MODELING AND SIMULATION OF TRACTION OF POWER SUPPLY SYSTEM CASE STUDY: MODJO-HAWASSA RAILWAY LINE IN ETHIOPIA

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ABSTRACT

This paper presents the modeling and simulation of traction power supply system case study of Modjo-Hawassa railway line in Ethiopia. In this particular case a 2 × 25 kV autotransformer fed traction power supply system model is developed. The traction performance of a railway system depends on the variation of the voltage along the overhead lines. In this study, the voltage profile along the traction line has been evaluated. The system has been investigated by applying different headway distances on the trains and by increasing the number of train along the feeding section. The load flow simulation results using Matlab software show that the voltage profile in the feeding circuit differs substantially depending upon the train positions, train current, numbers of trains in the same power-feeding section and track impedance. The minimum calculated pantograph voltage and maximum percentage voltage regulation are 22.93 kV and 8.28% respectively with a maximum rail line potential of 66.2 V. These computed values are within the tolerable voltage range of the industry standard. Hence, the simulation result verified the validity of the system being adopted.

Keywords: *Modeling Autotransformer, Traction power supply, Voltage profile, Simulation*

NTRODUCTION

For more than a decade, railway system has experienced a renaissance in many countries after several years of stagnation period, including in Ethiopia. The main reasons for the renewed interest in the railway development are economic, environmental, and safety related. This has quite naturally, in turn, increased both passenger and freight transports on railway. In order to cope with this increase, large railway infrastructure expansions are expected. An important part of this infrastructure is the railway power supply system

without it, only the weaker and less energy efficient steam and diesel locomotives could be used [1]. An accurate power supply system analysis offers important information for planning, operation and design. Almost in all traction power supply system analysis or study the widely adopted trend and cost effective mechanism is simulation. In [2], widespread application which deals with many railway power feeding system simulators were discussed. In this study, a different approach, using MATLAB, is developed.

Several alternatives analysis methods are used for designing the electrical scheme of the power supply system for electrified railways [3]. Selection of an appropriate supply system is always very dependent on the railway system objectives. Many studies show that direct linking of the feeding transformer to the overhead catenary system and the rails at each substation is relatively simple and economical. Nevertheless, there are some drawbacks to this arrangement such as high impedance of feeders with high losses, high Rail-to-earth voltage and the interference to neighboring communication circuits [2,4].On the other hand, the autotransformer feeding configuration has many advantages and solves many disadvantages of the direct feeding system. The addition of autotransformer (AT) at every 8-15 km intervals improves the voltage profile along the traction line and increases the substation distance up to 50-100 km [4]. The electromagnetic interference in an AT system is normally much more lower compared with direct feeding (1 x 25 kV) system [5]. In addition, for high power locomotive and high speed trains direct feeding system is out of choice because of this most countries are replacing 1 x 25 kV system with 2 x 25 kV [6]. In this paper, analysis, modeling and simulation of autotransformer traction power supply system is presented and a comparative analysis in terms of voltage profile along the traction network is performed.

SYSTEM CONFIGURATION

In this system, the traction transformers are supplied from state grid, at 132 kV voltage levels. This voltage is further stepped-down to 55 kV at traction substation by using 132/55 kV transformers with center tap on the secondary side to have ±27.5 kV between the center-tap and the respective terminals. Each traction substation has two 132 kV independent power lines. The secondary terminal of the traction power transformers are selected to give a voltage of ±27.5 kV in order to compensate for any voltage drop caused by power supply line prior to the catenary system. The nominal voltage of the catenary system is considered to be 25 kV.

In addition, the system consists of center-tapped autotransformers located at every 15 km of which the outer terminals are connected between the catenary and feeder wire. The autotransformer-fed system enables power to be distributed along the system at higher than the train utilization voltage. As a nominal value, power is distributed at 55 kV (line-to-line) while the trains operate at 25 kV (line-to-ground). The system voltages for the proposed system conformed to European standards EN 50163: 2004 [7] and its values given by the above standards are as follows.

1. The nominal voltage shall be 25 kV.
2. The maximum permanent voltage allowed in the supply line shall be 27.5 kV.
3. The maximum non-permanent voltage that should be allowed for a short period of time shall be 29 kV.
4. The minimum permanent voltage shall be 19.0 kV.
5. The minimum non-permanent voltage that should be allowed for a short period of time shall be 17.5kV.

FORCES ACTING ON THE TRAIN

Train, as a load, is on the move and considered to be one of the main problems of longitudinal rail dynamics and is governed by the Fundamental Law of Dynamics applied in the longitudinal direction of the train's forward motion [8].

$$F_t - F_{ex} = m^* a \qquad (1)$$

The term to the left of the equal sign as shown in equation (1) is the sum of all the forces acting in the longitudinal direction of the train, where " F_t" is the tractive or braking effort and "F_{ex}" is the forces that opposes the forward motion of the train. "m^*" is the total mass (train mass + passenger or freight mass) of the train but due to rotational inertia effect the effective linear mass of the train increases and this value varies from 5% to 15% depending on the number of motored axles, the gear ratio and the type of car construction and "a" is the longitudinal acceleration experienced by the train. Various literatures [9, 10, 11] shows different countries use different starting acceleration that ranges from $0.08\ \mathrm{m/s^2}$ to $0.25\ \mathrm{m/s^2}$ for freight trains.

Forces against the train

The total forces acting on a train against its direction of motion (F_{ex}) can be expressed mathematically as follows:

$$F_{ex} = F_r + F_{gr} + F_c \qquad (2)$$

Where F_r is mechanical and aerodynamics resistance, F_{gr} is gradient resistance,, and F_c is curves resistance.

