NEUTRAL AXIS DEPTH PROFILE OF REINFORCED CONCRETE BEAMS

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ABSTRACT

Many studies have been conducted for the estimation of effective moment of inertia expressions for Reinforced concrete (RC) beams. To compute deflection of reinforced concrete beams, study results show that the expressions for effective moment of inertia, in each case, are different and related to loading positions. As a result deflection of reinforced concrete beams using the empirical equations vary from the experimental values. In this study, variation in neutral axis depth profile of a simply supported reinforced concrete beam is studied and verified using experimental investigation. The experimental result reveal that neutral axis (NA) depth profile of a reinforced concrete beam varies longitudinally, moves with the load and in good agreement with the analytical curve. Moreover, a neutral axis depth profile and moment of inertia expression with parabolic functions for simply supported beam are proposed. These equations are to be used for the computation of deflection of reinforced concrete beams.

Keywords: Neutral axis depth, RC beam, effective moment of inertia, deflection

INTRODUCTION

Background

The variation in the modulus of elasticity with the increasing load is caused by the inelastic stress-strain behavior of concrete beyond the elastic limits, while the variation in the moment of inertia is associated with the cracking of concrete due to the tensile strains greater than the cracking strain of concrete. The cracked zones in a concrete beam are ineffective in resisting stresses originating from applied loads and moments [1]. The overall moment of inertia of a concrete beam decreases gradually from the uncracked moment of inertia (I_{ucr}) to the fully-cracked moment of inertia (I_{cr}), as flexural cracks form at discrete locations along the span [1]. Deflections may be computed using the modulus of elasticity for concrete as specified in AASHTO [2, 3] by taking the effective moment of inertia expression proposed by Branson [4] and it is given in Eq. (1) by setting the value of m = 3.

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^m I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] I_{cr} \leq I_g \qquad (1)$$

where:

I_{eff} is effective moment of inertia (mm^4)
M_{cr} is cracking moment (kN-m)
I_g is the gross moment of inertia (mm^4)
y_t is distance from the neutral axis to the extreme tension fiber (mm)
I_{cr} is moment of inertia of the cracked ection (mm^4)
M_a is maximum moment in a component at the stage for which deformation is computed (kN-m) m is a constant

In cases when the effective flexural stiffness is assumed to be a function of flexural moment, the same expression as that of Eq. (1) with different exponent (*m*=4) is given in JSCE Standard Specifications for Concrete Structures [5]. In some cases, the value of *m* decreases as the reinforcement ratio increases [1]. In this study, expressions for neutral axis depth and moment of inertia for simply supported beam are presented and verified experimentally.

Based on EC-2, reinforced concrete members behave in a manner intermediate between the uncracked and fully cracked sections and, the average curvature is given as follow [6]:

$$k = (1-\xi)k_1 + \xi k_2 \qquad (2)$$

where:
k is the average curvature
k_1 is the curvature in the uncracked regions
k_2 is the curvature in the fully cracked regions
ξ is distribution coefficient indicates how close the stress-strain state is to the condition causing cracking. It takes a value of zero at the cracking moment and approaches unity as the loading increases above the cracking moment.

Moreover, statistical parameters have been established for reinforced concrete beam specimens and deflection predictions made by the finite element package and design code methods ACI and EC2 was investigated. The result shows that, in most cases deflections were overestimated at the initial load intervals close to the cracking load. Justification for that were large standard deviations [7].

Computation of Neutral Axis Depth and Moment of Inertia

Based on AASHTO LRFD [2] and Chen et al. [8], for cracked section, the neutral axis depth and moment of inertia are given in Eqs. (3a) and (3b), respectively.

$$y_0 = \sqrt{0.5b(nA_s(d-y_0)+(n-1)A'_s(d'-y_0))} \qquad (3a)$$

$$I_{cr} = \frac{by_0^{\,3}}{3} + nA_s(d-y_0)^2 + (n-1)A'_s(y_0-d')^2 \qquad (3b)$$

where:
d is effective depth (mm)
b is width (mm)
d' is position of compression steel, measured from the top fiber (mm)
A_s is area of steel in tension (mm^2)
A'_s is area of steel in compression (mm^2)
n is modular ratio, E_s/E_c
y_0 is neutral axis depth of the fully cracked section, measured from the top fiber (mm)
I_{cr} is moment of inertia of the cracked section (mm^4)

