MULTI-OBJECTIVE OPTIMIZATION OF TRAIN SPEED PROFILES ON THE AYAT-MEGENAGNA LINE

Tamirat Gebremariam and Mengesha Mamo

ABSTRACT

In this paper, a new approach has been developed for train speed profile optimization, discrete space based modeling followed by the determination of an optimal set of riding modes using multi-objective optimization techniques. The optimization problem is formulated by making energy and time as the components of the two element objective vector function. A point mass model of the operation of trains is developed by considering all the important force components acting on the train. The distance to travel between stations is discretized into 20 equal length elements where a two stage solution procedure has been applied to get to the final results. The first stage of the solution procedure is the application of a Non dominated Sorting Genetic Algorithm II (NSGA II) based optimization technique taking vector of riding modes as the decision variable. Using the developed algorithms for the calculation of cost functions, a Pareto-optimal set of riding modes are determined. The second stage of the solution process smoothes out the results found in the previous stage without bringing about considerable change in the values of the cost functions. Various speed profiles are generated as optimal for the case of Ayat to Megenagna line of Addis Ababa Light Rail Transit (AALRT). The speed profiles that are generated as the fastest can bring about up to 30% reduction in headway over the plan. Furthermore, by choosing the slowest trajectories over the fastest ones, it is possible to save up to 38.18% of energy, while 23.98% of reduction in riding time can be achieved by preferring the fastest profiles over the slowest ones.

Keywords: *Speed profile, energy consumption, running time, multi-objective, optimization, AALRT*

INTRODUCTION

The speed profile of a train is the speed versus distance or speed versus time curve that it undertakes while riding between stations.

There is usually a tradeoff between riding time and energy consumption of a train [1]. Since it is difficult to minimize both of these objectives at the same time, it has been important to come up with driving strategies that can be used to improve both of these antagonistic parameters.

The main objective of this research is to model train operation using point mass approach and distance discretization to determine multiple optimal trajectories for each section from Ayat to Megenagna. The scope of this research is limited to the case where a single train runs between stations.

The speed profile optimization problem considers a lot of constraints. These include maximum acceleration rate, maximum braking rate, maximum jerk, track alignment, speed restrictions, loading, train resistance, inter-station distance, headway, and signaling.

LITERATURE REVIEW

It has been shown that the optimal trajectory consists of only four types of riding modes [2]. These are motoring (acceleration), cruising, coasting and braking. Ko *et al*. [3] used dynamic programming in the optimization of train speed profiles. Wong and Ho [4] worked on the optimization of train running trajectory by the determination of multiple coasting points on an inter-station run. Analytical methods of solving the optimization problems were not accurate while genetic algorithm based techniques were not fast enough. R´emy Chevrier [1] used evolutionary algorithm to optimize the problem formulated using two objectives, energy and time. The distance to ride between consecutive stations is partitioned into sub-sections. The evolutionary algorithm is used to calculate three speed values within the discrete sections.

TRAIN KINEMATICS MODEL

The force components that act on the train include weight of the train, Tractive Effort (TE), train resistance, brake effort and adhesion [5, 6]. Let P be the maximum power developed by a motor in watts, n the total number of electric motors, η the transmission efficiency, M effective mass in kg, v speed of train in m/s, μ the adhesion coefficient, W the mass of the train in tons, then [4, 7, 8, 9 and 14]:

$$TE = min\left\{\left(\sum_{i=1}^{n} P_i\right) * \frac{\eta}{v}, \mu * M * 9.81 \; [N]\right. \quad (1)$$

$$M = 1.04 * Train\,Mass \; [kg] \quad (2)$$

$$\mu = 7.5/(v+44) + 0.16 \quad (3)$$

The most common train resistance calculation equation is formulated by the Canadian National Railway and is expressed as [10]:

$$r_r = 1.5 + 18n/W + 0.03V + CAV^2 / (10000W) \; [kg/ton] \quad (4)$$

$$R_r = 9.81 * W * r_r \; [N] \quad (5)$$

Where r_r is the rolling resistance of the vehicle in lb/ton, n is number of axles, W is the total weight of train in tons, V is the velocity in mph, A is the cross-sectional area of the train in square feet, while C=2.0 is the usual value for modern lightweight Passenger Equipment. It was found out that the resistance due to horizontal curvature was 0.8 lb/ton per track curvature (in degrees). The resistance due to vertical track gradient is given in equation 6 while Equation 7 shows the equation for the resistance component associated with horizontal track curvature. The vertical gradient angle is represented by θ. The total resistance as shown in equation 8 is the sum of all resistive components. Equation 9 represents the brake effort needed to be applied to bring the train to a stop [11].

