CHART DEVELOPMENT FOR THE ESTIMATION OF DEFLECTION IN PRE-STRESSED RAILWAY GIRDER

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ABSTRACT

In designing pre-stressed girder, calculating deflection is the necessary and rigorous step which one has to calculate at the end of the design process and tell whether the design girder is serviceable or not. If it is not, the designer has to change the cross-sectional dimension and repeat the step once again. But if the designer can certainly know the deflection of that structure in advance of those steps, he/she can eliminate such iteration. This research is intended to develop a program that plot a deflection chart for simply supported pre-stressed railway girder. To achieve the objective of the research, a simple user friendly and versatile program is developed that plots a deflection chart for simply supported pre-stressed railway girder. With the help of the findings and using the program one can control the deflection or camber of the pre-stressed member with the magnitude of the applied load and varying the eccentricity of the tendon profile both at the support and at the mid-span. And for a given cross-sectional geometry of a bridge, a pre-stressed member has not only a maximum,, but also a minimum span length limit in order to serve the intended function without failure.

Keywords: Tendon, Pre-stressed girder, camber, deflection

INTRODUCTION

To increase the span of a member and to use the depth of the girder effectively, Addis Ababa LRT uses pre-stressed concrete girder. However, due to design limitation, difficulty of construction, and difficulty of hauling heavy pre-cast girder, and various factors, the span of the pre-stressed girder is limited. This leads to more closely-spaced piers in the road. And the main reason behind limited depth span is a deflection requirement by the pre-stressed girder. An accurate and precise determination of a beam-like structure helps the designer to limit the span of the structure based on the allowable deflection for that specific purpose. In designing a pre-stressed girder, calculating deflection is a necessary and rigorous step which one has to calculate at the end of the design process and tell whether the design girder is within the allowable limit. If it is not, the designer has to change the cross-sectional dimension and repeat the same step once again. But if the designer can certainly know the deflection of that structure in advance of those steps, he/she can eliminate such iteration. Therefore, providing a tool that can do this job is undoubtedly vital and has a great significance.

The objective of this research is, therefore, to provide a computer program that plots a deflection chart for a simply supported pre-stressed railway girder.

LITERATURE REVIEW

Pre-stressing is more a question of philosophy than a specific technique. It means preparing a structure to receive a load by applying a pre-emptive countervailing load. For instance, if it is known that a column will be deflected 100 mm to the left by applied loads, the designer can arrange a condition to bend it 50 mm to the right; the column then only has to be designed to resist a deflection of +/- 50 mm, rather than the full 100 mm [1].

Pre-stressed concrete is basically one in which internal stresses of a member has a suitable magnitude and distribution that are introduced so that the stresses resulting from external loads are counteracted to a desired degree [2].

A. Materials for Pre-Stressed Concrete

Concrete: Pre-stressed concrete requires concrete which has a high compressive strength at a reasonably early stage, with comparatively higher tensile strength than ordinary concrete. Low shrinkage, minimum creep characteristics and a high value of Young's modulus are generally deemed necessary for concrete used for pre-stressed members [2].

High tensile steel: For pre-stressed concrete members, the high-tensile steel used generally consists of wires, bars, or strands. The higher tensile strength is generally achieved by marginally increasing the carbon content in steel in comparison with mild steel.

B. Types of Pre-Stressing

Pre-stressing can be either Pre-tensioned or Post-tensioned based on method (system) of pre-stressing. *Pre-tensioning* – a method of pre-stressing concrete in which the tendons are tensioned before the concrete is placed. In this method, the pre-stress is imparted to concrete by bond between steel and concrete. *Post-tensioning* – a method of pre-stressing concrete by tensioning the tendons against hardened concrete. In this method, the pre-stress is imparted to concrete by bearing [2]. based on the interaction of tendon and concrete, they may be classified as Bonded and Non-bonded; based on the allowable tensile strength of concrete in the design, they may be classified as full pre-stressing and limited or partial pre-stressing; and based on the tendon profile, they can be classified as axial, eccentric, concordant, non-distortional, uniaxial, biaxial and triaxial, circular pre-stressing.

C. Advantages of Pre-Stressing

- Tension and cracking under service loads may be avoided or reduced to a low level, depending on the magnitude of the pre-stressing force. Avoiding of tensile cracks increase the efficiency of utilization the section compared to reinforced concrete [3].

