$$

Comparing Eq.27 with Eq.33, we observe that

$$g_k(\overline{W})=|y(k)|-1=\left|\overline{W}^H\overline{X}(k)\right|-1 \tag{34}$$

Then $g(\overline{W})$ becomes

$$g(\overline{W})=\begin{bmatrix} g_1(\overline{W}) \\ g_2(\overline{W}) \\ \vdots \\ g_K(\overline{W}) \end{bmatrix}=\begin{bmatrix} |y(1)|-1 \\ |y(2)|-1 \\ \vdots \\ |y(K)|-1 \end{bmatrix} \tag{35}$$

The gradient vector of $g_k(\overline{W})$ is given by [8]

$$\nabla(g_k(\overline{W}))=2\frac{\partial g_k(\overline{W})}{\partial \overline{W}^*} \quad =\overline{X}(k)2\frac{y^*(k)}{|y(k)|} \tag{36}$$

Substituting Eq.36 into Eq.29 results in:

$$D(\overline{W})=\left[\overline{X}(1)2\frac{y^*(1)}{|y(1)|},\overline{X}(2)2\frac{y^*(2)}{|y(2)|},...,\overline{X}(K)2\frac{y^*(K)}{|y(K)|}\right]$$

$$D(\overline{W})=XY_{CM} \tag{37}$$

Where: $X=\left[\overline{X}(1),\overline{X}(2),...\overline{X}(K)\right]$ is the input data matrix, and

$$Y_{CM}=\begin{bmatrix} \frac{y^*(1)}{|y(1)|} & 0 & \cdots & 0 \\ 0 & \frac{y^*(2)}{|y(2)|} & \cdots & 0 \\ \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & \frac{y^*(K)}{|y(K)|} \end{bmatrix} \tag{38}$$

is the output data matrix. Using Eq.34 and Eq.37 we have:

$$D(\overline{W})D^H(\overline{W})=XY_{CM}Y^H{}_{CM}X^H \quad =XX^H \tag{39}$$

and

$$D(\overline{W})g(\overline{W})=XY_{CM}\begin{bmatrix} |y(1)|-1 \\ |y(2)|-1 \\ \vdots \\ |y(K)|-1 \end{bmatrix}$$

$$=X\begin{bmatrix} y^*(1)-\frac{y^*(1)}{|y(1)|} \\ y^*(2)-\frac{y^*(2)}{|y(2)|} \\ \vdots \\ y^*(K)-\frac{y^*(K)}{|y(K)|} \end{bmatrix}$$

$$=X(\overline{Y}-\overline{P})^* \tag{40}$$

Where: $\overline{Y}=\left[y(1) \quad y(2) \quad \cdots \quad y(K)\right]^T$

$$\overline{P}=\left[\frac{y(1)}{|y(1)|} \quad \frac{y(2)}{|y(2)|} \quad \cdots \quad \frac{y(K)}{|y(K)|}\right]^T$$

The vectors $\overline{Y}(l)$ and $\overline{P}(l)$ are called the output data vector and complex-limited output data vector, respectively. Substituting Eq.39 and Eq.40 into Eqn.32 we obtain [8]:

$$\overline{W}(l+1)=\overline{W}(l)-\left[XX^H\right]^{-1}X(\overline{Y}(l)-\overline{P}(l))^*$$

$$=\overline{W}(l)-\left[XX^H\right]^{-1}X\overline{Y}^*(l)+\left[XX^H\right]^{-1}X\overline{P}^*(l)$$

$$=\overline{W}(l)-\left[XX^H\right]^{-1}XX^H\overline{W}(l)+\left[XX^H\right]^{-1}X\overline{P}^*(l)$$

$$=\left[XX^H\right]^{-1}X\overline{P}^*(l) \tag{41}$$

Where $\overline{Y}(l)=\left[W^H(l)X\right]^T$ then $\overline{P}(l)=\boldsymbol{L}\overline{Y}(l)$.

Note that $\boldsymbol{L}\overline{Y}(l)$ places a hard limit on $\overline{Y}(l)$. Since the algorithm iterates using a single block of K data vectors, $[x(k)]$, it is called static LS - CMA. The LS-CMA can be implemented both statically and dynamically.