$$R_g = 9.81 * M * sin\theta \; [N] \quad (6)$$

$$R_c = M * 9.81 * 700/radius [N] \quad (7)$$

$$R_{tot} = R_r + R_g + R_c \; [N] \quad (8)$$

$$B = 9.81 * M * 0.09 \; [N] \quad (9)$$

OPTIMIZATION PROCESS

a) Problem Formulation

The distance to travel is discretized into 20 elements. Let

$$j \in \{1,2,3,...20\} \quad (10)$$

$$\forall i \in \{1,2,3,...N_j\} \quad (11)$$

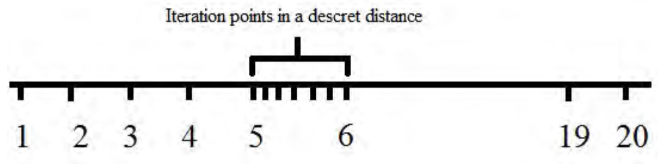

Figure 1. Distance discretization with 20 elements

Here, j represents the discrete distance element; i represents iteration points within a discrete distance element; while N_j is the maximum number of iterations within a discrete distance element. Let the total number of iterations in the last discrete element N_{20}=k, the optimization problem formulation can be expressed as:

$$Min(e,t) \quad (12)$$

Subject to:

Speed restrictions

$$v_{ji} \le v_{ji,max} \quad (13)$$

Boundary conditions

$$v_{11} \le v_{20k} = 20 \quad (14)$$

$$S_{20k} \le S_{final} \quad (15)$$

Non zero intermediate speed

$$v_{ji} > 0 \forall j \in \{2,3,...19\} \; and \; \forall i \in \{1,2,3,...N_j\} \quad (16)$$

Minimum average acceleration

$$\frac{v_{ji} - v_{11}}{t_{ji} - t_{11}} \ge \{\bar{a}_{min}\} \quad (17)$$

Minimum average deceleration

$$\left|\frac{v_{ji}-v_{20k}}{t_{ji}-t_{20k}}\right| \geq \bar{b}_{min} \tag{18}$$

Jerk limit

$$\left|\frac{a_{j+1i}-a_{ji}}{t_{j+1i}-t_{ji}}\right| \leq Jerk_{max} \forall j \in \{1,2,3,...20\} \; and \; \forall i \in \{1,2,3,...N_j\} \tag{19}$$

Where,

$$E = \sum_{j=1}^{20} \sum_{i=1}^{N_j} \left(TE_{ji} * v_{ji} * \left(v_{ji+1} - v_{ji}\right)/ a_{ji}\right) \tag{20}$$

$$a_{ji} = \left(TE_{ji} - R_{ji} - B_{ji}\right)/ M \tag{21}$$

$$t = \sum_{j=1}^{20} \sum_{i=1}^{N_j} \left(v_{ji+1} - v_{ji}\right)/ a_{ji} \tag{22}$$

b) Algorithms for the Speed Regimes

The motoring phase requires the application of maximum TE; while the Coasting and Braking phases have no need for the application of TE [1, 10]. Cruising phase requires the application of some amount of TE to make the train move at constant speed. If the train resistance is positive, the TE should be equal to the resistance. Otherwise, if the resistance is negative, the applied TE must be zero and braking force should be applied to compensate for the resistance value.

The following algorithms are used for the determination of cost for each type of driving regime in a discrete distance element. The output from each is used in the determination of the total cost using the flowchart in Figure 2. The tractive effort (T), Resistance (R) and Brake force (B) are calculated by using equations 1, 8 and 9, respectively.