Mechanical and Aerodynamic Resistance

The force created due to mechanical and aerodynamic resistance, F_r is given by:

$$F_r = A + Bv + cv^2 \qquad (3)$$

Generally $A + Bv$ is rolling resistance and cv^2 is aerodynamic resistance. The value of A can be approximately computed as [12]:

$$A \approx 2450 + 175 N_{axle} (\mathrm{N}) \qquad (4)$$

N_{axle} is number of trailing car axle and coefficient B can be expressed as a function of total train length rather than train mass.

$$B \approx -22 + 0.58 L_T (N\, s/m) \qquad (5)$$

Where,L_T is the total length of the train. The aerodynamics drag, the part which dependent upon the speed squared is usually written for no wind condition as:

$$F_D = \frac{1}{2} \rho A_f C_D v^2 = cv^2 \qquad (6)$$

Where A_f is the projected cross sectional area, C_D is the air drag area and ρ is the air density which is equal to $1.3\mathrm{kg/m^3}$.

Gradient Resistance

Gradient resistance is also the component of the train load against the direction of travel. It is positive for uphill gradients and negative for downhill gradients (i.e. pushes the train forward). Thus the gradient resistance is determined by:

$$F_{gr}= i10^{-3}mg \qquad (7)$$

Where $i$the percentage gradient, m is is the mass of the train and g is gravitational acceleration. For freight and passenger lines, the location where the maximum gradient needs to be used in design. Therefore, for the profile design of railway, we need to lower the maximum gradient to ensure the freight train passes through this section at no less than calculated speed.

Curve Resistance

Additional curving resistance F_c mainly corresponds to the increased energy dissipation that occurs in the wheel rail interface, due to sliding motions (creep) and friction phenomena, at curve negotiation. It is dependent on wheel rail friction and the stiffness and character of the wheel set guidance. The resistive force produced by the curve is modeled by the following equation [12]:

$$F_c = \frac{k_e}{r} 10^{-3}mg \qquad (8)$$

Where k_e (m) is the track gauge coefficient and r is the radius of the curve.

Maximum Tractive Effort

The tractive effort can be increased by increasing the motor torque but only up to a certain point. Beyond this point any increase in the motor torque does not increase the tractive effort but merely cause the driving wheels to slip. The transmitted force is limited by adhesion and the maximum force that can be transmitted can be written as [13]:

$$F_{t-max} = \mu_a . mg \qquad (9)$$

Where m, μ_a are mass of the train in ton (t) and coefficient of adhesion respectively. The adhesion coefficient μ_a can be found based on Curtius and Kniffler derived adhesion curve in [14].

$$\mu_a = 0.161 + \frac{7.5}{3.6v+44} \qquad (10)$$

Power Demand of the Train

The maximum tractive force of the locomotive multiplied by the train velocity gives the maximum power that a locomotive consumed. The mechanical tractive power of the motor is computed by [15]:

$$P_{motor} = F_{t-max}. v. (1 + \xi) \qquad (11)$$

Where v is the speed of the train, F_{t-max} is the maximum tractive force of the locomotive and ξ is slippage ratio. In order to obtain more realistic results some losses and auxiliary power consumptions must be taken into account. These losses will be modeled with the parameter η_{loco} which is the locomotive's efficiency.

$$P_D = \frac{P_{motor}}{\eta_{loco}} \qquad (12)$$

The auxiliary power consumption, P_{aux} will be considered as well in the calculation and includes the cooling systems, train heating and the power available for travelers (for the case of passenger train). The electrical active power demand will be:

$$P_D = \frac{P_{motor}}{\eta_{loco}} + P_{aux} \qquad (13)$$

Fig. 1 shows Matlab simulation results of tractive effort and resistance force with respect to the speed of the train.

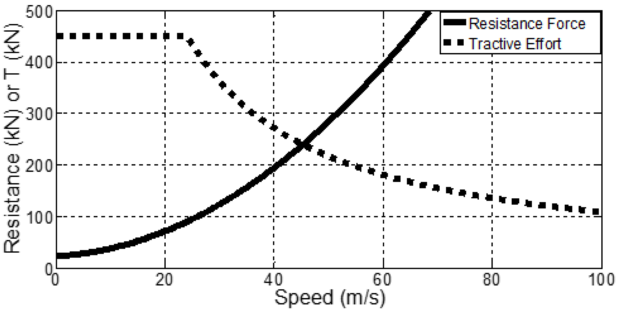

Fig. 1: Tractive effort (Resistance) Vs speed

MODELING OF TRACTION POWER SUPPLY SYSTEM

Mathematical Model

Electrical power system analysis is always depends on a mathematical model which is mainly comprises a mathematical equations that defines the relationship between various electrical power quantities with the required precision. Therefore, based on the objective of the electrical power system analysis various models for a given system may be applicable. In this section, the traction power supply system mathematical model is presented.

Train (Locomotive) Model

In order to reduce complexity in the analysis, a constant power model is used, in which the train power and power factor are assumed constant.

$$\text{Train current} = I_{TR} = \left(\frac{S_{TR}}{V_C - V_R}\right)^* \quad (14)$$

$$\text{Contact line current} = I_C = I_{TR} \quad (15)$$

$$\text{Rail line current} = I_R = -I_{TR} \quad (16)$$

$$\text{Negative feeder current } I_F = 0 \quad (17)$$

$$\begin{bmatrix} I_C \\ I_R \\ I_F \end{bmatrix} = I_{TR} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad (18)$$

Where S_{TR}, V_R, V_C are power demand of the train, rail voltage and the contact line voltage respectively.