The static LS-CMA repeatedly uses one data block X, which contains K snapshots of the input data vectors, in the updating of the weight vector $\overline{W}$. In the static LS-CMA, after a new weight vector $\overline{W}(l+1)$ is calculated using Eq.41, this new weight vector is used with the input data block X, which was also used in the last iteration, to generate the new output data vector $\overline{Y}(l+1)$ and the complex-limited output data vector $\overline{P}(l+1)$. The new complex-limited output data vector is then substituted into Eq.40 to generate a new weight vector.

In dynamic LS-CMA, however, different input data blocks are used during the updating of the weight vector. Let $X(l)$ denote the input data block used in the l^{th} iteration. $X(l)$ can be expressed as [8]

$$X(l)=\left[\overline{X}(1+lK),\overline{X}(2+lK),...\overline{X}((1+l)K)\right] \quad \text{For } l=0,1,2,\cdots L \tag{42}$$

Where L is the number of iterations required for the algorithm to converge. Using $X(l)$ we can describe the dynamic LS-CMA by the following equations

$$\begin{aligned}\overline{Y}(l)&=\left[\overline{W}^{H}(l)X(l)\right]^{T}\\&=\left[y(1+lK),y(2+lK),\cdots y((l+1)K)\right]^{T}\end{aligned} \tag{43}$$

$$\overline{P}(l)=\begin{bmatrix}\dfrac{y(1+lK)}{|y(1+lK)|},\dfrac{y(2+lK)}{|y(2+lK)|},\\ \cdots\dfrac{y((l+1)K)}{|y((l+1)K)|}\end{bmatrix}^{T} \tag{44}$$

$$\overline{W}(l+1)=\left[X(l)X^{H}(l)\right]^{-1}X(l)\overline{P}^{*}(l) \tag{45}$$

From the above equations we see that while the steepest descent CMA updates the weight vector on a sample-by-sample basis, the dynamic LS-CMA adjusts the weight vector on a block-by-block basis.

Finally, the sample mean estimate of the correlation matrix of the input data and the cross-correlation between the input data and the output for the block of data available at the l^{th} iteration can be constructed as [8]:

$$\hat{R}_{xx}=\frac{1}{K}X(l)X^{H}(l) \tag{46}$$

$$\hat{r}_{xd}=\frac{1}{K}X(l)P^{\bullet}(l) \tag{47}$$

Where K is the block size.

Then Eq. 45 becomes:

$$\hat{W}=\hat{R}_{xx}^{-1}\hat{r}_{xd} \tag{48}$$

COMPUTATIONAL COMPLEXITY ANALYSIS

Computational complexity can be expressed in terms of time and space complexity. But the analysis in terms of these two parameters is very complex. It is rather better to discuss the computational complexity of the above adaptive beamforming algorithms in terms of the two fundamental mathematical operators (addition and multiplication operators) performed per iteration. Using the latter concept, the computational complexity of the adaptive beamforming algorithms studied in this work are summarized in the following tables.

Table 1: Computational complexity of SMI Algorithm

Procedures	Multiplication per iteration	Addition per iteration
$\hat{R}_{xx}=\frac{1}{K}\sum_{k=1}^{K}\overline{X}^{*}(k)\overline{X}^{T}(k)$	KN+1	K+N
$\hat{r}_{xd}=\frac{1}{K}\sum_{k=1}^{K}d(k)\overline{X}^{T}(k)$	K^2N+1	K
$W=\hat{R}_{xx}^{-1}\hat{r}_{xd}$	N^2	N^2
Total operation	K^2N+KN+N^2+ 2 + Matrix Inversion operation	N^2+ 2K+N

Table 2: Computational complexity of LMS algorithm

Procedures	Multiplication per iteration	Addition per iteration
$y(k)=\overline{W}^{H}(k)\overline{X}(k)$	N	N
$e(k)=d(k)-y(k)$	-	1
$\overline{W}(k+1)=\overline{W}(k)+\mu\overline{X}(k)e^{*}$	N+1	N+1
Total operation	2N+1	2N+2

Where: *K is length of observable data.*
N is the number of array elements.