Algorithm 1: Motor

Input: s_0: initial position of train, v_0: initial speed, t_0: initial time, e_0: initial energy, L: section length, M: mass of train, T: tractive effort, R: resistance

Output: Delta_e: energy expense, Delta_t: time travelled

$i = 1$

$Delta\_e = 0$

$while \; S_i < (S_0 + L) \;\&\&\; v_i <= 70kph$

$k = v_i$

$if \; T(k) > R(k)$

$v_{i+1} = v_i + 1$

$t_{i+1} = t_i + M / (T(k) - R(k))$

$S_{i+1} = S_i + v_i * (t_{i+1} - t_i)$

$Delta\_e = Delta\_e + T(k) * v_i * (t_{i+1} - t_i)$

$else\,if \; T(k) < R(k)$

$v_{i+1} = v_i - 1$

$t_{i+1} = t_i - M / (T(k) - R(k))$

$S_{i+1} = S_i + v_i * (t_{i+1} - t_i)$

$i = i + 1$

$Delta\_t = t_i - t_0$

Algorithm 2: Brake

Input: s_0: initial position of train, v_0: initial speed, t_0: initial time, e_0: initial energy, L: section length, M: mass of train, T: tractive effort, R: resistance

Output: Delta_e: energy expense, Delta_t: time travelled

$i = 1$

$Delta\_e = 0$

$$while\ S_i < (S_0 + L)\ \&\&\ v_i <= 70kph$$

$$k = v_i$$

$$if\ B(k) + R(k) > 0$$

$$v_{i+1} = v_i - 1$$

$$t_{i+1} = t_i + M/(B(k) + R(k))$$

$$S_{i+1} = S_i + v_i * (t_{i+1} - t_i)$$

$$else\ if\ B(k) + R(k) < 0$$

$$v_{i+1} = v_i + 1$$

$$t_{i+1} = t_i - M/(B(k) + R(k))$$

$$S_{i+1} = S_i + v_i * (t_{i+1} - t_i)$$

$$i = i + 1$$

$$Delta\_t = t_i - t_0$$

Algorithm 3: Cruise

Input: s_0: initial position of train, v_0: initial speed, t_0: initial time, e_0: initial energy, L: section length, M: mass of train, T: tractive effort, R: resistance

Output: Delta_e: energy expense, Delta_t: time travelled

$$i = 1$$

$$Delta\_e = 0$$

$$while\ S_i < (S_0 + L)\ \&\&\ v_i <= 70kph$$

$$k = v_i$$

$$v_{i+1} = v_i$$

$$t_{i+1} = t_i + 1$$

$$S_{i+1} = S_i + v_i$$

$$Delta\_e = Delta\_e + T(k) * v_i * (t_{i+1} - t_i)$$

$$i = i + 1$$

$$Delta\_t = t_i - t_0$$

Algorithm 4: Coast

Input: s_0: initial position of train, v_0: initial speed, t_0: initial time, e_0: initial energy, L: section length, M: mass of train, T: tractive effort, R: resistance