For uncracked section, the neutral axis depth and moment of inertia [2,] are given in Eqs. (4a) and (4b), respectively.

$$y_1 = \frac{0.5bh^2 + (n-1)(A_s d) + (n-1)(A'_s d')}{bh + (n-1)A_s + (n-1)A'_s} \qquad (4a)$$

$$I_{unc} = \frac{bh^3}{12} + bh(\frac{h}{2} - y_1)^2 + (n-1)A_s(d-y_1)^2 + (n-1)A'_s(y_1-d')^2 \qquad (4b)$$

where:
h is total depth (mm)
y_1 *is* neutral axis depth of the uncracked section, measured from the top fiber (mm)

I_{unc} is moment of inertia of the uncracked section (mm^4)

Variation in Neutral Axis Depth Profile

To compute the effective moment of inertia, the variation in the neutral axis depth and moment of inertia along the span is taken into account. Actually, the neutral axis along the longitudinal line is not constant due to the tensile strength of concrete and the variation in effective reinforcement ratio in the section. For uniformly distributed loads, since the neutral axis depth is related to bending moment, a parabolic neutral axis and variable moment of inertia along the longitudinal direction are assumed [9]. The neutral axis depth profile be expressed by a quadratic equation given in Eq. (5).

$$\bar{y} = ax^2 + bx + c \tag{5}$$

where:

$\bar{y}$ is neutral axis depth at a section, measured from the top fiber (mm)

x is distance measured from the left support of the beam (m)

a, b and c are constants

For the derivations of neutral axis depth variation and to obtain expressions for the moment of inertia along the line of a beam of new structures, consider the longitudinal cross section of a simply supported beam shown in Fig. 1. In the figure, y_1 the neutral axis depth of the uncracked section and $\bar{y}_0$ is the neutral axis depth of the fully cracked section, measured from the top fiber (mm).

The basic assumption considered is that the neutral axis profile varies with the load, depends on its position and crack occurs at points where the bending moment is sufficiently large [9].

For new RC structures, the neutral axis depth profile is not steady and it moves with the load. In such a case, for the computation of moment of inertia, envelope for neutral axis depth profile is important. For old structures, for the derivation of neutral axis depth and moment of inertia at a section, the neutral axis depth is assumed to be independent of the location of the load and the section is fully cracked at the mid span [9]. Combining Eqs. (3a), (3b), (4a), (4b) and inserting to Eq. (5) gives a simplified neutral axis depth profile expression. The expression for the neutral axis depth profile is given in Eq. (6). In Eq. (6), the constants a, b and c are determined from boundary conditions. The boundary conditions are: at $x = 0$, $\bar{y} = c = y_1$, at $x = x_1$, $\bar{y} = y_0$ and at $x = L$, $\bar{y} = y_1$. Upon substitution, the following expression for the neutral axis depth, except at $x_1 = 0$ is obtained.

$$\bar{y} = \frac{(y_0 - y_1)}{x_1(L - x_1)}(Lx - x^2) + y_1 \tag{6}$$

where:

L is length of the beam (m)

x_1 is location of load position, measured from the left support of the beam (m)

Upon substitution and simplification of Eq. (6), the expressions for NA depth and moment of inertia of a simply supported reinforced concrete beam at a section are expressed as follows.

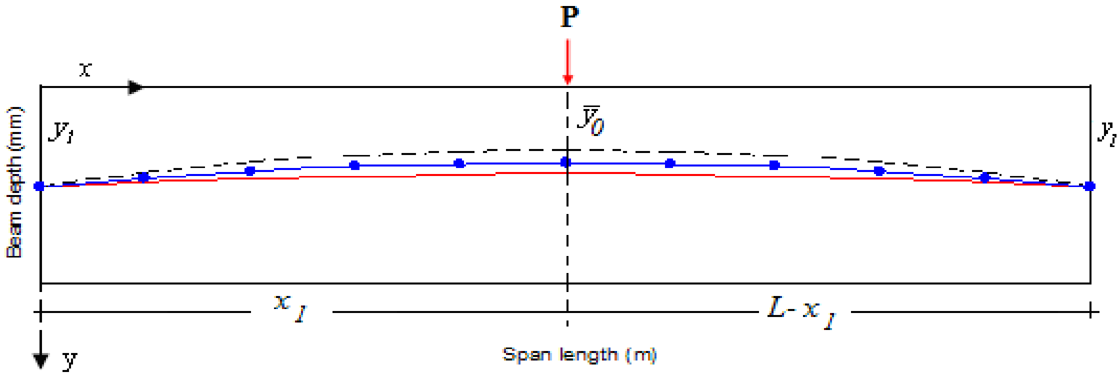

Fig. 1 Concept of variation in neutral axis profile at different loading condition