- Downward deflections of beams and slabs underservice loads maybe avoided or greatly reduced [3].

- Fatigue resistance (i.e. the ability to resist the effect of repeated live loading due to, for instance, road and rail traffic) is considerably enhanced [3].

- Enable Segmental forms of construction in which different members of a structure This research is principally concerned with the design of pre-tensioned structures (sleepers), although mention is made of the option of post-tensioning when appropriate. Are constructed separately and joined to become a monolithic structure [3].

- Very high strength steel may be used to form the tendons. This results in lighter and slender members than is possible with reinforced concrete. The two structural features of pre-stressed concrete, namely high-strength concrete and freedom from cracks, contribute to the improved durability of the structure under aggressive environmental conditions. Pre-stressing of concrete improves the ability of the material for energy absorption under impact loads [3].

- Beam and slab sections may be smaller than in reinforced concrete, due mainly to the capacity to reduce deflection. In the long-span range, pre-stressed concrete is generally more economical than reinforced concrete and steel [3].

- Pre-stressed concrete possess improved resistance to shearing forces due to the effect of compressive pre-stress, which reduces the principal tensile stress. The use of curved cables, particularly in long-span members, helps to reduce the shear forces developed at the support sections [2].

- A pre-stressed concrete flexural member is stiffer under working loads than a reinforced concrete member of the same depth. However, after the onset of cracking, the flexural behavior of a pre-stressed member is similar to that of reinforced concrete member [2].

D. Pre-stress Loss

The initial pre-stress in concrete undergoes a gradual reduction with time from the stage of transfer due to various causes. This is generally referred to as 'loss of pre-stress'. A reasonably good estimate of the magnitude of loss of pre-stress is necessary from the point of view of design.

Types of pre-stress loss in pre-tensioned and post-tensioned members are summarized as follows. These values are assuming that over tensioning is done to counteract the effect of anchorage and shrinkage losses in a post-tensioning member.

Table 1 Pre-stress loss

Type of loss	Pre-tensioned		Post-tensioned	
1. Elastic shortening	Yes	4	No if all cables are simultaneously tensioned	0
			Yes if sequential jacking	1
2. Anchorage slip	No	0	Yes	
3. Friction	No		Yes	
4. Creep of concrete	Yes	6	Yes	5
5. Shrinkage of concrete	Yes	7	Yes	6
6. Relaxation of tendon	Yes	5 - 8	Yes	6 - 8
Total	22-25%		18-20%	

METHODOLOGY

First, different literatures have been reviewed and then Hand Calculation have been done using ACI (435) procedure. Then coding followed with an Excel Template (which also served as a means of checking the program) and a program is developed using MATLAB program language. Finally chart development using the program and analyzing their property.

L < 4m	I = 60
4 m < L < 39 m	$I = 125/\sqrt{L}$
L > 39 m	I = 20

ANALYSIS

A. Deflection of Pre-stressed Concrete Beam

$$\delta pi = -\frac{pi * L^2}{48 * Ec * Ig} * (-5 * ec + ee) \tag{1}$$

In the design of pre-stressed concrete structures, the deflections (i.e. both short-term and long -term) are usually the governing criteria in the determination of the required member sizes. The use of high strength materials which result in slender members leads to higher deflection. Because of this deflection, pre-stressed member should be carefully examined [4].

Critical variables that affect the magnitude of short-term deflection of a pre-stressed beam are the magnitudes of the strain or stress gradient or the curvature at a section and its variation along the span, which are a function of:

- The magnitude and distribution of the load
- The magnitude and eccentricity of the pre-stress,
- The length of the span,
- The size and configuration of the cross- section,
- Boundary conditions
- the concrete properties [4]

The four basic stress equation of pre-stress member are shown below. Using this, we can tell whether the section has cracked or crushed.