Output: Delta_e: energy expense, Delta_t: time travelled

$$i = 1$$

$$Delta\_e = 0$$

$$while\ S_i < (S_0 + L)\ \&\&\ v_i <= 70kph$$

$$if\ R(k) > 0$$

$$v_{i+1} = v_i - 1$$

$$t_{i+1} = t_i + M / R(k)$$

$$S_{i+1} = S_i + v_i * (t_{i+1} - t_i)$$

$$else\ if\ R(k) < 0$$

$$v_{i+1} = v_i + 1$$

$$t_{i+1} = t_i - M / R(k)$$

$$S_{i+1} = S_i + v_i * (t_{i+1} - t_i)$$

$$i = i + 1$$

$$Delta\_t = t_i - t_0$$

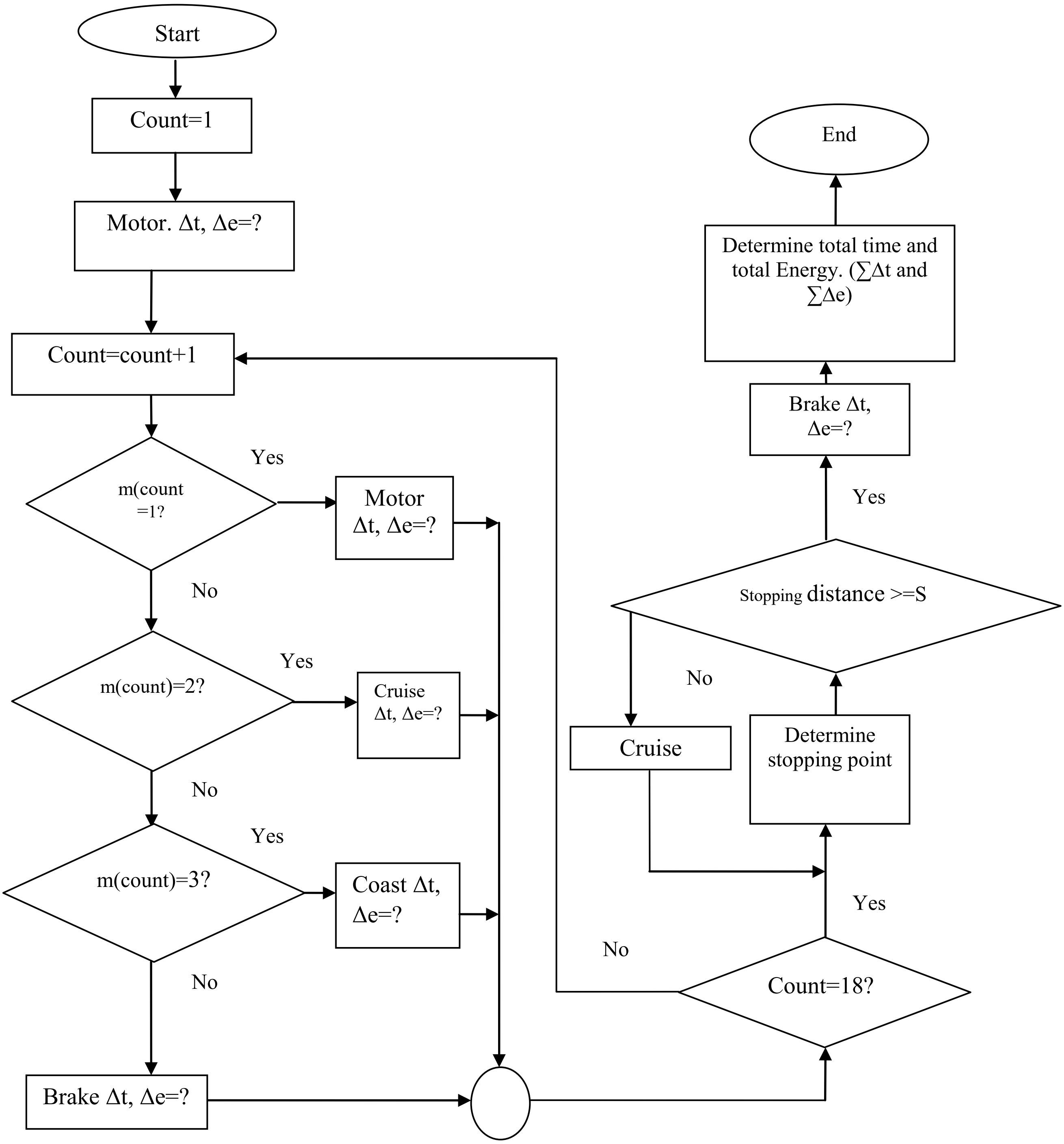

Figure 2. Flowchart of Speed profile construction and cost calculation

c) Vector of Riding Modes as the Decision Variable

The decision variable is made to be a vector of riding modes. Each riding mode is represented by an integer value ranging from 1 to 4, representing motoring, cruising, coasting and braking, respectively. Since a 20 section discrete space is used, the decision vector is a 20 element vector.

It is seen that this method of optimization of train speed profile is faster to converge as compared to the case where a vector of speed values is taken as the decision variable.

It is assumed that the starting mode of every journey should be motoring, while the final mode should be braking, to ensure that the train stops at the next station.

Non dominated Sorting Genetic Algorithm II (NSGA II) is used to arrive at well distributed set of riding modes as Pareto-optimal solutions. Constraints are handled implicitly within the calculation of costs and by using the smoothing procedure.