$$\bar{y} = y_1\left(\frac{2x}{L}\right)^2 + y_0\left(1-\left(\frac{2x}{L}\right)^2\right) \quad (7)$$

$$I(x) = I_1\left(\frac{2x}{L}\right)^2 + I_{cr}\left(1-\left(\frac{2x}{L}\right)^2\right) \quad (8)$$

Due to the variation in applied load and cracking moment of concrete, the neutral axis depth, y_0 is not constant and hence the effect of applied load has to be considered. The concept of variations in neutral axis profile due to a change in the applied load is shown in **Fig. 1**. By considering the variation in y_0 as a second degree equation, the modified neutral axis depth of the cracked section is expressed as follow.

$$\bar{y}_0 = a_1\beta^2 + c_1 \quad (9)$$

where:

a_1, c_1 are constants

$\bar{y}_0$ is modified neutral axis depth of the cracked section, measured from the top fiber (mm)

β is ratio of M_a / M_{cr}

The boundary conditions are: at $\beta = 0$, $\bar{y}_0 = y_1$ and at $\beta \geq 1$, $\bar{y}_0 = y_0$. Expressions for the modified neutral axis and moment of inertia of the cracked section are given in Eq. (11) and Eq. (12). The modified neutral axis profile at any section is given in Eq. (10).

$$\bar{y}_0 = \beta^2 y_0 + (1-\beta^2)y_1 \geq y_0 \quad (10)$$

$$\bar{I}_{cr} = \beta^2 I_{cr} + (1-\beta^2)I_{unc} \geq I_{cr} \quad (11)$$

$$\bar{y} = \frac{(\bar{y}_0 - y_1)}{x_1(L-x_1)}(Lx - x^2) + y_1 \quad (12)$$

where:

$\bar{I}_{cr}$ is modified moment of inertia of the cracked section corresponding to $\bar{y}_0$ (mm^4)

A similar method is used to get an expression for moment of inertia at a section.

$$I(x) = \frac{(\bar{I}_{cr} - I_{ucr})}{x_1(L-x_1)}x(L-x) + I_{unc} \quad (13)$$

where:

$I(x)$ is moment of inertia at a section (mm^4)

For old (already cracked) structures, the distribution of the neutral axis profile is independent of load position, and it does not move with load and is assumed to be unchanged since the section is already cracked by the maximum possible load experienced in the past [9]. In this case, the boundary conditions are: at x=0, and at $x = L,\ \bar{y} = y_1$, at $x = L/2,\ \bar{y} = \bar{y}_0$.

Upon substitution and simplification, the expressions for the neutral axis depth and moment of inertia at a section are given in Eqs. (14) and (15), respectively.

$$\bar{y} = \left(\frac{4x}{L^2}(L-x)\right)\bar{y}_0 + \left(1 - \left(\frac{4x}{L^2}(L-x)\right)\right)y_1 \quad (14)$$

$$I(x) = \left(\frac{4x}{L^2}(L-x)\right)\bar{I}_{cr} + \left(1 - \left(\frac{4x}{L^2}(L-x)\right)\right)I_{unc} \quad (15)$$

For uniformly distributed loads, since the neutral axis depth is related to bending moment, a parabolic neutral axis profile and variable moment of inertia along the longitudinal direction are assumed [9].

Experimental Investigation of Na Depth Profile of RC Beam

To verify the variation in the NA depth profile with the load position a RC beam was prepared, experimentally tested and the result was analyzed.

Materials

Concrete with a 28 days characteristic compressive strength, f'_c, of 31.82MPa and steel bar with yield strength of 528MPa were used.

Test Specimen

A test beam specimen with rectangular cross section of b x h= 500 x 485mm, with overall length of 3200mm and 2800mm distance between supports was prepared. Four deformed bars on the bottom and three deformed bars on top surfaces with 35mm in diameter were provided. For the stirrups, 16mm diameter deformed bars with a spacing of 200mm were used. Fig. 2 shows the cross section of the beam. Strain gauges for steel bars and concrete are attached at 0.4m intervals.