A. Tensile Stress at Transfer

$$-ftt \le \frac{pi}{Ac} - \frac{pi * e}{Zt} + \frac{Mg}{Zt} \tag{2}$$

Compressive Stress at Transfer

$$fct \ge \frac{pi}{Ac} + \frac{pi * e}{Zb} - \frac{Mg}{Zb} \tag{3}$$

Compressive Stress in Service

$$fcw \ge \frac{pe}{Ac} - \frac{pe * e}{Zt} + \frac{Mtot}{Zt} \tag{4}$$

Tensile Stress in Service

$$ftw \le \frac{pe}{Ac} + \frac{pe * e}{Zb} - \frac{Mtot}{Zb} \quad (5)$$

Based on these equations, we can tell whether the section has cracked or not. If the section has cracked, the effective moment of inertia has to be used and if not the gross moment of inertia of the section has to be used in deflection calculation.

The program works for all AASHTO I beams as presented in Appendix B of PCI Bridge Design Manual, i.e. from type I to type VI plus another custom-made beam is added in which it is important that the necessary dimensions are also incorporated by the user. In this case, the number of dimensions that the users are going to feed into the program is kept to the minimum to simplify the programs application. This custom-made I beam can have any dimension and shape (i.e. rectangular, or T-shape, or trapezoidal).

C. Loading

Super imposed dead-load – the values given in AREMA MRE volume 2 has been used.

Live load – Cooper loading E80 and E60 has been used for live rail loading as provided in AREMA manual for railway engineering volume 2.

Impact Load – the impact shall be equal to the following percentages of the live load [5]:

D. Deflection Calculation

Deflection calculation using elastic analysis

1. ***Due to initial pre-stressing***
2. ***Due to self-weight***

a. If the section is un-cracked

$$\delta dl = \frac{5wdl * L^4}{384 * Ec * Ig} \quad (6)$$

b. If the section is cracked

$$\delta dl = \frac{5wdl * L^4}{384 * Ec * Ie}$$

$$Ie = Icr + \left(\frac{Mcr}{Ma}\right)^3 * \left(Ig - Icr\right) \le Ig \quad (7)$$

$$\left(\frac{Mcr}{Ma}\right) = \left(1 - \frac{ftl - fr}{fl}\right) \quad (8)$$

ftl
$= total\ calculated\ stress\ in\ member$
$= fbim$

$$fl = calculated\ \ stress\ \ due\ to\ self\ \ weight\ \ load\ \ only = Mdl / Zb$$

$$Where\ np = Eps/Ec\ and\ \ \rho p = Aps/(b * dp) \quad (9)$$

$$dp = ec + yt + ds \quad (10)$$

Therefore net deflection at transfer is

$$\delta nett = \delta i + \delta dl \quad (11)$$

After an un-shored slab is cast,

3. ***Due to Super imposed dead-load.***

a. If the section is un-cracked

$$\delta sl = \frac{5wsl * L^4}{384 * Ec * Ig}$$

b. If the section is cracked

$$\delta sl = \frac{5wsl * L^4}{384 * Ec * Ie}$$

$$Ie \;=\; Icr + \left(\frac{Mcr}{Ma}\right)^{3} * \left(Ig - Icr\right) \;\le\; Ig$$

$$\left(\frac{Mcr}{Ma}\right) = \left(1 - \frac{ftl - fr}{fl}\right)$$

ftl
$= total\ calculated\ stress\ in\ member$
$= fbsm$

$fl = Mdl\ /\ Zb\ if\ the\ section\ cracks\ in\ this\ stage$

$fl = (\ Mdl + MSd\ )\ if\ the\ section\ cracked\ in\ the\ transfer\ stage$

$Where\ np = Eps\ /\ Ec\ and\ \rho p = Aps\ /(\ b * dp\ )$

$$dp = ec + yt + ds$$

$Therefore\ net\ deflction\ after\ un-shorted\ cast\ is$

$$\delta nets = \delta nett + \delta sl \quad (12)$$

Immediate service load deflection;

4. *Due to live load*

a. If the section is un-cracked

$$\delta ll = \frac{5wll * L^{4}}{384 * Ec * Igc}$$

b. If the section is cracked

$$\delta ll = \frac{5wll * L^{4}}{384 * Ec * Ie}$$

$$Ie \;=\; Icr + \left(\frac{Mcr}{Ma}\right)^{3} * \left(Igc - Icrc\right) \;\le\; Ig$$

$$\left(\frac{Mcr}{Ma}\right) = \left(1 - \frac{ftl - fr}{fl}\right)$$

$ftl = total\ calculated\ stress\ in\ member$
$= fbem$

$fl = Mll/Zb\ if\ the\ section\ cracks\ in\ this\ stage$

$fl = (\ Msl + Mll)/Zb\ if\ the\ section\ cracks\ after\ the\ slab\ is\ cast$

$fl = (\ Mdl + Msl + Mll)\ if\ the\ section\ is\ cracked\ in\ transfer\ stage$

$Where\ np = Eps/Ec\ and$
$\rho p = Aps/(b * dp)$

$$dp = ec + yt + ds$$

$Therefore\ net\ immediate\ defection\ at\ service\ is$

$$\delta netl = \delta nets + \delta ll \quad (13)$$

Long-term deflection:

Using Table 2-3 of PCI multipliers

- Net deflection at transfer ***(δnet) = δnet***
- Net erection deflection before superimposed dead-load ***(δne)***

$$\delta ne = \delta pi * 1.8 + \delta dl * 1.85 \quad (14)$$

- Net erection deflection immediately after super imposed dead-load (***δnes)***

$$\delta nes = \delta ne + \delta sl \quad (15)$$

- Net erection deflection immediately after service load (***δnel)***

$$\delta nel = \delta nes + \delta ll \quad (16)$$

- Net final deflection before superimposed dead-load ***(δnf)***

$$\delta nel = \delta pi * 2.2 + \delta dl * 2.4 \quad (17)$$

- Net final deflection immediately after superimposed dead load (***δnfs)***

$$\delta nef = \delta nf + \delta sl * 3 \qquad (18)$$

- Net final deflection immediately after service load (***δnfl)***

$$\delta nfl = \delta nfs + \delta ll \qquad (19)$$

Note:- The user puts in the necessary values and can read the deflection vs span length at different stages in Figure 1 of the program.

Note: - The upward deflection (camber) is assigned negative values and the downward deflection is assigned positive values.

For example, for the beam properties described in the graph and a span length of 18.29 m, the deflections at different stages are as follows:

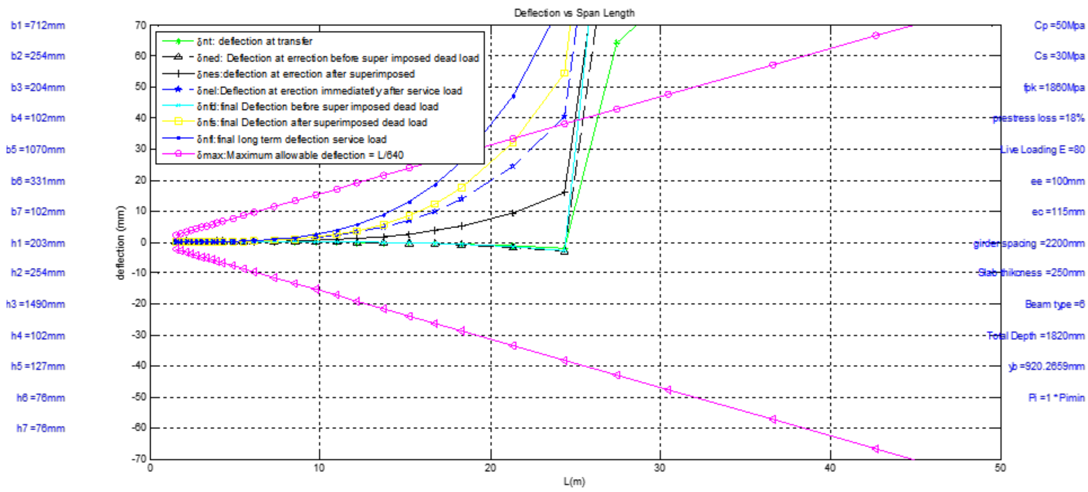

Figure 1 Typical Deflection Chart

Table 2 Deflection values at different stage

Load	At Transfer	PCI Multiplier	Erection	PCI Multiplier	δ Final	
	Deflection/Camber (mm)					
Pre-stressing Load Only (P)	δ_i = -2.40	1.80	-4.32	2.20	-9.51	
Self-Weight (Wd)	2.82	1.85	5.21	2.40	12.52	
δ_{net}	δ_{nt}= 0.42	Downward	δ_{ned}= 0.89	Downward	δ_{nfd}= 3.01	Downward
Super Imposed DeadLoad (Wsd)			6.01	3.00	18.02	
δ_{net} =			δ_{nes}= 6.90	Downward	δ_{nfs} = 21.03	Downward
Service Load (Wl)			16.99		16.99	
δ_{net}	0.42	Downward	δ_{nel} = 23.89	Downward	δ_{nfl}= 38.02	Downward