The algorithm is defined by the flowchart in **Figure 3**. A random population is initially generated. The population is sorted using the so- called fast-non-dominated-sort. To this purpose, for each vector of riding modes i, an integer value holding the number of solutions that dominate i is created (domination count) and a set with the individuals dominated by the individual i is calculated. With those parameters, each individual is assigned a rank representing the front to which it belongs. The Pareto front has rank 0. Those individuals dominated only by individuals from the Pareto front have rank 1. The best solutions have always rank 0 with this approach, so elitism is naturally fitted within the sort.

The diversity of the population is preserved by a parameter less crowded-comparison approach. The density of individuals surrounding a particular individual i is calculated as the perimeter of the hypercube formed by taking the nearest individuals to i as the hypercube's vertices. This quantity is called the crowding distance. An individual is considered to be better than another if and only if it has a lower rank or, having the same rank, if it has a higher crowding distance.

The best N (population) chromosomes are picked from the current population and put into a mating pool where tournament selection, cross over and mating is done. The mating pool and current population is combined. The resulting set is sorted, and the best N chromosomes make it into the new population. This procedure is repeated until a maximum number of generations have been reached [12].

d) Smoothing of Speed Profiles

The output of the optimization process is generally a speed profile that can have multiple switching points that are difficult to use in reality. Therefore, some techniques are used to make some changes on the trajectories so that applicable profiles can be generated without considerable variation on the cost metrics.

The coasting phase is usually a very small positive or negative acceleration. Hence, coasting regimes with small acceleration values can be approximated by a cruising regime. The cruising regime can be approximated by continuous acceleration and coasting regimes. Finally, shifting of riding modes can be applied without resulting in considerable deviation in cost values.

RESULTS

Some of the most important information about the Light Rail Vehicles (LRV) was collected. There are 3 cars per train with two power bogies. There are two axles per bogie which contain two electric motors. Power per electric motor is 130 KW.

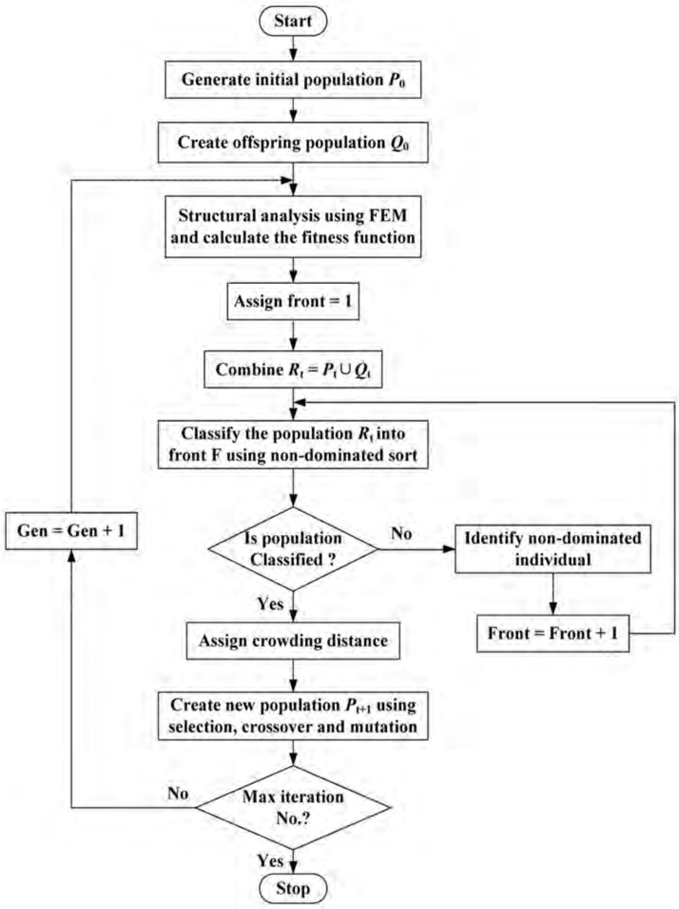

Figure 3. Flowchart of Non-dominated Sorting Genetic Algorithm II

Total laden mass of train is estimated to be 63.02 ton while mass per axle is 10.05 ton. The maximum speed of the train on a level track is 70 kph while level crossing speed limit is 50 kph.