Table 3: Computational complexity of RLS Algorithm

procedures	**Multiplication per iteration**	**Addition per iteration**
$\overline{T}(k)=\dfrac{\lambda^{-1}P(k-1)\overline{X}(k)}{1+\lambda^{-1}\overline{X}^{H}(k)P(k-1)\overline{X}(k)}$	$2N^2+3N+1$	$2N^2+2N+1$
$e(k)=d(k)-\overline{W}^{H}(k-1)\overline{X}(k)$	N	N+1
$\overline{W}(k)=\overline{W}(k-1)+\overline{T}(k)e^{*}(k)$	N	N
$P(k)=\langle\ \lambda^{-1}P(k-1)-\lambda^{-1}\overline{T}(k)\overline{X}^{H}(k)P(k-1)\ \rangle$	N^2+2N+1	N^2+N+1
Total operation	$3N^2+7N+2$	$3N^2+5N+3$

Table 4: Computational complexity of CMA

Procedures	**Multiplication per iteration**	**Addition per iteration**
$y(k)=\overline{W}^{H}(k)\overline{X}(k)$	N	N
$e(k)=y(k)-\dfrac{y(k)}{\lvert y(k)\rvert}$	1	1
$W(k+1)=\overline{W}(k)-\mu\overline{X}(k)e$	N+1	N
Total operation	2N+2	2N+1

Table 5: Computational complexity of LS-CMA

Procedures	**Multiplication per iteration**	**Addition per iteration**
$\overline{Y}(l)=\left[\overline{W}^{H}(l)X(l)\right]^{T}$	NK	NK
$\overline{P}(l)=\left[\dfrac{y(1+lK)}{\lvert y(1+lK)\rvert},\dfrac{y(2+lK)}{\lvert y(2+lK)\rvert},\cdots\dfrac{y((l+1)K)}{\lvert y((l+1)K)\rvert}\right]^{T}$	K	-
$\overline{W}(l+1)=\left\{\left[X(l)X^{H}(l)\right]^{-1}X(l)\overline{P}^{*}(l)\right\}$	$N^2K + N^2 + NK +$ inversion operation	$N^2K + N^2 + NK$
Total operations	$N^2K + N^2 + 2NK + K +$ inversion operation	$N^2K + N^2 + 2NK$

From the above computational complexity table, the algorithms can be arranged in the order of decreasing computational complexity as follows SMI, LS-CMA, RLS, LMS and CMA.

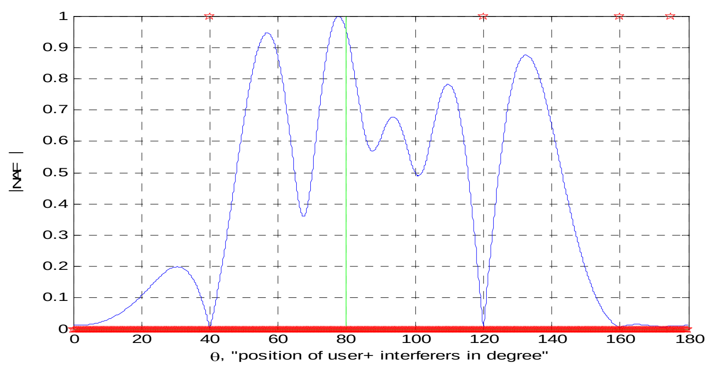
To see the performance in terms of convergence rate, we need to make simulation for the corresponding beamforming algorithms. This is presented in the next section with brief discussion whenever required.