Model of Substation Power Transformer

Today, there are many transformers which are used in railway power supply system such as open delta or Vv, Scott, YNd11, Wood-Bridge, single phase transformers, etc [15]. These different transformers are used in various feeding configurations, for example, in direct feeding system, Vv transformers are the best choice whereas in an autotransformer feeding arrangement, the three winding specially built single phase traction power transformer are used. In this type of transformer arrangement, the primary terminals of the transformer have a voltage rating of may be 132, 275 or 400 kV and the secondary terminal voltage is 55 kV. For the line being studied, the single phase three-winding transformer is used with the voltage designated as 132 kV/27.5 kV-0-27.5 kV. Fig. 2 illustrate such connections.

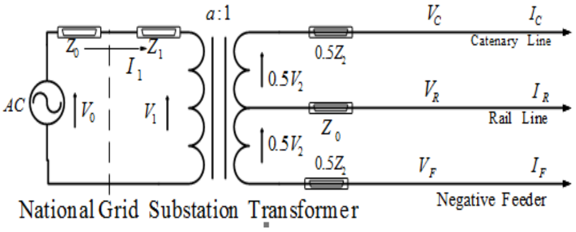

Fig. 2: Substation Transformer

For the substation power transformer, the Norton equivalent circuit in the multi-conductor model is formed as given in equation 3 defining $Z_A = Z_O + Z_1 + 6Z_2$ and $Z_B = 6Z_2 + 24Z_e$, where Z_O is short circuit impedance of the high voltage grid, Z_1 and Z_2 are impedance of the primary and secondary windings respectively, Z_e is impedance connected between the center tap of the second winding and the rails and a is Transformer's turn ratio. voltage respectively. In addition, J_{SS} is the substation current injected model, VSS is the substation voltage and Y_{SS} is the substation admittance. The short circuit impedance of the grid is assumed zero, this is equivalent to assuming the primary voltage of the substation transformer an infinite bus with a voltage of 132 kV. The no load secondary voltage between the overhead catenary system and feeder is 55 kV with a grounded center tap. The nameplate data of the substation transformer are to be 132/55 kV, 60 MVA calculated based on annual transportation demands, X/R ratio equal to 10 based on ANSI/IEEE C37.010-1979 and the impedance of the traction transformer will be 15 % based on IEC 60076 standards [16].

Model of Autotransformer

The autotransformer has a single winding connected between the catenary and the feeder wires. The rail system (rails and grounded wires) is connected to a center point on the winding. The usual voltage rating is 50 kV supply between the catenary and feeder with a transformation ratio of 2:1 to obtain a 25 kV catenary to rail and from rail to the return feeder.

The station where the autotransformer is located is also called paralleling station because the two tracks (C wire and F wire) are connected in parallel. Thus, the admittance matrix of the autotransformer becomes:

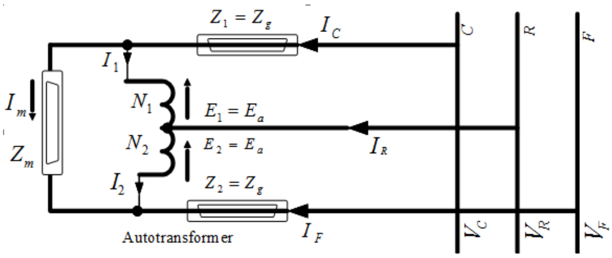

Fig. 3: Autotransformer model

Z_1, Z_2 are primary and secondary leakage impedances, Z_m is magnetizing impedance, I_m is magnetizing current, I_1, I_2 are primary and secondary current, E_1, E_2 are electromotive forces windings and N1, N2 are turns of the two windings. Assuming that $N_1 = N_2$ and $Z_1 = Z_2 = Z_g$, therefore

$$I_C + I_R + I_F = 0 \quad (27)$$

$$\left[V_F - V_C = 2E_a + Z_g(I_F - I_C) = Z_m I_m + 2Z_g(I_F - I_C)\right] \quad (28)$$

Substituting equation (27) to (28), we can easily find the following equations

$$\left[I_C = \left(\frac{1}{2Z_g} - \frac{1}{Z_m+2Z_g}\right)V_C - \frac{1}{Z_g}V_R + \left(\frac{1}{2Z_g} + \frac{1}{Z_m+2Z_g}\right)V_F\right] \qquad (29)$$

$$I_R = -\frac{1}{Z_g}V_C + \frac{2}{Z_g}V_R - \frac{1}{Z_g}V_F \qquad (30)$$

$$\left[I_F = \left(\frac{1}{2Z_g} - \frac{1}{Z_m+2Z_g}\right)V_C - \frac{1}{Z_g}V_R + \left(\frac{1}{2Z_g} + \frac{1}{Z_m+2Z_g}\right)V_F\right] \qquad (31)$$