DISCUSSION

As the chart was developed for different beams something unexpected happened and that is, the deflection vs. span length chart indicated some strange behavior after a certain span length. It suddenly changes its dimensions and direction in an unanticipated manner. To know the reason behind this sudden shift, different values have been tried for the eccentricity of the tendon profile. For this reason, it was necessary to know the maximum and the minimum allowable range both at the support and mid- span. The algorithm was developed and the code was written for those parameters and a chart was plotted. Surprisingly, the maximum and minimum limits of the eccentricity will cross each other at some point both for mid-span and support section; that means there is a span length for which the maximum limit becomes less than the minimum, which is practically impossible.

At this stage, the calculation was repeated manually using a Microsoft Excel Template which was developed for checking purposes. But the result was still the same. That means the section has a span length limit for a given load beyond which it will no longer work or it will fail if one chose to build it with some eccentricity.

The reason behind the maximum span length limit is to avoid tensile failure of concrete at mid-span in which for a given pre-stressing force and given self-weight, the maximum eccentricity is limited to avoid the tensile failure at the top and the compressive failure at the bottom during transfer. But in this case, the tensile strength of concrete always dictates the maximum possible eccentricity. In other words, the maximum eccentricity is set to avoid tensile failure at the top during transfer. On the other hand, for a given pre-stressing force and a given service load, the minimum eccentricity is set to avoid the tensile failure at the bottom and compressive failure at the top during service. Yet again, the tensile strength of concrete comes to dictate the minimum possible eccentricity, which means that the minimum possible eccentricity is set to avoid the tensile failure at the bottom during service. However, after a certain span length, it becomes impossible to avoid tensile failure at the one surface while maintaining the safety of the other. It means that, after a certain span length, the maximum allowable eccentricity set to avoid the tensile failure at the top during transfer will be less than the minimum allowable eccentricity set to avoid the tensile failure at the bottom during service and vice versa. In addition to that, the maximum allowable pre-stressing force becomes less than the minimum allowable span length after this point and that was the reason behind this sudden shift of the slopes of the deflection curvem i.e. the member has tensioned beyond its tensile capacity and stated to develop a tensile crack.

Note: - the same thing happens for the support section, too. Since it happens after the mid-span section, it does not govern the maximum span length.

What is more interesting in this chart was the shift of the maximum and minimum eccentricity that will happen also prior to some span length, i.e. the maximum allowable eccentricity will be less than the minimum up to a certain span length. This means unlike in reinforced concrete which has no minimum span length limit structurally speaking, pre-stressed member has a minimum limit of span length due to the applied axial compression force.

The reason for the minimum span length limit is to avoid the crushing of concrete. As discussed above, for a given pre-stressing force, and given self-weight, the maximum outcome of eccentricity is limited to avoiding the tensile failure at the top and the compressive failure at the bottom during transfer. In this case the maximum eccentricity is set to avoid compressive failure at the bottom during transfer (The tensile failure at the bottom will also occur just right before the minimum span length). For a given pre-stressing force and a given service load, the minimum eccentricity is set to avoid the tensile failure at the bottom and compressive failure at the top during service. In this case, the minimum possible eccentricity is set to avoid the compressive failure at the top during service. However, before a certain span length the eccentricity needed to avoid the compressive failure at the bottom during transfer will be less than the minimum allowable eccentricity required to avoid the compressive failure at the top during service and vice versa.

If one uses the minimum pre-stressing force as an applied force, the maximum allowable mid- span eccentricity (e_{maxm}), the minimum allowable mid-span eccentricity (e_{minm}) and the maximum mid-span eccentricity corresponding to the minimum pre-stressing force (e_{maxc}) will be the same for a working range. Before or beyond that range, the maximum allowable mid -span eccentricity (e_{maxm}) will be less than the maximum mid-span eccentricity corresponding to the minimum pre-stressing force (e_{maxc}) which in turn is less than the minimum allowable mid-span eccentricity (e_{minm}).

