EVALUATION OF APPROXIMATE DESIGN PROCEDURES FOR BIAXIALLY LOADED RECTANGULAR REINFORCED CONCRETE COLUMNS

Girma Zerayohannes and Teferra Desta
Departments of Civil Engineering and Building Technology
Addis Ababa University

ABSTRACT

Many building codes including the Ethiopian Building Code Standard, EBCS 2 [1], recommend different approximate procedures for the design of biaxially loaded reinforced concrete columns. The Ethiopian code and other codes such as the British Standard, BS 8110 [2], recommend the design of biaxially loaded column for uniaxial bending using an "equivalent" uniaxial eccentricity of load along the axis parallel to the larger relative eccentricity. Another commonly used approximate design procedure adopted by many codes [2,3] is based on the use of simplified expressions for the normal load contour of the failure surface. Such interaction curves have at least two rigorously determined points corresponding to the design values of the ultimate uniaxial moment capacities of the cross-sections under different levels of normal forces. The approximation according to the ACI [4,5], can also be categorized in this group. However it involves the determination of an additional point on the actual interaction diagram where the magnitudes of the moment components related to the respective uniaxial capacities are equal.

Although one or the other approximate procedure is recommended by the different building code standards, the extent to which such procedures may lie on the safe or the unsafe side or relative merits of the different approaches is lacking in the literature. The aim of the paper is to evaluate the different approximate procedures by comparing the results with the more rigorous solution for biaxially loaded columns [7, 11].

The comparative result of the investigation shows that the ACI's approach [4, 5] represents the most accurate approximation for biaxial bending. The approximation according to the Ethiopian Building Code Standard, EBCS - 2 gave mostly conservative results. Based on the investigation, improvements on the γ- factors have been suggested to give less conservative results.

INTRODUCTION

The relationship between moment and curvature of a reinforced concrete section is non-linear because of the non-linear relationship between stress and strain of the constituent materials and cracking of the cross-section. Therefore, the design of a reinforced concrete section subject to normal load and biaxial bending, involve iteration and require the use of computers and relevant software. In order to simplify the design process, many building codes (ACI, EC2, BS8110, CP110, DIN1045, and EBCS 2) resort to approximate procedures for the design of biaxially loaded reinforced concrete columns. Most of the approximate methods can be classified in to the following three groups:

In the first group, columns of rectangular cross-sections may be checked separately for uniaxial bending in each respective direction provided the ratio of the relative eccentricities is less than 0.2 or equivalently, the point of application of the design normal force N lies within the shaded area in Fig.1 (DIN1045-1 [6], EBCS-2 [1]).

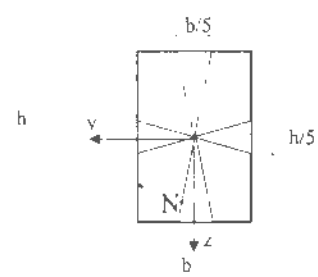


Figure 1

In the second group, the approximations are based on simplified expressions for the normal load contours of the failure surface and used for the design of cross-sections by a trial and adjustment procedure. In the third group of approximate methods, the biaxial moments are converted in to an equivalent uniaxial bending moment, for which the cross-section is designed with the total reinforcement distributed along each face of the column or at each corner [7].

In spite of the availability of different approximate procedures for the design of biaxially loaded reinforced concrete columns, the relative merits of each of these methods with regard to proximity to the rigorous solution or suitability as a design tool is not available in the literature. In this paper, the approximate methods of design according to the American Concrete Institute (ACI) and the British Standard (CP110) both from the second group and the Ethiopian Building Code Standard (EBCS 2) from the third group are evaluated. Based on the results of the evaluation, a modification has been recommended to improve the approximate procedure according to EBCS 2.

APPROXIMATIONS BASED ON SIMPLIFIED EXPRESSIONS FOR THE LOAD CONTOURS OF THE FAILURE SURFACE

a) General

Design values of the ultimate relative normal load and moment capacity of a reinforced concrete column under biaxial bending can be represented by an interaction surface as shown in Fig. 2(a). The column interaction surface can alternatively be plotted as a function of the related axial load and bending moments as shown in Fig. 2(b). In the Figures, m_{uy} and m_{uz} represent the relative uniaxial moment capacities at different normal load level and n_u equals the relative axial load capacity of the column cross-section.

The general forms of the load contours in Fig. 2(b) can be approximated by a non-dimensional interaction equation [8,4,5]

$$\left(\frac{m_y}{m_{uy}}\right)^{\alpha_1} + \left(\frac{m_z}{m_{uz}}\right)^{\alpha_2} \qquad (1)$$

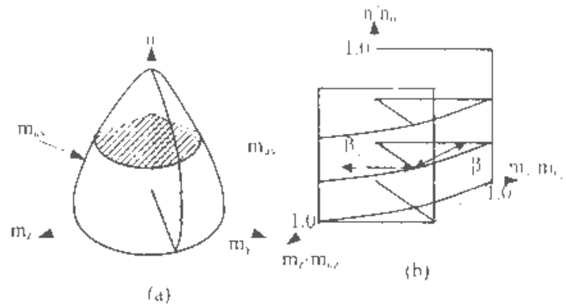


Figure 2 Alternative representations of interaction surfaces of a reinforced concrete sections.