Thus,

$$\mathrm{Y_{AT}} = \begin{bmatrix} \frac{1}{2\mathrm{Z_g}} - \frac{1}{\mathrm{Z_m}+2\mathrm{Z_g}} & -\frac{1}{\mathrm{Z_g}} & \frac{1}{2\mathrm{Z_g}} + \frac{1}{\mathrm{Z_m}+2\mathrm{Z_g}} \\ -\frac{1}{\mathrm{Z_g}} & \frac{2}{\mathrm{Z_g}} & -\frac{1}{\mathrm{Z_g}} \\ \frac{1}{2\mathrm{Z_g}} - \frac{1}{\mathrm{Z_m}+2\mathrm{Z_g}} & -\frac{1}{\mathrm{Z_g}} & \frac{1}{2\mathrm{Z_g}} + \frac{1}{\mathrm{Z_m}+2\mathrm{Z_g}} \end{bmatrix} \qquad (32)$$

$$\begin{bmatrix} \mathrm{I_C} \\ \mathrm{I_R} \\ \mathrm{I_F} \end{bmatrix} = \mathrm{Y_{AT}} \begin{bmatrix} \mathrm{V_C} \\ \mathrm{V_R} \\ \mathrm{V_F} \end{bmatrix} \qquad (33)$$

Autotransformer components are modeled by their equivalent circuits in terms of inductance, and resistance. The magnetizing impedance Z_m of the autotransformer is taken as infinite and also the impedance $Z_1 = Z_2$ equal because the two windings are similar. An earthling resistance Z_e is assumed to form the center-tap to remote earth. The calculated results of the AT ratings are: 50/25 kV, 10 MVA, with 7.5 % impedance and an X/R ratio of 10.

Calculation of Impedance of Overhead Conductors

In 1923 Carson published an impressive paper which discussed the impedance of the overhead conductor with earth return [6]. This paper has been used in many researches for the calculation of the impedance of the overhead power supply line in cases current flows through the earth especially in the railway system where significant amount of current flows via the earth to the traction substation [17][18]. In this paper Carson line model has been used for impedance calculation of the traction network. The following Carson equation is used to $\mathrm{I_1}$ is the primary current of the transformer caused by two secondary current, the contact line current $\mathrm{I_C}$ and the negative feeder current $\mathrm{I_F}$ as follows.

$$I_1 = \frac{1}{2a}(-I_C + I_F) \qquad (19)$$

Consider the primary-side circuit, using Kirchhoff's law, and replacing $\mathrm{I_1}$ in equation (14), we will have,

$$V_1 = V_O - \frac{1}{2a}(Z_O + Z_1)(-I_C + I_F) \qquad (20)$$

Using Kirchhoff's law on the secondary-side contact to rail line (C-R) circuit,

$$V_C - V_R + \frac{1}{2}V_2 + \frac{1}{2}Z_2 I_C + Z_e(I_C + I_F) = 0 \qquad (21)$$

By defining

$$Z'_1 = Z_O + Z_1 \qquad (22)$$

Based on the assumption that all currents are supplied by the substation and eventually returned to the substation, that is

$$I_C + I_R + I_F = 0 \qquad (23)$$

By defining

$Z_A = Z'_1 + 6Z_2$ and $Z_B = 6Z_2 + 24Z_e$ thus,

$$[(-Z_A + Z_B)I_C + (Z_A + Z_B)I_F = 24V_F - 24V_R + 5V_O] \qquad (24)$$

The above Equations (19-24) can be written in the other form as the Norton equivalent circuit as shown in equation (25)

$$\begin{bmatrix} \mathrm{I_C} \\ \mathrm{I_R} \\ \mathrm{I_F} \end{bmatrix} = 2.5\frac{\mathrm{V_O}}{\mathrm{Z_A}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} \frac{6}{\mathrm{Z_A}} + \frac{6}{\mathrm{Z_B}} & \frac{12}{\mathrm{Z_B}} & \frac{6}{\mathrm{Z_A}} - \frac{6}{\mathrm{Z_B}} \\ \frac{12}{\mathrm{Z_B}} & \frac{-24}{\mathrm{Z_B}} & \frac{12}{\mathrm{Z_B}} \\ \frac{6}{\mathrm{Z_A}} - \frac{6}{\mathrm{Z_B}} & \frac{12}{\mathrm{Z_B}} & \frac{6}{\mathrm{Z_A}} + \frac{6}{\mathrm{Z_B}} \end{bmatrix} \begin{bmatrix} \mathrm{V_C} \\ \mathrm{V_R} \\ \mathrm{V_F} \end{bmatrix} \qquad (25)$$

$$\mathrm{I_{SS}} = \mathrm{J_{SS}} + \mathrm{Y_{SS}}\mathrm{V_{SS}} \qquad (26)$$

Where $\mathrm{I_C}$, $\mathrm{I_R}$ and $\mathrm{I_F}$ are the contact line current, the rail line current and the negative feeder current and $\mathrm{V_O}$, $\mathrm{V_C}$, $\mathrm{V_R}$ and $\mathrm{V_F}$ are the nominal voltage, contact line voltage, rail line voltage and negative feeder voltage respectively. In addition, $\mathrm{J_{SS}}$ is the substation current injected model, VSS is the substation voltage and $\mathrm{Y_{SS}}$ is the substation admittance. The short circuit impedance of the grid is assumed zero, this is equivalent to assuming the primary voltage of the substation transformer an infinite bus with a voltage of 132 kV. The no load secondary voltage between the overhead catenary system and feeder is 55 kV with a grounded center tap. The nameplate data of the substation transformer are to be 132/55 kV, 60 MVA calculated based on annual transportation demands, X/R ratio equal to 10 based on ANSI/IEEE C37.010-1979 and the impedance of the traction transformer will be 15 % based on IEC 60076 standards [16].

