INVESTIGATION ON APPLICABILITY OF SUBSTITUTE BEAM - COLUMN FRAME FOR DESIGN OF REINFORCED CONCRETE SWAY FRAMES

Abrham Ewnetie and *Girma Zerayohannes
School of Civil and Environmental Engineering, Addis Ababa Institute of Technology

*Corresponding Author's Email: zerayohannes@yahoo.com

ABSTRACT

The paper deals with the evaluation of the sway frame moment magnification provision for the design of slender reinforced concrete columns in sway frames according to EBCS 2: 1995.

A special feature of the moment magnification method in EBCS 2: 1995 is the introduction of the concept of substitute frame for the determination of the storey buckling load. The evaluation is carried out by comparing the magnified column moment with the corresponding values obtained from the more rigorous second order iterative P-Δ second order analysis.

The magnified column moments are also compared with the corresponding values determined using the ACI's sway moment magnification provision. The results of the evaluation show that the sway moment magnification method according to EBCS 2: 1995 yields design moments close to the iterative P-Δ solutions. However, the results lie on the unsafe side and the percentage deviation is found to be the highest for irregular frames.

Key Words: Sway frames, slender columns, substitute beam-column frame, ETABS, critical load, sway moment magnification, second-order analysis.

INTRODUCTION

Since the Ethiopian Building Code Standard, EBCS-2: 1995 [4] is based on Eurocode 2: ENV 1992 [5]; the two Codes are very similar with only few exceptions such as provisions for the design of columns in sway frames.

ENV 1992 [5] gives detailed simplified design provisions for slender reinforced concrete columns that may be considered as isolated columns. These are individual columns with articulation in non-sway structures, slender bracing elements, and columns with restrained ends in a non-sway structure. Corresponding provisions for the design of columns in sway frames are not provided by ENV 1992 [5]. According to ENV 1992 [5], such columns are to be designed using the more rigorous approach based on the results of a second order global analysis.

The EBCS-2: 1995 [4] seems to be more complete in this respect, because it gives additional simplified procedures for the design of columns in sway frames. A closer look into the provisions reveals that they are based on the corresponding procedures according to the American Concrete Institute, ACI [1]. The provisions in ACI and EBCS, however, have significant differences in the procedures such as the concept of the substitute frame adopted by EBCS-2 [4] for column stiffness computation.

Therefore the design of slender reinforced concrete columns in sway frames has long been a controversial subject among practicing structural engineers with lack of consensus with regard to its suitability as a design tool or even the validity of the results [10].

It is thus very important to make a detailed investigation on the validity of the results obtained from the provision in EBCS-2 [4] by comparing them with the corresponding results using the more rigorous second order iterative P-Δ analysis and ACI's sway moment magnification method.

DESIGN PROVISIONS

Slender Columns Design in Sway Frames According to ACI and EBCS: 1995 Codes

The Ethiopian Building Code Standard, EBCS 2: 1995 seems to have similar provisions for design of slender columns in sway frames with the American Concrete Institute (ACI). However they have some clear differences. One of these is the introduction of the substitute beam-column frame in the EBCS 2: 1995 for the determination of the effective column stiffness in sway frames to calculate the critical buckling loads.

The following is a summary of the steps followed in the moment magnification procedure for the design of slender columns in sway frames based on the EBCS 2: 1995 provision. It may be observed that similar steps are followed for the design of slender columns in sway frames based on the ACI provision.

Moment Magnification Procedure for Sway Frames According to EBCS

Step 1: Check for Storey Sway

According to Section 4.4.4.2 of EBCS-2, 1995, a storey in a given frame may be classified as non-sway storey if:

$$\frac{N_{Sd}}{N_{cr}} \leq 0.1 \qquad (1)$$

Beam-and-column type plane frames in building structures with beams connecting each column at each storey level may be classified as non-sway storey if:

$$\frac{N\delta}{HL} \leq 0.1 \qquad (2)$$

Where, in both equations,

N_{Sd}, N = total factored axial load in the storey,

N_{cr} = storey buckling load,

H = total horizontal reaction (shear) at the bottom of the storey,

δ = first-order relative deflection between the top and bottom of that storey due to the design loads (vertical and horizontal),plus the initial sway imperfection,

L = storey height.

The displacement δ shall be determined based on stiffness values for beams and columns appropriate to Ultimate Limit State.

Step 2: Check for Slenderness

(i) Generally, the slenderness ratio of concrete columns should not exceed 140.

(ii) According to section 4.4.6 of EBCS-2, second order effects for columns in sway frames need not be taken into account if:

$$\lambda \leq \text{Max}\,(25,\ 15/\sqrt{\nu_d}\,) \qquad (3a)$$

Where $\nu_d = N_{Sd}/(f_{cd}A_c)$, (3b)

f_{cd}= design compressive strength of concrete,

A_c = gross cross-sectional area of the columns

Step 3: Effective Buckling Length Factors

The effective buckling length factors of columns in a sway frame shall be computed by using approximate equations given in EBCS-2 Section 4.4.7 based on EI values for gross concrete sections provided that the α values do not exceed 10. For higher values of α_1 or α_2 more accurate methods must be used.

$$k = \frac{L_e}{L} = \sqrt{\frac{7.5 + 4(\alpha_1 + \alpha_2) + 1.6 \times \alpha_1 \times \alpha_2}{7.5 + \alpha_1 + \alpha_2}} \geq 1.15 \qquad (4)$$