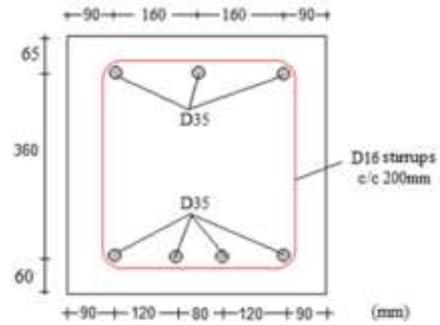

Fig. 2 Cross section of RC beam [9]

Strain gauges for steel and concrete at both top and bottom parts are attached. Locations of strain gauges are shown in Fig. 3.

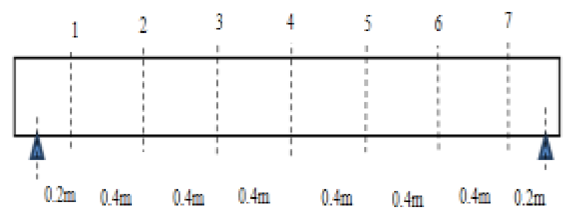

Fig. 3 Locations of strain gauges

Methods

To test the RC beam specimen, as per the recommendation of ASTM C 78 – 02, a standard test method for flexural strength of concrete beams using simple beam with three-point loading was used [10]. The specimen was simply supported at both ends and tested for with loading points symmetrically spaced at 400mm, 1200mm and 2000mm apart. The different load positions are shown in Table 1.

Table 1 Loading positions

Load position	1st load from left support (x)	Load spacing (y)
Position 1	1.2m	0.4m
Position 2	0.8m	1.2m
Position 3	0.4m	2.0m

RESULTS AND DISCUSSIONS

By varying the magnitude and position of loads, nine different cases (by varying load position and magnitude) have been considered. Initially, at the specified load positions, the beam was loaded with 70kN load (below cracking load). This test was repeated (Fig. 4) and subsequently a load beyond cracking load is applied at the same load positions. The maximum load applied was 300kN.

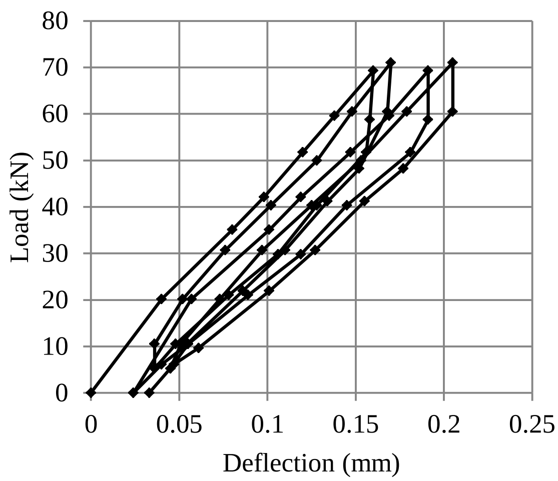

Fig. 4 Load deflection diagram of the test beam due to repeated loading

The neutral axis depth of reinforced concrete specimen at a particular section for different loading position is calculated from the strain distributions shown in Figs. 5-7 and the corresponding neutral axis depth profile is plotted. For other loading positions and values, similar procedure has been followed.

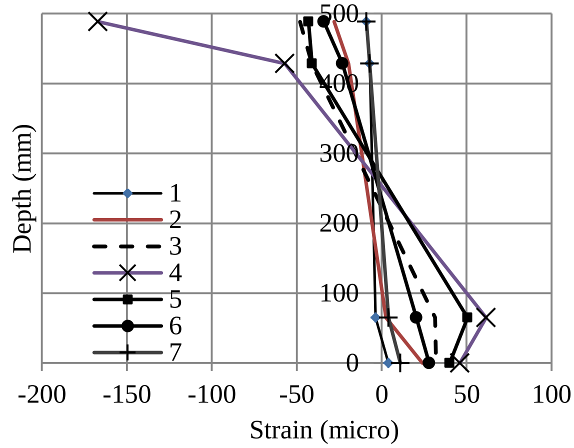

Fig. 5 Stain distribution at 70kN (Position-1)