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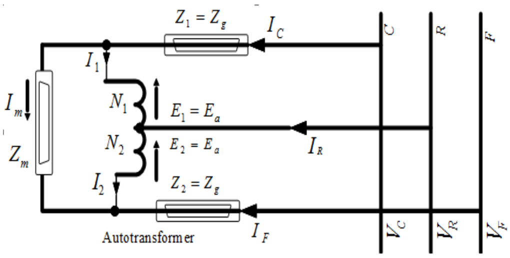

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$$r_d = \pi^2.10^{-4}.f\ ohm/km \tag{34}$$

The self-impedance (Z_{aa}) of wire a with earth return can be expressed as:

$$Z_{aa} = (r_d + r_a) + j2\omega k\ ln\left({}^{D_{ad}}\!/_{r_o}\right)\Omega/L \tag{35}$$

Where r_o is the radius of the conductor (m) and D_{ad} is equivalent conductor at depth. Also, $k = 2.10^{-7}$,$\boldsymbol{\omega} = 2\pi f$ finally, the mutual impedance (Z_{ad^1}) is stated as:

$$Z_{ad^1} = j\omega k(ln\left({}^{1}\!/_{D_{sa}}\right))\ ohm/L \tag{36}$$

The quantity $\mathbf{D_{ad}}$ is a function of both the earth resistivity ρ_e and the frequency **(f)** and is defined by the relation

$$D_{ad} = 1.309125 \times \delta \tag{37}$$

$$D_{ad} = 658.87 \times \sqrt{\frac{\rho_e}{f}}\ m \tag{38}$$

If no actual earth resistivity data is available, it is a common practice or a thumb rule to consider ρ_e **as** 100 ohm-meter. The earth resistivity ρ_e depends on the nature of the soil.

1. **Contact wire (C)**: Part of the overhead contact line system which establishes contact with the current collector. To avoid errors, the impedances of the messenger and the contact wire are calculated independently.
2. **Messenger wire (M)**: Parts of the overhead contact line system used to support the contact wire.
3. **Rails (R)**: The two rails in the same track are also treated as an independent conductor. They are connected to autotransformers at the center tap.
4. **Negative Feeder (F)**: Besides the catenary, another outer tap of the autotransformer is connected to the feeder wire..

The conductor configuration or arrangement of the overhead line is based on the industry standards conductor clearances. According to IEC the minimum electrical clearances of the conductor must be maintained under all line loading and environmental conditions. Since the actual sag clearance of conductors on overhead contact line is seldom monitored, sufficient allowance for this clearance (safety buffer) must be considered in the process of the initial design.

Minimum horizontal and vertical distances from energized conductor ("electrical clearances") to ground, other conductors, vehicles, and objects such as buildings, are defined based on three parameters. Clearances are defined based on the transmission line to ground voltage, the use of ground fault relaying, and the type of object or vehicle expected within proximity of the line. The IEC 270 Rules cover both vertical and horizontal clearances to the energized conductors [19].

The electric static clearance, which is the minimum distance required between the live parts of the overhead wire equipment and structure or the earthed parts of the overhead wire equipment under 25 kV must be at the minimum 320 mm as per IEC 270. The minimum electrical clearance to earth or another conductor is 150 mm under adverse condition and the minimum clearance between 2 parallel wires in open overlaps is 250 mm but may be reduced to 150 mm absolute minimum under the worst case.

The international standards covering most conductor types are IEC 61089 (which supersedes IEC 207, 208, 209 and 210) and EN 50182 and EN 50183[20][21]. In this paper, for all negative feeder wire, earth wire and messenger wire aluminum conductor steel reinforced (ACSR) has been used. ACSR has been widely used because of its mechanical strength, the widespread manufacturing capacity and cost effectiveness. Hard Copper wires are used for the overhead contact lines, which has a very high strength, corrosion resistance and is able to withstand desert conditions under sand blasting.

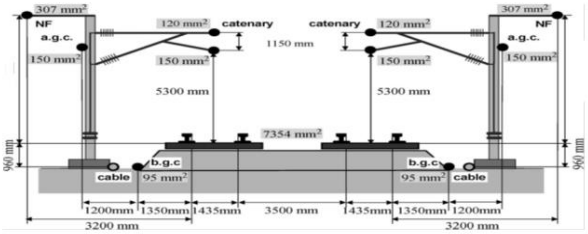

Fig. 4: Configuration of the catenary system

Modeling Using Matlab Autotransformers

Matlab software does not have an autotransformer model in its library for this reason a two winding linear transformer as shown below in figure 5 connected and used as autotransformer [7].

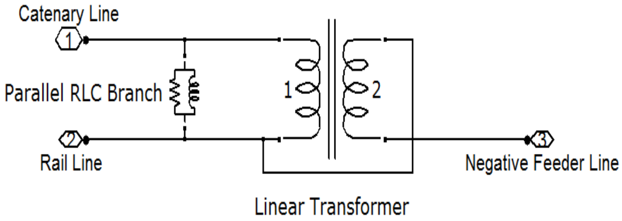

Fig. 5: Autotransformer model using two winding transformer

Power Transformer Model

In this section the traction substation power transformer model as shown on the figure 5 below is presented. The Matlab Simulink library does not have an exact substation transformer that have seen in the mathematical modeling section but the linear transformer which has three windings, the primary winding at the input side and two secondary winding is appears to the perfect match. The secondary windings are connected in such a way as shown in Fig. 6 to form the center tap.