SIMULATION RESULTS

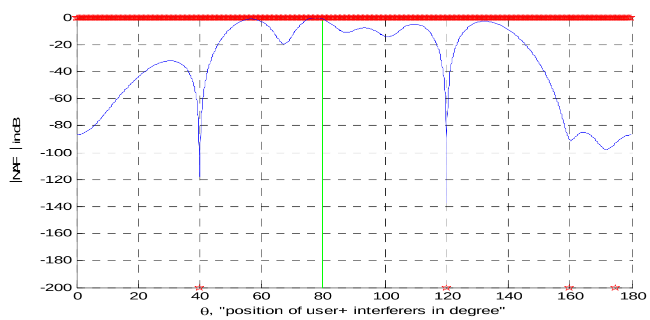
In this section, simulation results of different adaptive beamforming algorithms used in this work are presented. All the adaptive beamforming

In this section, simulation results of different adaptive beamforming algorithms used in this work are presented. All the adaptive beamforming simulations are done for 8 array elements and 5 users, where one intended user is located at 80^0 and the rest four users are interferers which are located at 40^0, 120^0, 160^0, and 175^0. Moreover, on the simulation output, the solid line position at 80^0 infer to the position of intended user and the locations of the asterisks (*) correspond to the locations of the interferers.

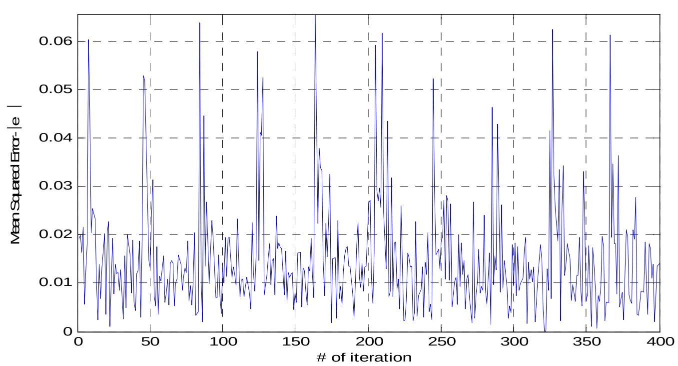
SMI Algorithm Simulation Results



a) Radiation pattern rectangular plot



b) Radiation pattern rectangular plot in dB

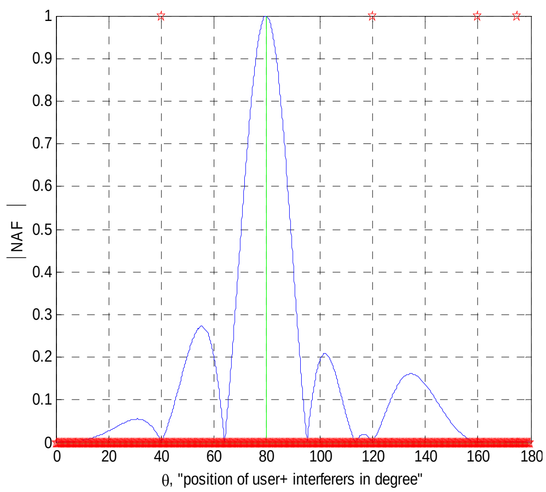


c) Plot of mean squared error (MSE)

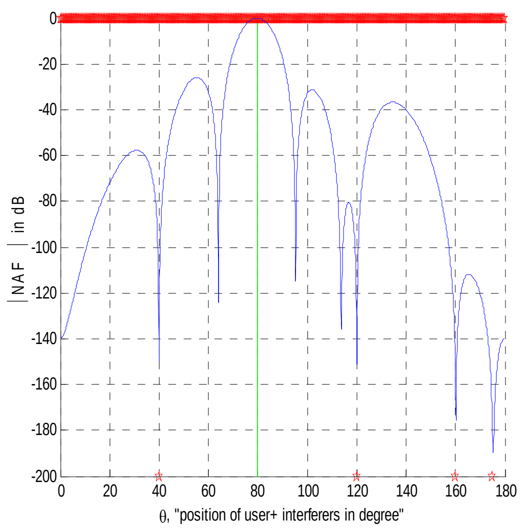
Figure 3 Simulations of Radiation Pattern and MSE, synthesized by using SMI algorithm for SNR=30 dB, and block size=40.

The simulation for SMI algorithm is carried out for SNR= 30dB, for 10 blocks with block size=40 snap shot (samples). As can be seen from the simulation, the SMI algorithm converges quickly.