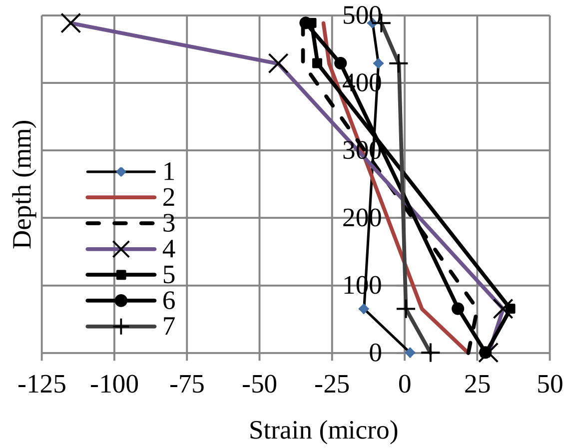

Fig. 6 Stain distribution at 70kN (Position-2)

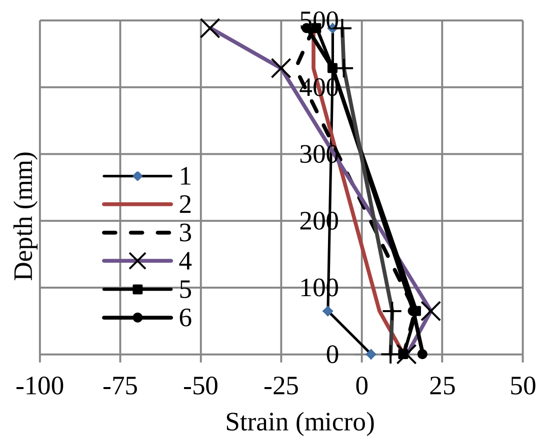

Fig. 7 Stain distribution at 70kN (Position-3)

The beam is unloaded and loaded to *P*=300kN until the section has fully cracked. The crack pattern of the test beam specimen at 300kN is shown in Fig. 8.

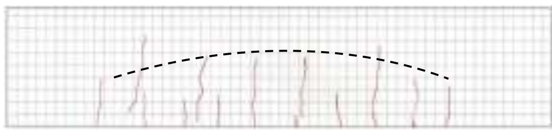

Fig. 8 Crack pattern at 300kN

As shown in Fig. 8, the crack pattern of the test beam at 300kN is parabolic and follows the assumption. Based on the experimental results, variations of NA profile for each loading case is drawn along the longitudinal profile of the beam are plotted and shown in Figs. 9-11. As shown in these figures, the neutral axis depth profile varies with the magnitude of the load. After the section has fully cracked the NA depth profile remains constant along the longitudinal profile of the beam.

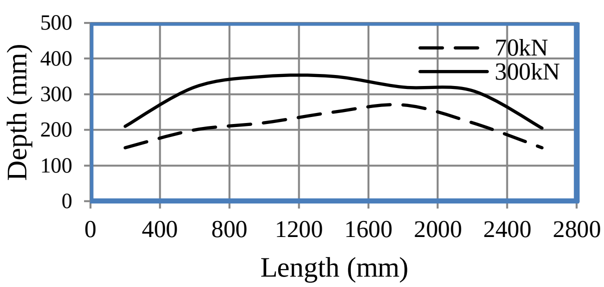

Fig. 9 Variation of NA Profiles (Position-1)

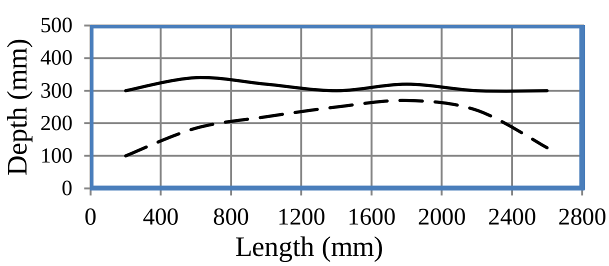

Fig. 10 Variation of NA Profiles (Position - 2)

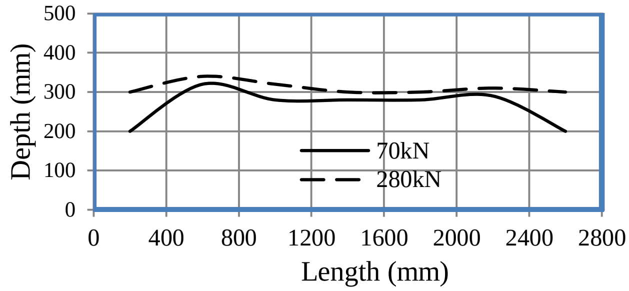

Fig. 11 Variation of NA Profiles (Position3)

As shown in Fig.11, as the spacing between the concentrated loads increases (1200mm), the NA depth profiles are almost in similar position.