The maximum jerk value is 1m/s^3. The maximum average acceleration value is limited to be 0.5 m/s^2 for $0<v<40$ kph, 1 m/s^2 for $0<=v<=70$ m/s^2 while the minimum average deceleration is limited to be 1m/s^2. Average dwelling time is 30 sec.

The maximum tractive effort of the train is calculated with respect to the speed of the train using equation 1.

Table 1. Track information of the Ayat-Megenagna railway line

Section	Location 1 (m)	Location 2(m)	Level Crossing Range
EW1-EW2	21050	22300	21940-21960
EW2-EW3	19940	21050	20067-20216
EW3-EW4	19080	19940	-
EW4-EW5	18220	19080	18990-19010
EW5-EW6	17500	18220	-
EW6-EW7	16520	17500	17142-17287
EW7-EW8	15440	16520	15790-15810
EW8-EW9	14600	15440	-

Table 1 shows the location of railway stations on the line. The track geometry from Ayat station to Megenagna station is made up of various gradients and curves. These geometric values, together with the speed and other train parameters are used in the calculation of train resistance as per equations 4-8.

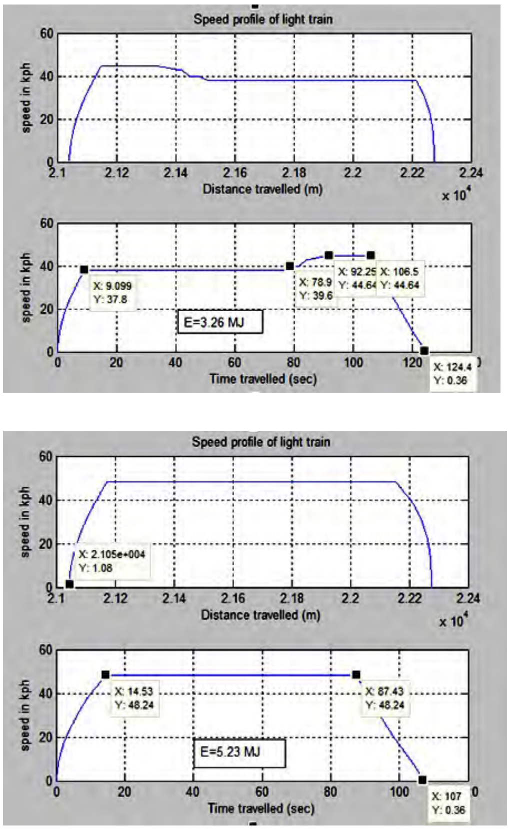

Figure 4: a) Speed profilea) Most energy efficient EW1-EW2 movement.b) Fastest EW1-EW2 movement.

CONCLUSIONS

The fastest riding times for the stations from EW1 to EW 9 are respectively, 108, 100, 79, 84, 81, 96, 105, and 75 seconds. Whereas the riding times for the sections from EW 9 to EW 1 are found to be 71, 99, 92, 114, 84, 70, 95 and 118 seconds, respectively.