That I, $e_{maxm} = e_{minm} = e_{maxc}$ for the possible working range and $e_{maxm} < e_{maxc} < e_{minm}$ before and beyond the possible working range,which means, if one has to choose the minimum pre-stressing force as an applied pre-stressing force in pre-stressing design, there is only one specific point that can be used as the eccentricity of tendon at the mid- span section. By range it is meant to say span length.

However, if one chooses to use the applied pre-stressing force higher than the minimum value. the maximum allowable mid-span eccentricity (e_{maxm}) will be higher than the minimum allowable mid-span eccentricity (e_{minm}) for the working range and the maximum allowable mid- span eccentricity (e_{maxm}) will be lower than the minimum allowable mid-span eccentricity (e_{minm}) outside of the working range.

Note: - the values on the chart after the break (the sudden shift) are just numerical and not true values.

- for the same input the user can also read the deflection envelope vs span length, the allowable eccentricities vs span length and the allowable pre-stressing force vs span length in Figure 2 of the program.

As can be seen in Figure, 2 the maximum allowable mid-span eccentricity/e_{max} mid-span is higher than the minimum allowable mid-span eccentricity/e_{min} mid-span in the range from 7m – 21 m. This means, with this type of loading and material property AASHTO type 6 I beam works in the range of 7.0 m to 21 m. When it comes to support eccentricity, the maximum allowable support eccentricity/e_{max} support is again higher than the minimum allowable support eccentricity/e_{min} support in the range of about from 7m to 33m. Based on the support eccentricity allowable working range from 7m to 33m however, since the minimum range governs, the working range of this beam is from about 7 m to 21 m.

As can be seen in Figure 3, the three eccentricities, - maximum allowable mid-span eccentricity (e_{maxm}), the minimum allowable mid-span eccentricity (e_{minm}) and the maximum mid-span eccentricity corresponding to the minimum pre-stressing force (e_{maxc}) - are the same for a working range. That means there is one possible tendon profile at mid-span, if we are going to use the minimum possible pre-stressing force as the one to be applied.

As the applied pre-stressing force increases, the upward deflection (camber) increases proportionally and the overall downward deflection decreases; and vice versa.

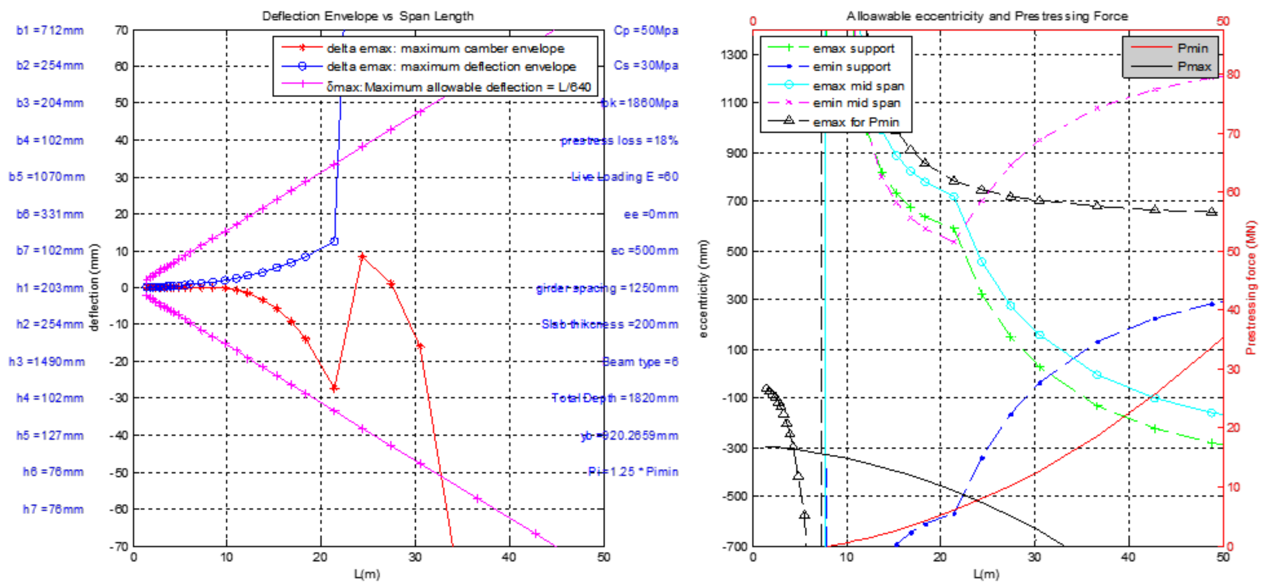

Figure 2 Deflection envelope (left), allowable eccentricity and pre-stressing force (right) for Pi = 1.25*Pimin