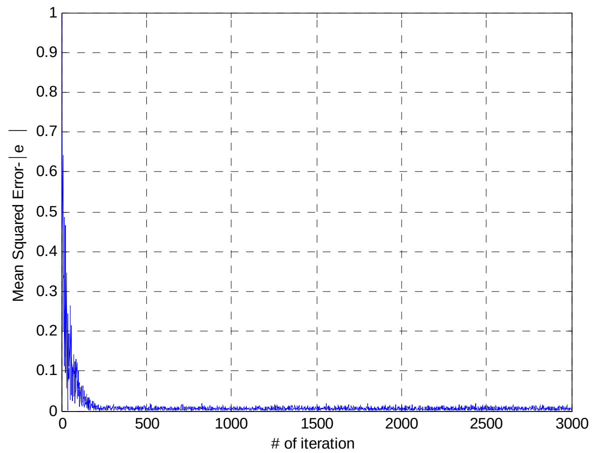
LMS Algorithm Simulation Results



a) Radiation pattern rectangular plot



b) Radiation pattern rectangular plot in dB

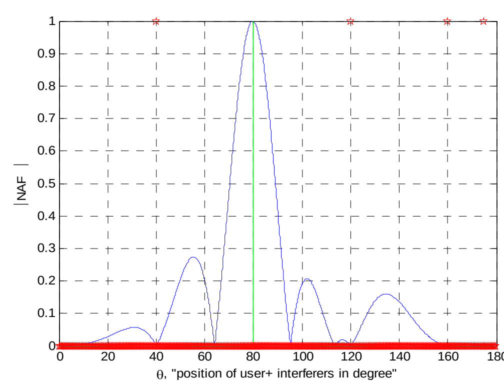


c) Plot of mean squared error (MSE)

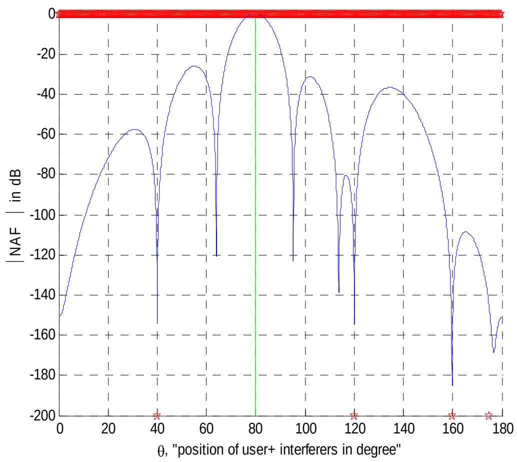
Figure 4 Simulation of radiation pattern and MSE,synthesized by using LMS algorithm for μ =0.0110,SNR=30dB, iteration=3000.

As can be seen from the simulation results, the LMS beamforming algorithm has less convergence rate than the SMI beamforming algorithm, but the latter one forms much stronger beams only to the direction of the intended user. Besides its ability to form much stronger beams only to the direction of the intended user, it also has less computational complexity. Because of these reasons the family of LMS beamforming algorithms is preferred over the SMI beamforming algorithm to implement in *existing* wireless communication scenario.

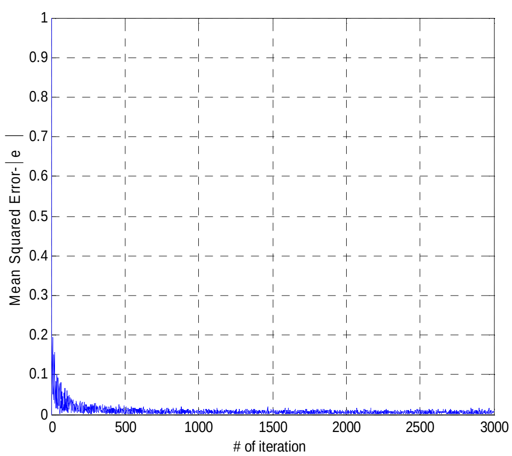
RLS Algorithm Simulation Results



a) Radiation pattern rectangular plot



b) Radiation pattern rectangular plot in dB

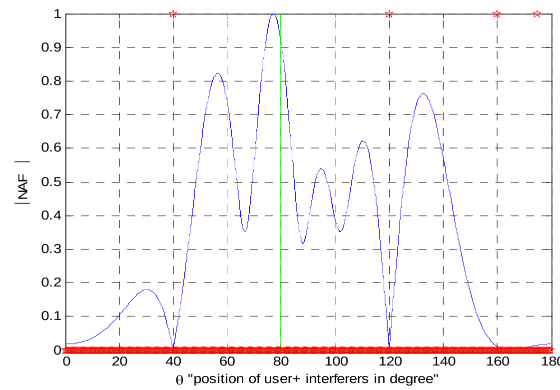


c) Plot of least squared error (LSE)