Or conservatively,

$$k = \frac{L_e}{L} = \sqrt{1 + 0.8\alpha_m} \geq 1.15 \qquad (5)$$

Where, for columns being designed and beams and columns just above them,

$$\alpha_1 = \frac{\sum (E_c I_c / l_c)}{\sum (E_b I_b / l_b)} \qquad (6)$$

For columns being designed and beams and columns just below them

$$\alpha_2 = \frac{\sum (E_c I_c / l_c)}{\sum (E_b I_b / l_b)} \qquad (7)$$

$$\alpha_m = \frac{\alpha_1 + \alpha_2}{2} \qquad (8)$$

Step 4: Magnified Moments

The magnified sway moments, $\delta_s M_s$, are computed using the amplified sway moments method given in EBCS-2 Section 4.4.11. The total design moments M_1 and M_2 at the ends of the columns shall then be obtained by adding the unmagnified non sway moments, M_{ns}, found by a first order analysis using member stiffness in EBCS-2, Section 3.7.6, and the magnified sway moments $\delta_s M_s$.

$$M_1 = M_{1ns} + \delta_s M_{1s} \qquad (9a)$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \qquad (9b)$$

The sway moment magnification factor δ_s shall be computed from

$$\delta_s = \frac{1}{1-\sum N_{Sd}/\sum N_{cr}} \qquad (10)$$

Where N_{Sd} is the design value of the total vertical load

N_{cr} is its critical value for failure in a sway mode

The amplified sway moments method shall not be used when the critical load ratio N_{Sd}/N_{cr}, is more than 0.25.

Step 5: Storey Buckling Load, N_{cr}

The approach used in this step is different from that of the ACI because of the introduction of the concept of substitute beam-column frame method for the determination of the critical buckling load. The approach allows a more accurate appraisal of the bending stiffness of the columns because the moments of inertia are determined including the contribution of steel designed using the substitute columns.

The substitute beam-column frame is a propped half portal made of substitute columns and beams as shown in Fig. 2.1. According to Section 4.4.12 (1) of EBCS-2, the buckling load of a storey may be assumed to be equal to that of the substitute beam-column frame. EBCS-2 Section 4.4.12(4) states that the equivalent reinforcement areas, $A_{s,tot}$, in the substitute column are obtained by designing the column at each floor level to carry the storey design axial load and magnified sway moment at the critical section.

$$N_{cr} = \frac{\pi^2 EI_e}{L_e^2} \qquad (11a)$$

Where, the effective stiffness of a column EI_e shall be taken from Section 4.4.12(1),

$$EI_e = \frac{0.2E_cI_c + E_sI_s}{1+\beta_d} \qquad (11b)$$

Or alternatively,

$$EI_e = \frac{M_{bal}/(1/r_{bal})}{1+\beta_d} \qquad (11c)$$

Where:

$$E_c = 1100 f_{cd} \qquad (11d)$$

E_c = modulus of elasticity of the concrete,

E_s = modulus of elasticity of the steel,

I_c =gross moment of inertia of the concrete section about its centroidal axis,

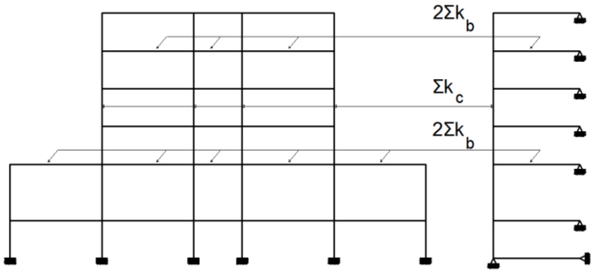
I_s = moment of inertia of the reinforcement about the centroidal axis of the concrete section,

M_{bal} = balanced moment capacity of the column,

$(1/r_{bal})$ = curvature at the balanced load and may be taken as:

$$\frac{1}{r_{bal}} = \frac{5}{d} * 10^{-3} \qquad (12)$$

The term $(1 + \beta_d)$ in both equations reflects the effect of creep on the column deflections as stated in Section 4.4.13(4)).



(a) Actual frame *(b) Substitute frame*

Fig. 1: Substitute Multi-storey Beam-Column Frame

Step 6: Location Check for the Maximum Column Moments

EBCS-2 Section 4.4.8.1(2) also requires checking whether moment at some point between the ends of the column exceeds that at the end of the column but does not give any explicit equation as in the ACI. The check is done by comparing the magnified moments for nonsway columns that are determined using the design procedure in EBCS Section 4.4.9 and 4.4.10 with those of the magnified column end moments.

Step 7: Stability Check under Gravity Loads Only

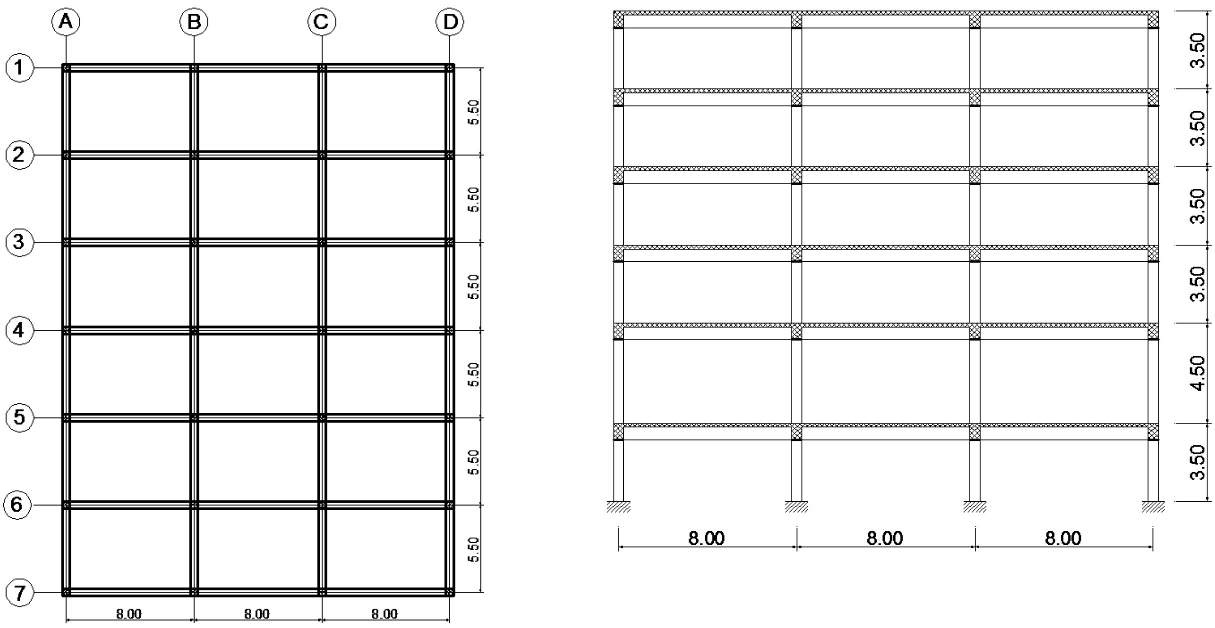
EBCS-2 section 4.4.8.1(1) states that all frames shall have adequate resistance to failure in a sway mode, but it does not place any explicit limit on $\delta_s$or the critical load ratio as in the ACI.