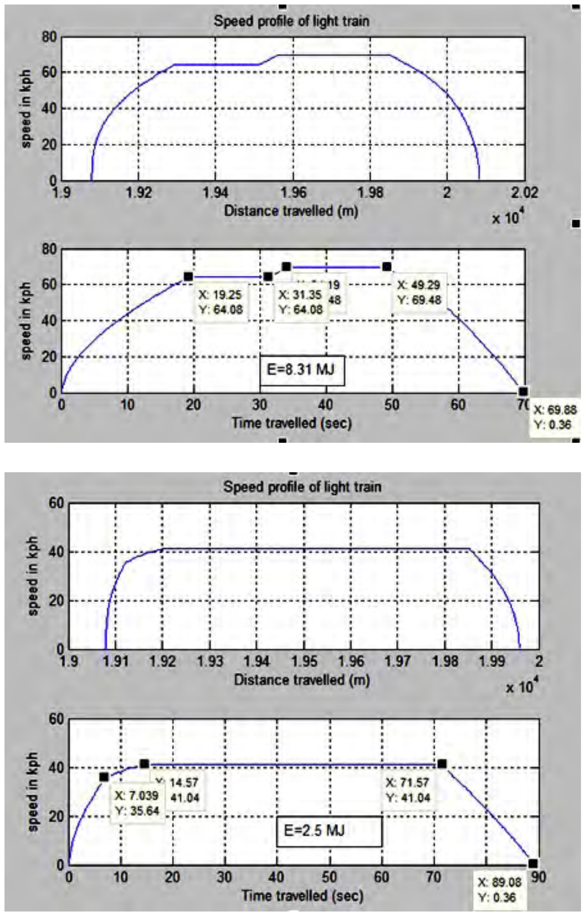

Figure 5. a) EW4-EW3 movement (the fastest of all) b) EW4 -EW3 movement (the most energy-saving)

Therefore, the mean fastest riding time between stations is 92 seconds. Considering a dwelling time of 30 seconds, total average riding time=122 seconds. By taking the operation of 41 trains on both lines in both directions, we will have a total of 21*4=84 sections. Therefore, headway between trains by using the fastest operation will be (84/41)*122 =250 seconds. This means that there is an improvement over the planned headway of 6 minutes. Percentage headway reduction = 30%.

Furthermore, by choosing the fastest trajectories over the slowest ones, it is possible to save up to 38.18% of energy, while 23.98% of reduction in riding time can be achieved by preferring the fastest profiles over the slowest ones.

RECOMMENDATIONS

The optimization techniques used in this paper can be applied for the optimal operation of trains on AALRT. In that case it can be possible to improve the capacity of the line as well as minimize the total energy consumption by the operation of trains.

The algorithms developed in this paper can also be used in other types of railway systems such as metro and Heavy Rail Transit (HRT). Further research can also be done to investigate the applicability of such techniques in Automatic Train Control (ATC) systems.

The speed profiles so generated by the optimization process can be used in the calculation of power supply demand and the specification of electrical equipment.

REFERENCES

[1] R. Chevier, "*An Evolutionary Multi-objective Approach for Speed Tuning Op timization with Energy Saving in Railway Management*", hal-00465116, Version 1, Mar. 19, 2012.

[2] P. Howlett, "An optimal strategy for the control of a train", J. Austral. Math. Soc., Ser. B, Vol. 31, ISSN 1442-4436, pp. 454 –471., 1990.

[3] H. Ko, T. Koseki, and M. Miyatake, "Application of dynamic programming to the optimization of the running profile of a train", Computers In Railways IX, vol. 15, pp. 103-112, 2004.

[4] K. K. Wong and T. K. Ho, "Dynamic coast control of train movement with ge netic algorithm", International Journal of Systems Science, 35(13-14). pp. 835-846, 200415, Universite Lille Nord de France, Oct. 2012.

[5] American Railway Engineering Associa tion, "*American Railway Engineering Man ual*", Washington, DC, 1999.

[6] Lipetz, "*Diesel Engine in rail Transporta tion*", Engineering Bulletin Research Se ries 49, Purdue University, Lafayette, In dia,1935.

[7] Remy Chevrier, Paola Pellegrini and Joaquin Rodriguez, "*Energy-saving in railway timetabling: A bi-objective evolutionary approach for computing alternative running times*", hal-00742039, version 1 –

[8] W. Hay, "*Railroad Engineering*", 2nd Ed., John Wiley & Sons, Inc., New York, 1982.

[9] Z. Filipovic, "*Electrische Bahnen", Springer-Verlag*, pp. 23-27, Berlin-Haidelberg-New York, 1995.

[10] Kitae Kim, "*Optimal Train Control on Various Track Alignments Considering Speed and Schedule Adherence*", Ph.D. dissertation, New Jersey Institute of Technology, Jan. 2010.

[11] Y. Wang *et al*. "*Optimal trajectory planning for trains using mixed integer linear programming*", Proceedings of the 14th international IEEE Conference on Intelligent Transportation Systems (ITSC 2011), pp. 1598–1603, Washington, DC, Oct. 2011.

[12] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. "*A fast and elitist multiobjective genetic algorithm: NSGA-II. Evolutionary Computation",* IEEE Transactions on, 6(2):182 -197, Apr. 2002.