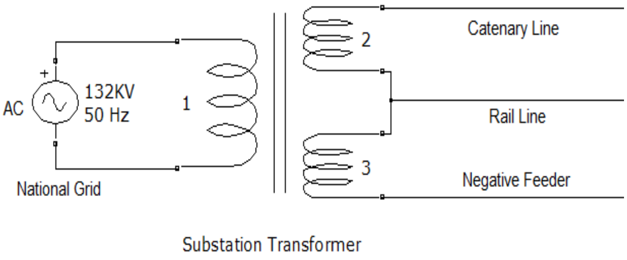

Fig. 6: A Substation transformer model

Overhead Catenary System Model

To model the catenary system MATLAB/ Simulink mutual inductance element which is shown in the Fig. 7 below is used. As explained earlier the traction power supply system uses five conductors, which give 25 full impedance matrixes, from which five of them are self-impedance and twenty of them are mutual impedance as shown in Table 1.

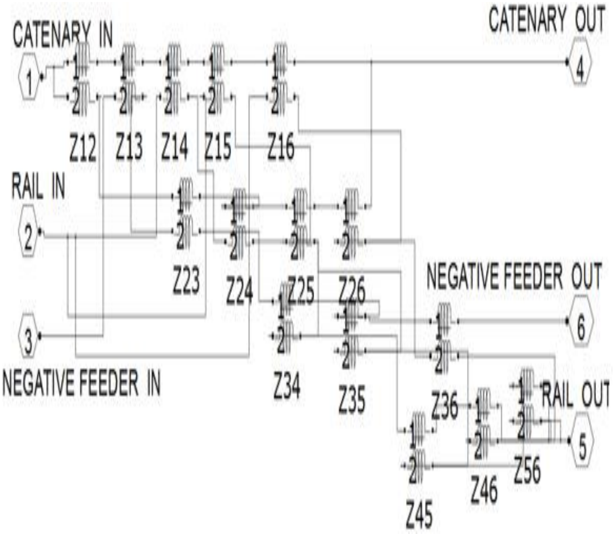

Fig. 7: The catenary system model

The six input line as shown in fig. 7 are connected in such a way to form to a six output line where the six input and output line combined to form three input and three output wire. These sets are appropriately called Catenary, Rail, and Feeder.

SIMULATION RESULT AND ANALYSIS

The purpose of this simulation is to evaluate the designed autotransformer-fed power supply system for Modjo-Hawassa railway line corridors. Since voltage profile and voltage regulation along the line are the most important parameters to evaluate the system performance, a computer-aided steady state load-flow simulation in terms of voltage were performed. The analysis is done for two different cases.

The performance of the traction power system has been investigated by applying different headway distances on the trains and by increasing the number of train along the feeding section. Tables 2-4 shows simulation findings based on a single locomotive which is moving along the 55 km long feeding section and results were taken one at a time at points 15 km, 30 km and 55 km from the traction substation. Table 5-7 shows simulation results based on three consecutive locomotives which are moving at the same time along feeding section at a distance of 15 km, 30 km and 55 km (end of the feeding section) from the traction substation.

Note that in Tables SS means substation and the first (AT1), the second (AT2), the third (AT3) and the fourth (AT4) autotransformers are located at a distance of 15 km, 30 km , 45 km and 55 km away from the traction substation respectively.

Note that in Table 1, C (C1, C2) means contact line wires, M (M1, M2) messenger wires, F (F1, F2) Negative feeders and R(R11, R12, R21, R22) one of the four rail line on the double track configuration.

Table 1: Ten by ten impedance matrix of the overhead catenary system

	C1	M1	F1	C2	M2	F2	*R11*	*R12*	*R21*	*R22*
C1	*0.218+j0.77*	*0.049+j0.42*	*0.049+j0.34*	*0.049+j0.33*	*0.049+j0.327*	*0.049+j0.29*	*0.049+j0.32*	*0.049+j0.32*	*0.049+j0.31*	*0.049+j0.30*
M1	*0.049+j0.42*	*0.239+j0.75*	*0.049+j0.34*	*0.049+j0.33*	*0.049+j0.329*	*0.049+j0.29*	*0.049+j0.31*	*0.049+j0.31*	*0.049+j0.29*	*0.049+j0.29*
F1	*0.049+j0.34*	*0.049+j0.34*	*0.152+j0.72*	*0.049+j0.29*	*0.049+j0.292*	*0.049+j0.27*	*0.049+j0.29*	*0.049+j0.29*	*0.049+j0.28*	*0.049+j0.27*
C2	*0.049+j0.33*	*0.049+j0.33*	*0.049+j0.29*	*0.218+j0.77*	*0.049+j0.420*	*0.049+j0.34*	*0.049+j0.31*	*0.049+j0.30*	*0.049+j0.32*	*0.049+j0.32*
M2	*0.049+j0.33*	*0.049+j0.33*	*0.049+j0.29*	*0.049+j0.42*	*0.239+j0.745*	*0.049+j0.34*	*0.049+j0.29*	*0.049+j0.29*	*0.049+j0.31*	*0.049+j0.31*
F2	*0.049+j0.29*	*0.049+j0.29*	*0.049+j0.27*	*0.049+j0.34*	*0.049+j0.342*	*0.152+j0.72*	*0.049+j0.28*	*0.049+j0.27*	*0.049+j0.29*	*0.049+j0.29*
R11	*0.049+j0.32*	*0.049+j0.31*	*0.049+j0.29*	*0.049+j0.31*	*0.049+j0.297*	*0.049+j0.28*	*0.073+j0.61*	*0.049+j0.41*	*0.049+j0.33*	*0.049+j0.31*
R12	*0.049+j0.32*	*0.049+j0.31*	*0.049+j0.29*	*0.049+j0.30*	*0.049+j0.294*	*0.049+j0.27*	*0.049+j0.41*	*0.073+j0.61*	*0.049+j0.35*	*0.049+j0.33*
R21	*0.049+j0.31*	*0.049+j0.29*	*0.049+j0.28*	*0.049+j0.32*	*0.049+j0.312*	*0.049+j0.29*	*0.049+j0.33*	*0.049+j0.35*	*0.073+j0.61*	*0.049+j0.41*
R22	*0.049+j0.30*	*0.049+j0.29*	*0.049+j0.27*	*0.049+j0.32*	*0.049+j0.312*	*0.049+j0.29*	*0.049+j0.31*	*0.049+j0.33*	*0.049+j0.41*	*0.073+j0.61*