RESULTS AND DISCUSSIONS

Four different types of frames have been analyzed according to the ACI and EBCS sway moment magnification provisions. The results obtained have been compared with iterative P-Δ analysis results for the corresponding load combinations. The analysis outputs of each frame have been summarized and discussed in the following sections.

A) Five-Storey Regular Building

The results obtained based on the ACI and EBCS sway moment magnification provisions as well as the iterative P-Δ analysis are summarized in Table 1 below. The comparison of the results is shown in the table as a percent deviation. Fig. 2 also shows the results in graphical form.



a) Plan *b) Section*

Fig. 2: Five-Storey Building Detail

Table 1: Comparison of sway moment magnification and iterative P-Δ analysis outputs

ACI	**δ_s**	**Design Action Effects**	**Exterior Columns**				**Interior Columns**			
			MM	**Iterative P-Δ Outputs**	**ETABS P-Δ Outputs**	**% Change**	**MM**	**Iterative P-Δ Outputs**	**ETABS P-Δ Outputs**	**% Change**
Load case 1	1.254	P (kN)	1184.26	1187.5	1180.38	-0.273	2175.68	2175.92	2175.91	-0.011
		M (kN-m)	157.4	154.12	152.06	2.128	97.57	92.61	89.03	5.356
Load case 2	1.254	P (kN)	1338.88	1351.77	1348.98	-0.954	2192.99	2194.03	2193.96	-0.047
		M (kN-m)	329.72	311.60	307.05	5.815	375.16	345.99	339.04	8.431

Table 1: Cont…

ACI	δ_s	Design Action Effects	Exterior Columns				Interior Columns			
			MM	**Iterative P-Δ Outputs**	**ETABS P-Δ Outputs**	**% Change**	**MM**	**Iterative P-Δ Outputs**	**ETABS P-Δ Outputs**	**% Change**
Load case 1	1.254	P (kN)	1184.26	1187.5	1180.38	-0.273	2175.68	2175.92	2175.91	-0.011
		M (kN-m)	157.4	154.12	152.06	2.128	97.57	92.61	89.03	5.356
Load case 2	1.254	P (kN)	1338.88	1351.77	1348.98	-0.954	2192.99	2194.03	2193.96	-0.047
		M (kN-m)	329.72	311.60	307.05	5.815	375.16	345.99	339.04	8.431

Where:

δ_s= Sway moment magnification factor

MM = Results of the Sway moment magnifier method provisions,

Iterative P-Δ= Results of iterative P-Δ analysis method (calculated manually)

Etabs P-Δ = Results of Etabs 9.7.4 software iterative P-Δ analysis

Load case 1 = gravity and wind loads

$$= \begin{cases} 1.05D + 1.275L \pm 1.3W, \\ \text{(according to ACI)} \\ \\ 1.20D + 1.20L \pm 1.3W, \\ \text{(according to EBCS)} \end{cases}$$

Load case 2 = gravity & earthquake loads

$$= \begin{cases} 1.05D + 1.275L \pm 1.0E, \\ \text{(according to ACI)} \\ \\ 0.975D + 1.20L \pm 1.0E, \\ \text{(according to EBCS)} \end{cases}$$

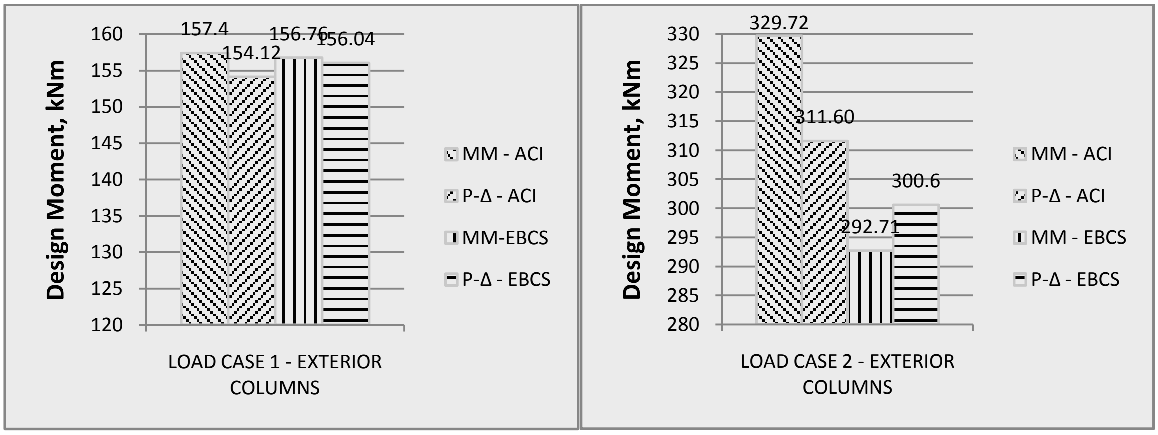
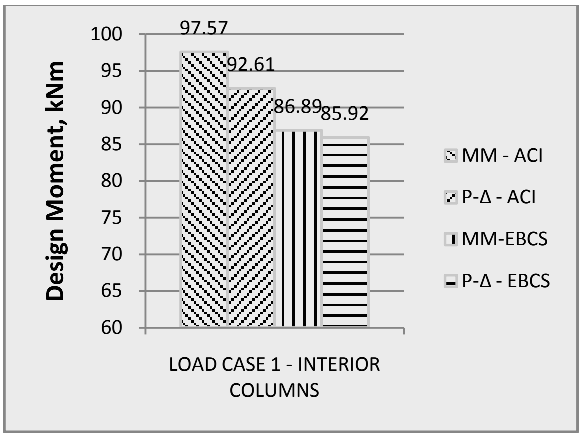