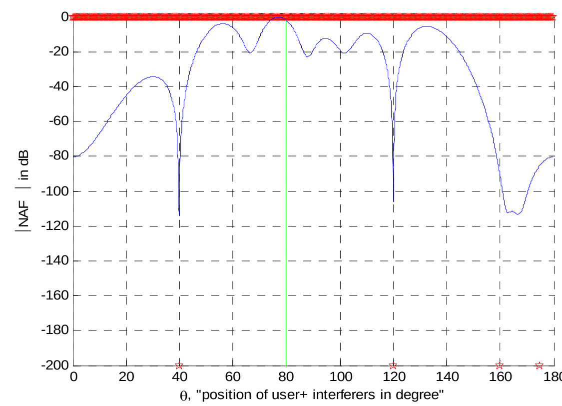
Figure 5 Simulation of radiation pattern synthesized by using RLS algorithm for SNR=30dB, δ =0.068, λ =1, and number of iteration= 3000.

We observe that the convergence of the RLS algorithm is better than that of the LMS algorithm, but this increase in convergence rate is obtained at the cost of increased computational complexity. In addition to the good convergence rate, RLS has the ability to retain information about the input data vector from the very beginning. Another important feature of the RLS algorithm is its ability to replace the inversion of the covariance matrix in the Weiner solution with a simple scalar division

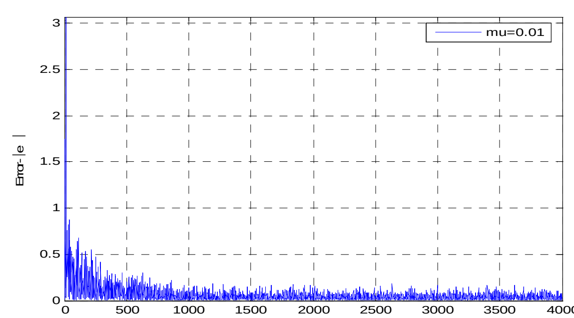
CMA Simulation Results



a) Radiation pattern rectangular plot



b) Radiation pattern rectangular plot in dB

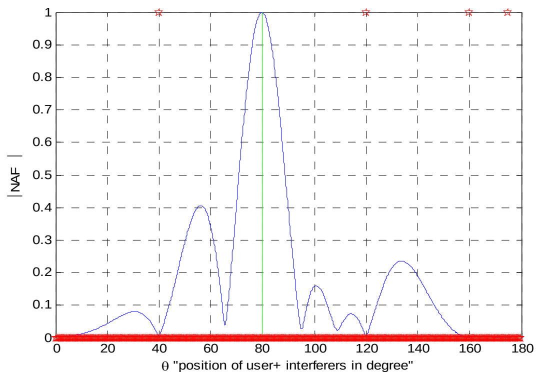


c) Plot of mean square error (MSE)

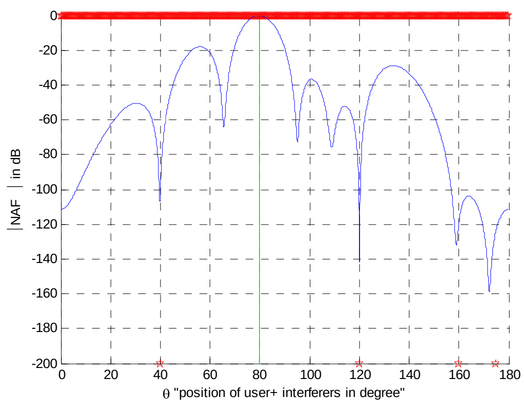
Figure 6 Simulation of radiation pattern and MSE,synthesized by using CMA for MSK signal with μ= 0.01,SNR=30 dB,iteration=4000.