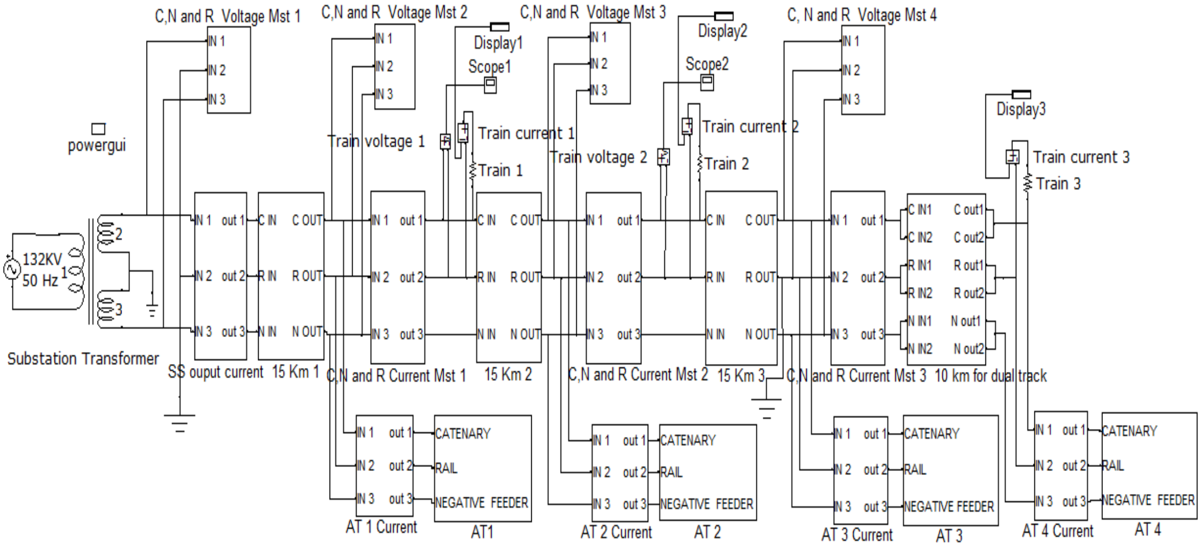

Fig. 8: Simulated power supply system having 2 x 25 kV AT, 2 conductors and a return wire

In this study, the train voltages will be the potential at the train's current collector (pantograph) or elsewhere on the catenary, measured between the catenary and the rail return circuit and also the train current is the current measured at the pantograph.

Train position	SS output voltage	Train voltage
15 km	27.41 kV	26.67 kV
30 km	27.38 kV	25.47 kV
55 km	27.36 kV	24.78 kV

Table 2**:** Train position and voltage profile in the case of single train

Table 2 shows that the train voltage decreased from 26.67 kV to 24.78 kV as the train position changed from 15 km to 55 km along the feeding section, which shows that as the train distance increases relative to the traction substation, the impedance of the traction network lifts up which in turn leads to reduction in voltage profile across the catenary system.

Table 3: Train position and current in the case of single train

Train Position	Substation output Current (A)	Train Current (A)
15 km	592.86	587.82
30 km	577.63	574.28
55 km	573.85	559.61

Table 3 indicates the train current decreases when the distance from the substation increases, this is because as the trains distance increases, small amount of current flows in different circuits such as autotransformers that does not have train in between (theoretically this current should not flow into this autotransformers but practically that is not the case because it only works for ideal autotransformers) and cause the current to return to the substation via the return conductors and it is found that the train current decreases from 587.82 A to 559.61 A along the feeding section.

Table 4: Train position and AT voltages in the case of single train

Train position	AT Terminal	AT 1 kV	AT 2 kV	AT 3 kV	AT 4 kV
30 km	C	26.72	26.08	25.47	25.54
	R	0.000	0.056	0.000	0.00
	N	26.82	26.37	26.05	25.79
55 km	C	26.69	26.06	25.47	24.77
	R	0.000	0.030	0.000	0.000
	N	26.72	26.16	25.67	25.26

Table 4 indicates that as the train distance with respect to the traction substation increases the autotransformer voltage decreases this is because when the distance increases the impedance of the traction network rises which in turn increase the voltage drop and o the maximum observed rail line potential for this case becomes 56.35 V.

Table 5: Train positions and voltage profiles in the case of three trains

Train Position	SS Output Voltage	Train Voltage	% Voltage regulation
15 km	27.08 kV	24.89 kV	0.64 %
30 km	26.96 kV	23.85 kV	4.60 %
55 km	26.95 kV	22.93 kV	8.28 %

Table 5 shows that the voltage at 15 km is 24.89 kV and at 55 km is 22.93 kV. The train voltage at 15 km and at 55 km are reduced by 6.67 % and 7.46 % respectively compared with previous case of single train shown in Table 2. This is because as the number of train increases, the current flowing through the traction catenary network increases as presented in Table 6, which leads to a higher voltage drop. This higher voltage variation across the line causes the percentage voltage regulation to rise from 0.64 % to 8.28 %.