Fig. 3: Comparison of ACI and EBCS Results with Iterative P-Δ Analysis Results



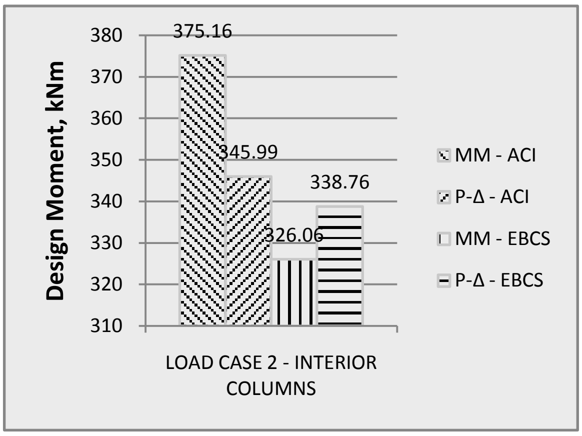
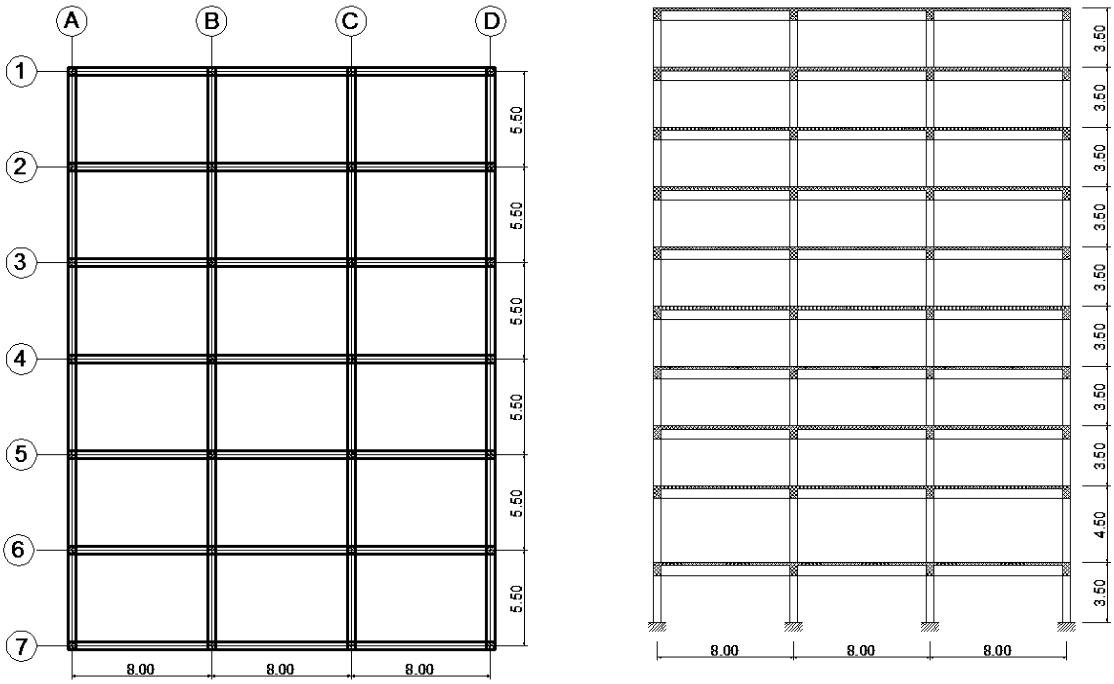


Fig. 3: Cont…

From Table 1 and Fig. 3 one can see that:

- ✓ The sway moment magnification method provision of the EBCS gives a closer result to the iterative P-Δ analysis results than the ACI provisions; numerically:
    - o For load case 1: 0.461% vs. 2.128% deviation for exterior columns, and 1.129% vs. 5.356% for interior columns.
    - o For load case 2: 2.625% vs. 5.815% deviation for exterior columns, and 3.749% vs. 8.431% for interior columns
- ✓ For load case 2, however, the results of the EBCS provision are smaller than the iterative P-Δ analysis results; unsafe.

B) Nine- Storey Regular Building Frame



a) Plan *b) Section*

Fig. 4: Nine-Storey Building Detail

Table 2: Comparison of sway moment magnification and iterative P-Δ analysis outputs

ACI	δ_s	Design Action Effects	Exterior Columns				Interior Columns			
			MM	Iterative P-Δ Output	ETABS P-Δ Output	% Change	MM	Iterative P-Δ Output	ETABS P-Δ Output	% Change
Load case 1	1.183	P (kN)	2436.04	2447.7	2443.39	-0.476	4172.81	4173.32	4173.36	-0.012
		M (kN-m)	235.91	224.14	225.85	5.251	195.84	187.91	178.52	4.220
Load case 2	1.183	P (kN)	2829.91	2866.42	2857.14	-1.274	4200.58	4202.42	4202.39	-0.044
		M (kN-m)	466.01	439	436.47	6.153	584.48	548.04	532.23	6.649
EBCS										
Load case 1	1.114	P (kN)	2620.1	2630.74	2626.78	-0.404	4478.64	4492.69	4492.75	-0.313
		M (kN-m)	225.4	229.4	224.03	-1.744	166.56	172.05	163.37	-3.191
Load case 2	1.087	P (kN)	2667.57	2700.83	2694.41	-1.231	3919.43	3921.09	3921.19	-0.042
		M (kN-m)	419.08	431.59	425.12	-2.899	518.97	536.16	526.18	-3.206