We observe that the CMA has similar behavior to that of SMI but with slow convergence rate. The convergence rate can be improved by increasing μ However; care must be taken not to use large value of μ that renders the algorithm unstable.

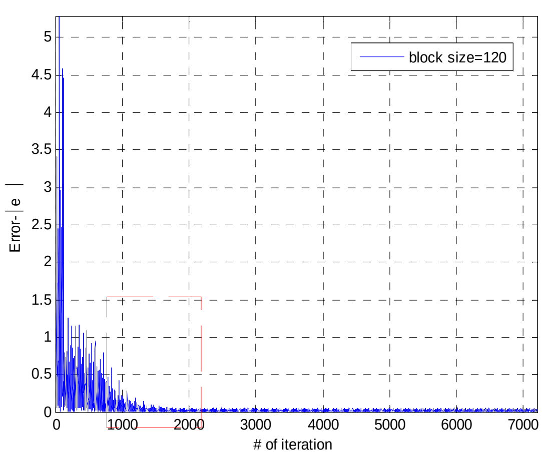
LS-CMA Simulation Results



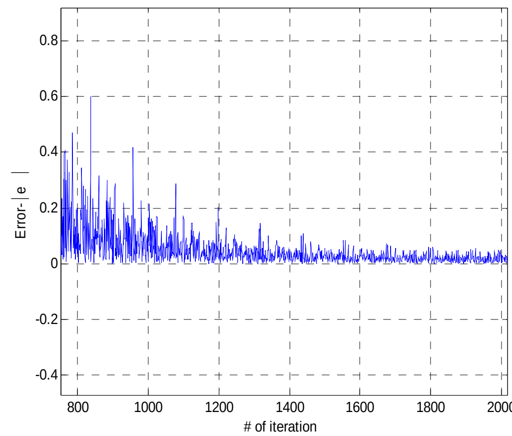
a) Radiation pattern rectangular plot



b) Radiation pattern rectangular plot in dB



c) Plot of least square error (LSE)



d) Partially enlarged view of the error graph shown in (c) above.

Figure 7 Simulation of radiation pattern synthesized by using LS-CMA for SNR = 30 dB, block size=120, no. of iterations = 7000.

As we can see from the above simulations, the LS-CMA has improved the performance of the CMA. Since the adaptation is made in block form, increasing the block size results in an increase of the performance of the algorithm.

CONCLUSION

In comparison to the existing wireless communication concept which virtually deploys fixed spectrum band for different wireless technologies, the concept of cognitive radio technology is indicative in bringing the wireless technology to new era. In this work, the performance of different beamforming techniques has been investigated. Although smart antennas technology has been used in the third generation communication, the way it is proposed for cognitive radio technology is slightly different from the way it is used earlier. The difference is mainly from the point of view of side lobes requirement. The existing wireless communication does not require any side lobe, if possible, whereas the cognitive radio technology takes as an advantage the generation of side lobes in all directions equally except to the direction of interferers so as to simplify the spectrum detection capability of the system. Therefore, the investigations in this work are made from this point of view. In accordance to the aforementioned ideas, the following conclusions are drawn from this work.

The investigation of different adaptive beamforming algorithms for the cognitive radio technology from both blind and non-blind algorithms has shown that the Sample Matrix Inversion (SMI) from the non-blind beamforming family and the Constant Modulus Algorithm (CMA) from the blind beamfoming family have better radiation pattern (beam pattern) that suits the cognitive radio technology. The others have low and dying side lobes and using them for detection in CR application could result in the scanned RF giving wrong information (false alarm) about the vacant and occupied spectrum holes. In comparing the overall performance, the SMI is preferred for CR applications as compared to the other adaptive beamforming algorithms. In fact, it has very fast convergence rate of all adaptive beamforming algorithms studied in this work which one big advantage for the cognitive radio system.

In general, it has been shown that smart antenna technology has the potential to be used in the next generation communication i.e. in cognitive radio.

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