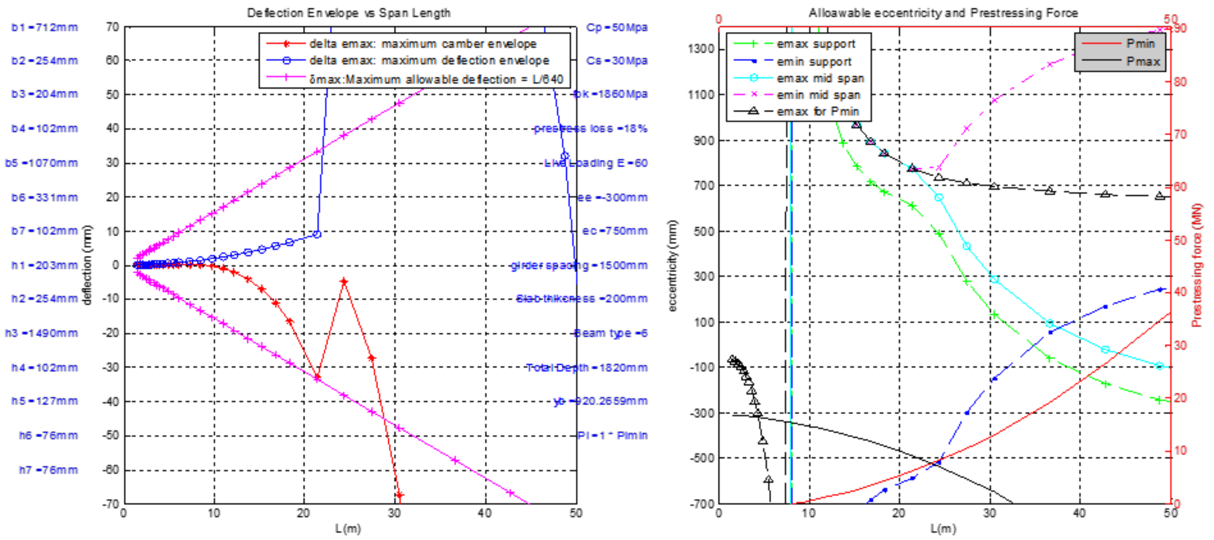

Figure 3 deflection envelope left, Allowable eccentricity and pre-stressing force right for $P_i = P_{imin}$

When the mid-span eccentricity increases to the positive value, the upward deflection (camber) increases proportionally and the overall downward deflection decreases, and vice versa. When the support eccentricity increases to the positive value, the upward deflection (camber) decreases proportionally and the overall downward deflection increases; and vice versa. When the support eccentricity increases in magnitude to the negative value, the upward deflection (camber) increases proportionally and the overall downward deflection decreases; and vice versa.

CONCLUSION

The objective of the research is achieved with a simple user friendly and versatile program that is developed to plot a deflection chart for a simply supported pre-stressed railway girder.

One can control the deflection or camber of the pre-stressed member with the magnitude of the applied load and varying the eccentricity of the tendon profile both at the support and at the mid-span points. And especially at the support area where we have a large range where we can fix the centroid of our tendon. When pre-stressing force increases, the upward deflection increases proportionally and the overall downward deflection decreases; and vice versa.

When the mid-span eccentricity increases to the positive value, the upward deflection increases proportionally and the overall downward deflection decreases, and vice versa. When the support eccentricity increases with the positive values, the upward deflection decreases proportionally and the overall downward deflection increases; and vice versa. When the support eccentricity increases in magnitude to the negative value, the upward deflection increases proportionally and the overall downward deflection decreases; and vice versa

For a given load and a given cross-sectional geometry, a pre-stressed member has not only a maximum span length limit, but also a minimum span length limit in order to serve successfully the intended function. This minimum limit is due to the applied axial compression force and upward deflection.

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