Where:

$$\text{Load case 1} = \begin{cases} 1.05D + 1.275L \pm 1.3W, \text{ (according to ACI} \\ 1.20D + 1.20L \pm 1.3W, \text{ (according to EBCS)} \end{cases}$$

$$\text{Load case 2} = \begin{cases} 1.05D + 1.275L \pm 1.0E, \text{ (according to ACI)} \\ 0.975D + 1.20L \pm 1.0E, \text{ (according to EBCS)} \end{cases}$$

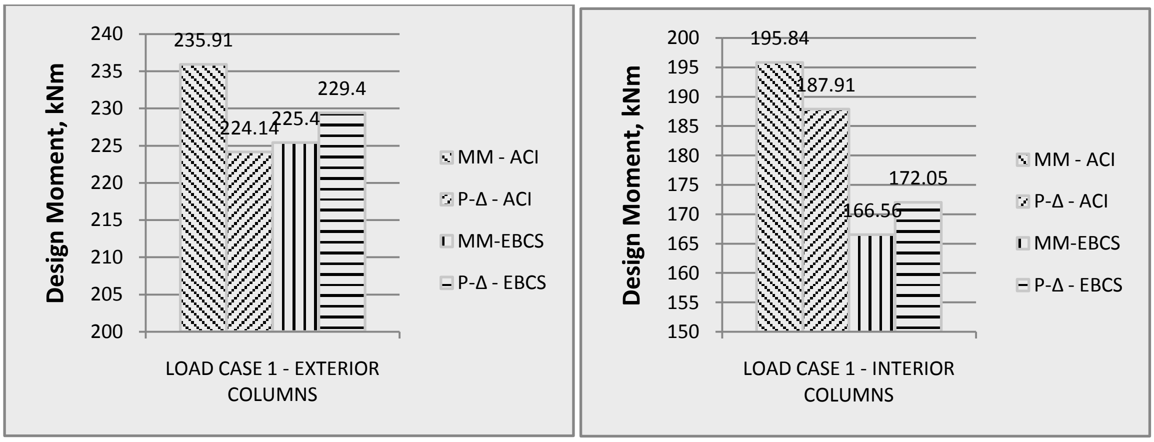


Fig. 5: Comparison of ACI and EBCS provision results with iterative P-Δ analysis results

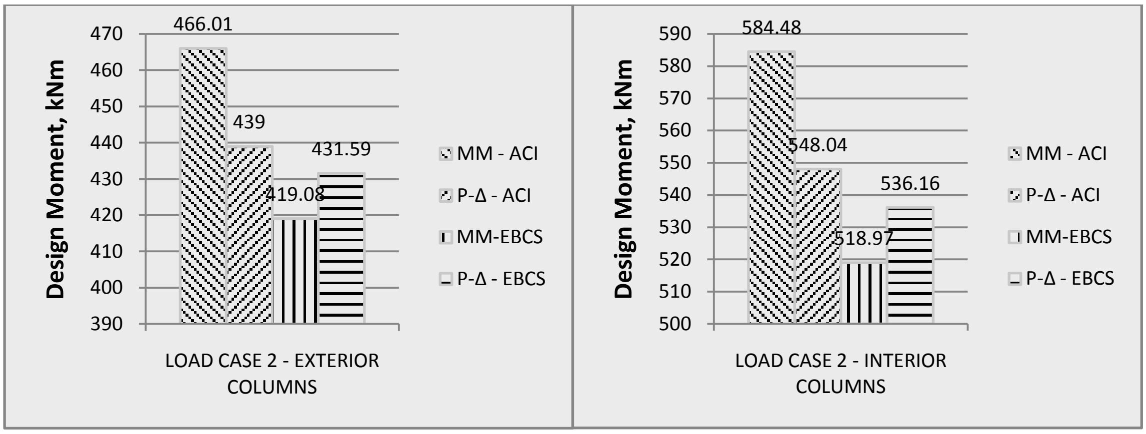


Fig. 5: Cont…

From Table 2 and Fig. 4 one can see that:

- ✓ The sway moment magnification method provisions of the EBCS give a closer result to the iterative P-Δ analysis results than the ACI provisions; numerically:
  - o For load case 1: 1.744% vs. 5.251% deviation for exterior columns, and 3.191% vs. 4.220% for interior columns.
  - o For load case 2: 2.899% vs. 6.153% deviation for exterior columns, and 3.206% vs. 6.649% for interior columns.
- ✓ In all cases above, however, the results of the EBCS provision are smaller than the iterative P-Δ analysis results; unsafe.

C) Five-Storey Building with Plan Irregularity

From Table 3 and Fig. 6 one can see that:

- ✓ The sway moment magnification provision of the ACI gives a closer result to the iterative P-Δ analysis results than the EBCS provisions. Numerically, 1.737% vs. 3.879% deviation for exterior columns, and 0.182% vs. 4.892% for interior columns.
- ✓ The results of the EBCS provision are smaller than the iterative P-Δ analysis results; unsafe.