Table 6: Train positions and current in the case of three trains

Substation output Current (A)	Train Position	Train Current (A)
1590.3	15 km	548.61
	30 km	524.04
	55 km	514.25

As shown in Table 6 the train currents decreases for the case three consecutive trains compared to a single train as shown in Table 3, this is due to the fact small amount of the autotransformer current returned to the substation via the rail. This reduction of current decreases the performance of the train or decreases the speed of the train because the speed of the train directly depends on the current that the train motor receives.

Table 7: AT voltage in the case of three trains

AT Terminals	AT 1 kV	AT 2 kV	AT 3 kV	AT 4 kV
C	24.89	23.81	23.42	23.34
R	0.000	0.066	0.000	0.000
N	25.57	24.66	24.05	24.06

Table 7 show the reduction of the voltage at autotransformer terminals with the increase of the number of trains along the given feeding section. The conclusion is that both the number of train and train distance affects the autotransformers voltage profile.

The following simulation results are found based on a model developed using Matlab as shown in Fig.8.

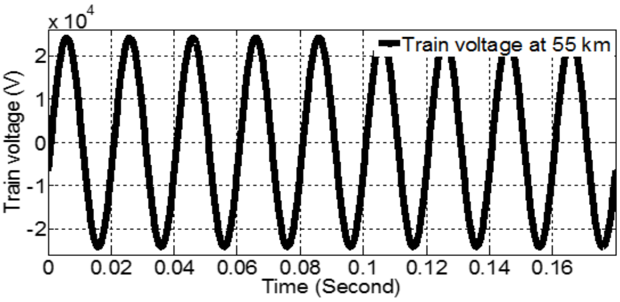

Fig. 9: Voltages of a single train on the same feeding section

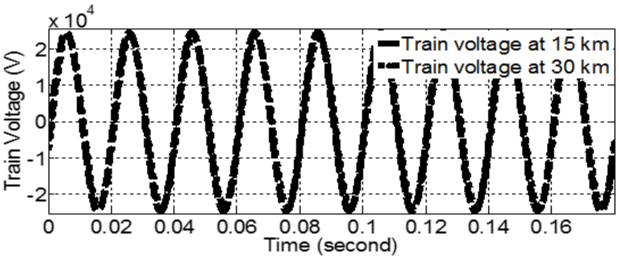

Fig.10: Voltage of two consecutive trains on the same feeding section

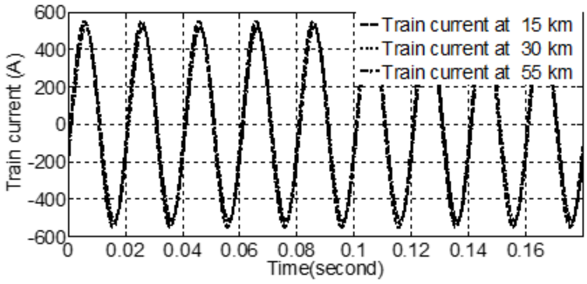

Fig. 11: Train currents of three consecutive trains on the same feeding section

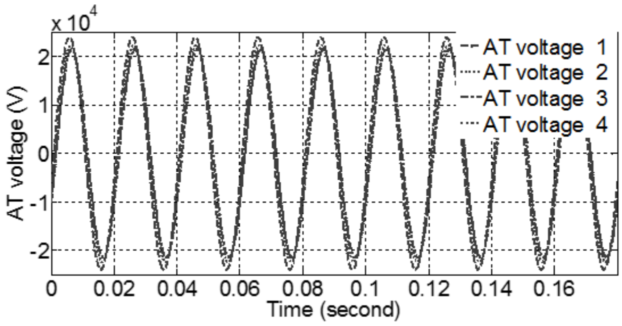

Fig.12: AT voltages of three consecutive trains on the same feeding section

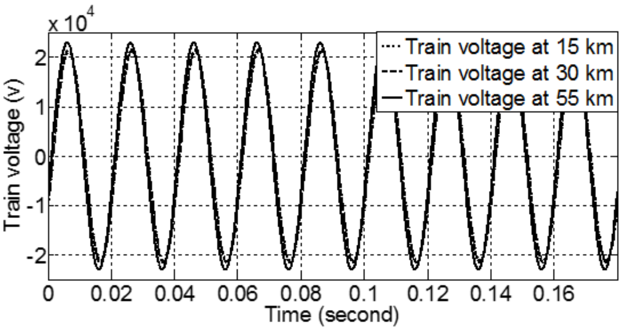

Fig. 13: Voltages of three consecutive trains on the same feeding section

CONCLUSIONS

In this paper, modeling of major components of traction power supply system has been done for autotransformer traction power supply system arrangement using Matlab and simulation of the supply system with variation of distance of electric locomotive and the number of train across the line has been conducted.

From the results obtained, it can be concluded that the magnitude of the train voltage decreases with increase in distance of locomotive from the traction substation and also decreases with increasing the number of train along the feeding section.

The minimum computed pantograph voltage for the train is 22.93 kV, which is within the tolerable voltage fluctuation range of BS EN 50163:2004 of overhead contact lines [22] and also the maximum rail line potential becomes 66.2 V which is also within the recommended limit of BS EN 50122-1 (IEC 62128-1) [23]. In addition, the maximum percentage voltage regulation is found to be 8.28 % with respect to the nominal voltage which is within the standard limit described by IEC 61000-2-2 [24].

Finally, the results obtained from the model confirm with the industry standards and this clearly indicates the successful use of developed mathematical model for simulating traction power supply system.

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