Based on the strain readings of concrete and steel bars of the experimentally tested beam specimen at different sections, the contour showing the strain distributions are plotted. The strain distribution of concrete at the top and bottom fibers of the beam at a loading stage of 300kN is shown in Fig. 12. Moreover, the strain distributions of steel at the top and bottom reinforcement zones at the loading stage of 70kN and300kN are shown in Figs. 13-14. As shown in the figures, the NA depth profile along the longitudinal axis of the beam is variable and follows a parabolic path.

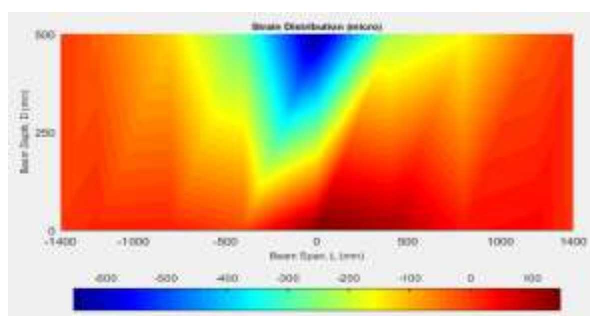

Fig. 12 Stress Distribution (Position-1, Concrete at 300kN)

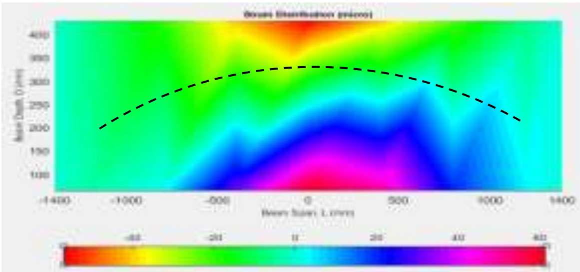

Fig. 13 Stress Distribution (Position-1, Steel at 70kN)

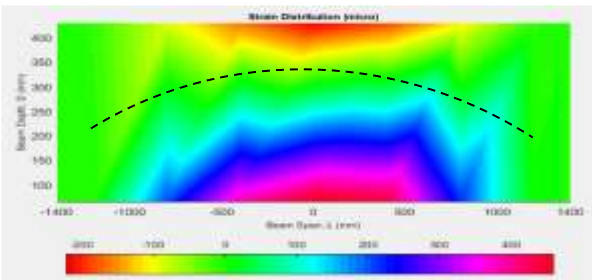

Fig. 14 Stress Distribution (Position-1, Steel at 300kN)

The NA depth for cracked and uncracked section become: $\bar{y}_0$ =156.72mm and y_1 =247.09mm, respectively. For different load positions, the NA depth profile of the RC beam was calculated numerically using Eq. (12) and compared with the experimental results (Fig. 15).

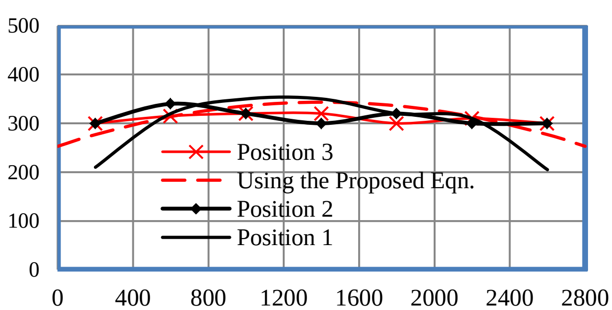

Fig. 15 Variation of NA Profiles

As shown in Fig. 18, it is observed that the NA depth profile of the experimental cases coincides with the values obtained using the numerical equation proposed by this study.

CONCLUSIONS

1) For limited data of experimental results, expression for the computation of neutral axis depth and moment of inertia of single-span simply supported RC beam have been obtained.
2) Comparison of empirical expressions of NA depth profile of a RC beam with experimental results has been carried out.
3) The experimental result reveal that NA depth profile of a RC beam varies longitudinally, moves with the load and in good agreement with the analytical curve.
4) During calculation of deflection of RC beams variation of NA depth profile along the longitudinal axis of the beam should be considered and variation of moment of inertia should be used accordingly.
5) Expressions for the computation of neutral axis depth and moment of inertia can be extended for beams with different end conditions.

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