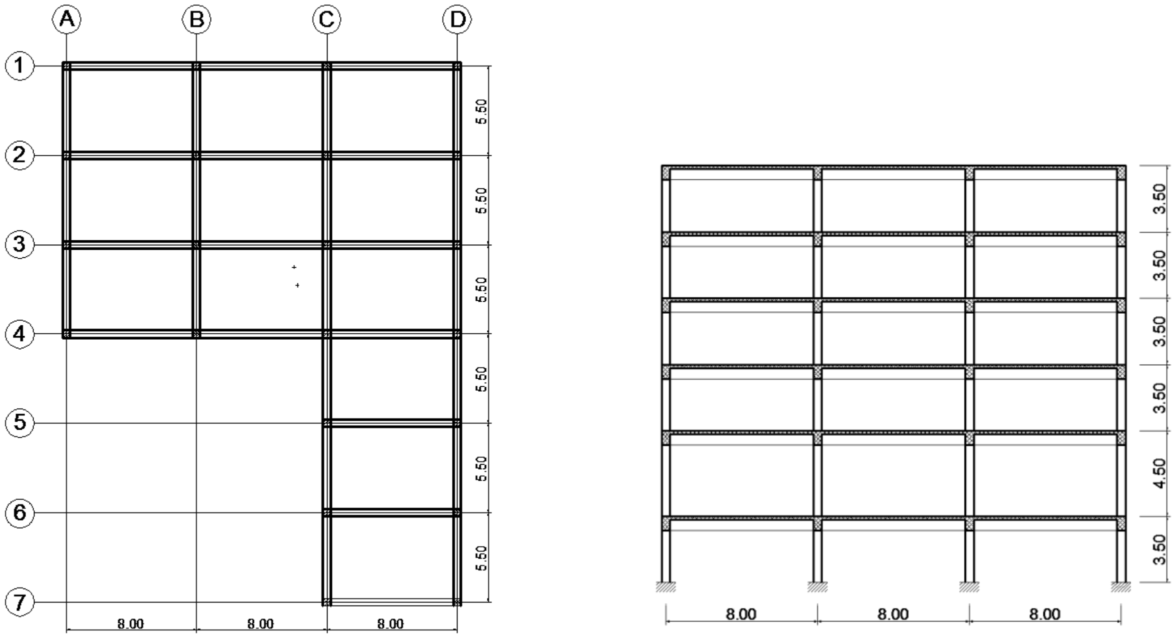


Fig. 6: Plan and Section of a Five-Storey Building with Plan Irregularity

Table 3: Comparison of sway moment magnification and iterative P-Δ analysis outputs

ACI	δ_s	Design Action Effects	Exterior Columns				Interior Columns			
			MM	Iterative P-Δ Output	ETABS P-Δ Output	% Change	MM	Iterative P-Δ Output	ETABS P-Δ Output	% Change
Load case 1	1.157	P (kN)	1424.28	1443.01	1438.57	-1.298	2312.79	2314.57	2314.43	-0.077
		M (kN-m)	392.45	385.75	385.45	1.737	451.27	450.45	440.91	0.182
EBCS										
Load case 1	1.088	P (kN)	1345.65	1362.97	1359.95	-1.271	2158.77	2160.41	2160.4	-0.076
		M (kN-m)	367.74	382.58	378.71	-3.879	424.04	445.85	440.2	-4.892

$$\text{Where: Load case 1} = \begin{cases} 1.05D + 1.275L \pm 1.0E, & \text{according to ACI} \\ 0.975D + 1.20L \pm 1.0E, & \text{according to EBCS} \end{cases}$$

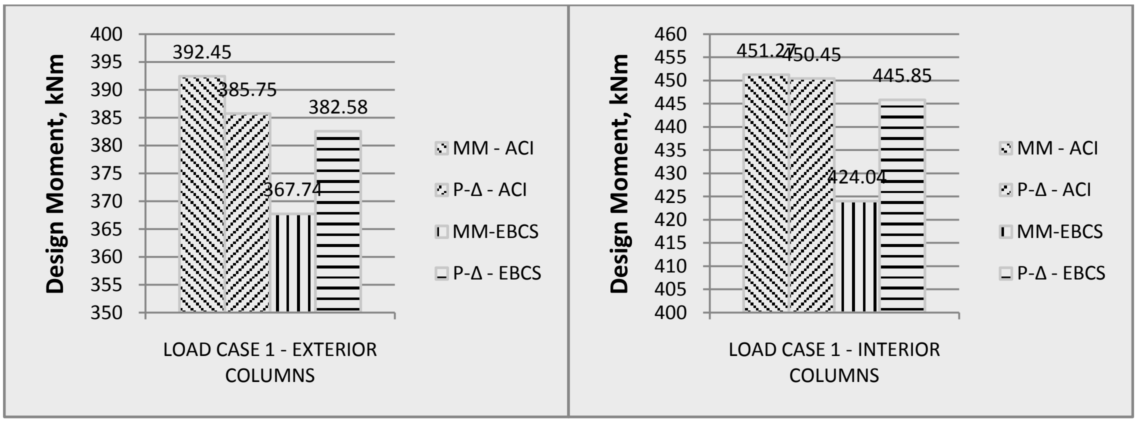
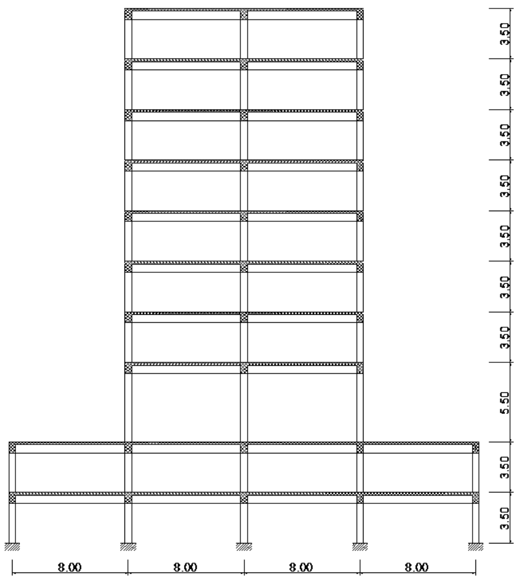


Fig. 7: Comparison of ACI and EBCS provision results with iterative P-Δ analysis results

D) *Nine-Storey Building with Elevation Irregularity*

The figure in the right shows the section of a 9-storey building and from Table 4 and Fig. 8 one can see that:

- ✓ The sway moment magnification method provisions of the EBCS give a closer result to the iterative P-Δ analysis results than the ACI provisions. Numerically, 6.365% vs. 10.724% deviation for exterior columns, and 6.292% vs. 14.629% for interior columns.
- ✓ In both cases, however, the results of the EBCS provision are smaller than the iterative P-Δ analysis results; unsafe.



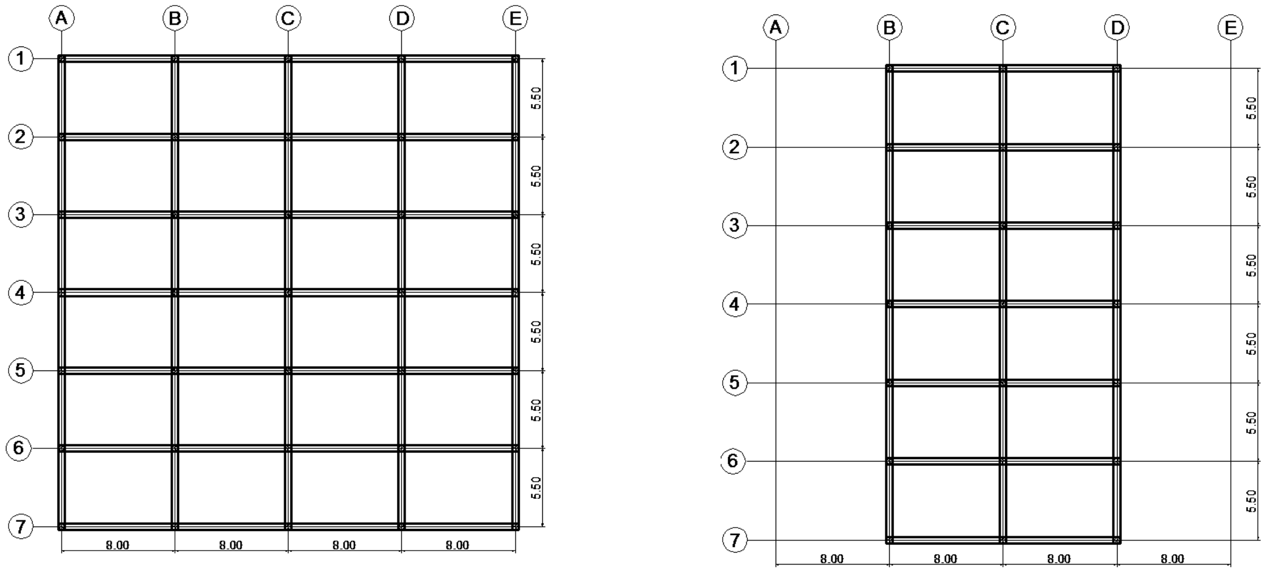


Fig. 8: Ground and First Floor Plan (left) and Typical Floor (right) of a Nine-storey Building with Elevation Irregularity

Table 4: Comparison of sway moment magnification and iterative P-Δ analysis outputs

ACI	δ_s	**Design Action Effects**	**Exterior Columns**				**Interior Columns**			
			MM	**Iterative P-Δ Output**	**ETABS P-Δ Output**	**% Change**	**MM**	**Iterative P-Δ Output**	**ETABS P-Δ Output**	**% Change**
Load case 1	1.319	P (kN)	2552.31	2589.54	2576.5	-1.438	3782.4	3782.43	3782.98	0.000
		M (kN-m)	565.51	510.74	468.54	10.724	634.08	553.16	518.86	14.629
EBCS										
Load case 1	1.097	P (kN)	2412.24	2450.65	2440.1	-1.567	3534.1	3534.12	3534.62	0.000
		M (kN-m)	473.56	505.75	468.34	-6.365	528.27	563.74	520.7	-6.292

$$\text{Load case 1} = \begin{cases} 1.05\text{D} + 1.275\text{L} \pm 1.0\text{E}, \text{ according to ACI} \\ 0.975\text{D} + 1.20\text{L} \pm 1.0\text{E}, \text{according to EBCS} \end{cases}$$

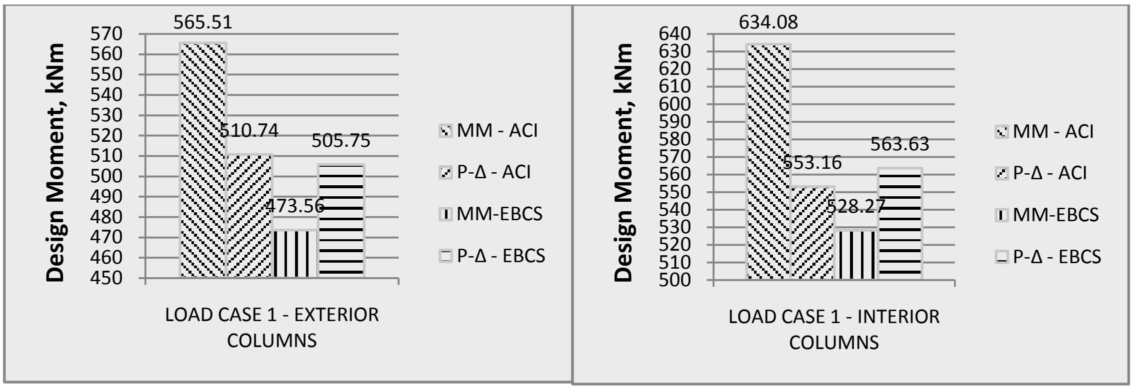


Fig. 9: Comparison of ACI and EBCS provision results with iterative P-Δ analysis results

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

From this research the following conclusions have been made.

1. Generally, the ACI provisions give more conservative results (higher design axial load and design moment) than those of the EBCS provisions reflecting the differences in load combinations used in the two codes. However, when designing structures for gravity and wind loads, the axial loads obtained from EBCS provisions are higher than those from ACI provisions.
2. In all the building frames considered, except the case with planar irregularity, the EBCS provision gives results closer to the iterative P-Δ analysis than the ACI provision, although the results are, almost always, on the unsafe side.
3. Unlike the ACI provision, the sway moment magnification provision of the EBCS gives design moments smaller than the iterative P-Δ analysis outputs, with maximum deviation of 6.365% for the nine storey frame with vertical irregularity.
4. Results of the design examples also show that the sway-moment magnification factors from EBCS provision are slightly less than the ACI sway moment magnification factors in all cases.
5. While using the sway moment magnification provision of the EBCS for designing slender columns in sway frames, one has to recall that the sway-moment magnification factor is different for different load conditions. This is because of the introduction of the substitute frame which has to be designed for the load combination under consideration to determine the effective stiffness, critical load and hence the sway moment magnification factor.
6. The provision in EBCS does not give any explicit limit as in the ACI for checking frame stability under gravity loads only; though it requires the check to be made.

Recommendations

1. When using the sway moment magnification method provisions of the ACI and the EBCS for the design of slender columns of sway frames with irregularities, precaution should be made since the reliability of the results decreases with irregularities.
2. The author recommends the following limits for checking the possibility of sideway buckling under gravity loads only, which are equivalent to the limits in ACI 318-05.

i) When $\delta_s M_s$ is computed from second-order elastic analysis, the ratio of second-order lateral deflections to first-order lateral deflections for factored dead and live loads plus factored lateral loads applied to the structure shall not exceed 2.5;

ii) When $\delta_s M_s$ is computed using the sway moment magnification procedure, δ_s computed by Equ. (10) using ΣN_{Sd} for 1.3D + 1.6L and ΣN_{cr} based on

$EI_e = \dfrac{0.2E_cI_c + E_sI_s}{1+\beta_d}$, shall be positive and shall not exceed 2.5.

iii) The critical load ratio N_{Sd}/N_{cr}, N_{Sd} computed using N_{Sd} for 1.3D + 1.6L and N_{cr} based on $EI_e = \dfrac{0.2E_cI_c + E_sI_s}{1+\beta_d}$ shall not exceed 0.60, which is equivalent to δ_s = 2.5.

In **i)**, **ii)** and **iii)** above, β_d shall be taken as the ratio of the total sustained axial loads to the total axial loads.

iv) As in ACI318-08, the above three checks can be ignored simply by limiting the ratio of the total moment including second-order effects to first-order moments to 1.40.

REFERENCES

1. ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary*, American Concrete Institute, Farmington Hills, MI, 2008.
2. ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05)*, American Concrete Institute, Farmington Hills, MI, 2005.
3. Bekele M., "*Effective Length and Rigidity of Columns*," ACI Journal, Proceedings V. 84, No. 3, pp. 316-329, July-August 1987
4. Ethiopian Building Code Standard, *EBCS 2-Part 1, Structural Use of Concrete*, Ministry of Works and Urban Development, 1995
5. *Eurocode 2: Design of concrete structures - Part 1-1: General Rules and Rules for Buildings*, 2004.
6. McGregor, J. G., J.K. Wight, *Reinforced Concrete Mechanics and Design*, 4th edition in SI units, Prentice-Hall, 2006, pp. 522-595
7. McGregor, J. G., Breen, J. E. and Pfrang, E. O., "*Design of Slender Concrete Columns*", ACI Journal, *Proceedings* V. 67, No. 2, Jan. 1970, pp. 6-28.
8. McGregor, J. G., "*Design of Slender Concrete Columns-Revisited*," ACI Journal, *Proceedings* V. 90, No. 3, May-June. 1993, pp. 302-309.
9. Zerayohannes, G., *Ethiopian Building Code Standard, EBCS 2-Part 1 and Part 2, Design Aids for Reinforced Concrete Sections*, Ministry of Works and Urban Development, 1998.
10. Zerayohannes G., "Influence of ACI Provisions for the Design of Columns in Sway Frames on EBCS-2:1995", ACI-ETHIOPIA CHAPTER Journal, proceedings, 2009, pp.24-48
11. Ewnetie A., *Investigation on Applicability of Substitute Beam-Column Frame for Design of Reinforced Concrete Sway Frames*, MSc Thesis, 2012.

Notations

ACI: American Concrete Institute

$A_{s,tot}$: Theoretical area of reinforcement required by the design

D: Dead (permanent) load

E: Earthquake load

EBCS: Ethiopian Building Code Standard

E_c: Modulus of elasticity of concrete

E_s: Elastic modulus of reinforcement steel

EI: Flexural Stiffness

EI_e: Effective flexural stiffness

f_{cd}: Design compressive strength of concrete

I_c, I_b: Gross Moment of inertia of column & beam cross sections respectively

I_g: Gross moment of inertia of a member

I_s, I_{se}: Moment of inertia of reinforcement steel of the column with respect to the centroid of the concrete section

k: Effective buckling length factor

L: Live (variable) load

l_b: Length of beams

l_c, L: Storey height

L_e: Effective buckling length

M_1: The algebraically smaller of M_{top} and M_{bottom}

M_2: The algebraically bigger of M_{top} and M_{bottom}

M_{bal}: Balanced moment capacity of a column

M_{bottom}: Moment at the bottom of a column

M_{ns}: Nonsway moment

M_s: Sway moment

M_{top}: Moment at the top of a column

$\delta_s M_s$: Magnified sway moment

N_{cr}: Critical buckling load, storey buckling load

N, N_{Sd}: Total factored axial load in the storey

P-Δ: Second order moments which result from lateral deflections, Δ, of the beam–column joints from their original un-deflected locations

Q: Stability index

U, S_d: Factored load combination

V_u, H: Total factored shear in all frames in the storey under consideration

W: Wind load

$(1/r_{bal})$: Curvature at the balanced load

δ_s: Sway moment magnification factor for the same load combination

Δ: Relative deflection between the top and bottom of a storey

λ: Slenderness ratio

α_1, α_2: Relative stiffness of columns to beams at the top and bottom of a storey