The exponents α_1 and α_2 in Eq. (1) depend on column dimensions, amount and distribution of steel reinforcement, stress-strain characteristics of steel and concrete, amount of concrete cover and size of lateral ties.

Bresler [8] used a single exponent α for the interaction equation as given by Eq. (2).

$$\left(\frac{m_y}{m_{uy}}\right)^{\alpha} + \left(\frac{m_z}{m_{uz}}\right)^{\alpha} = 1.0 \qquad (2)$$

His comparison calculation with experimental values of biaxial strengths resulted in values of α in the range of 1.15 to 1.55. For practical purposes, α may be taken as 1.5 for rectangular sections and between 1.5 and 2.0 for square sections [12].

Many codes [2,3,4,5] have adopted the simple interaction Eq. (2) with $\alpha_1 = \alpha_2 = \alpha$. They differ mainly by their approximation for the exponent α.

b) Approximation According To The ACI

The approximation according to the ACI is based on the work by Parme [9] who chose to approximate α as a logarithmic function of a parameter β representing an actual point on the non-dimensional load contour, where the two moment components, related to the respective uniaxial capacities are equal, i.e. $\beta = m_y/m_{uy} = m_z/m_{uz}$

Thus the method involves the rigorous determination of a third point that lies on the true interaction diagram other than the uniaxial capacities about the principal axes, which have the value of unity. Inserting β in Eq. (2), and rearranging terms, α can be shown to be a logarithmic function of β given by Eq. (3).

$$\left(\frac{m_1}{m_{u1}}\right)^{\alpha} + \left(\frac{m_2}{m_{u2}}\right)^{\alpha} = 1.0$$

$$\beta^{\alpha} + \beta^{\alpha} = 1.0$$

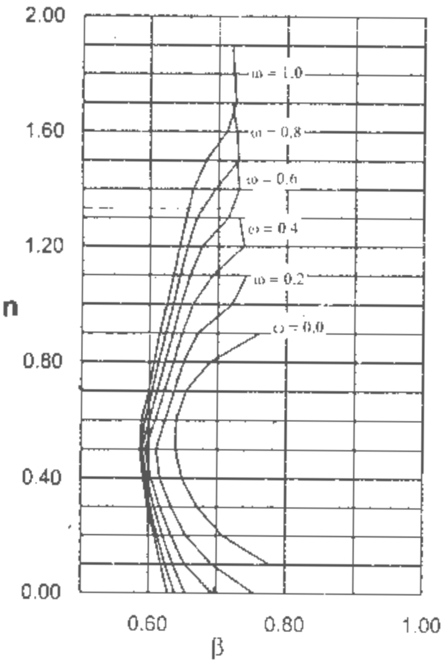
$$\beta^{\alpha} = 0.5$$

$$\alpha = \frac{\log 0.5}{\log \beta} \qquad (3)$$

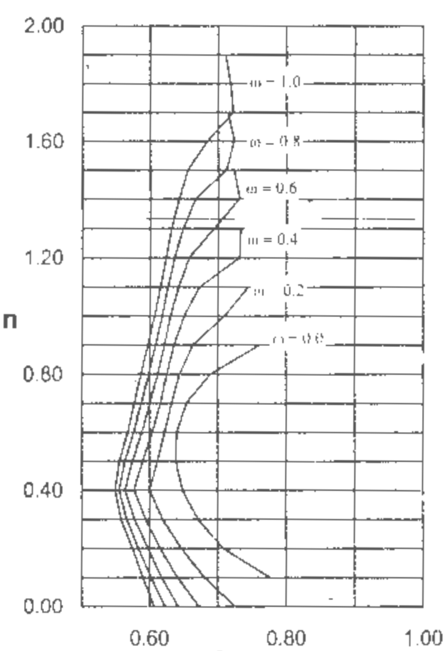
Although β-values for different cross-sections and reinforcement patterns are available in the form of design charts in the ACI Design Handbook [5], they were not useful for the purpose of the investigation, because:

(a) the charts allow for only very approximate reading, thus are not suitable for critical evaluation and
(b) they include strength reduction factors for columns.

Therefore β-values had to be rigorously determined as functions of the normal load and mechanical reinforcement ratio using the computer program [1,] developed for the preparation of biaxial charts [7]. The results have been compiled in the form of β-n charts [10]. The scope of the study has limited the reinforcement pattern to the three most commonly-used types involving corner reinforcements, eight-bar arrangement and uniform distribution along the four faces. Some of the charts are reproduced and shown in Fig. 3.



(c) *Bars uniformly distributed along the four sides (Cover ratio = 0.10)*



(d) *Bars uniformly distributed along the four sides (Cover ratio = 0.20)*

Figure 3 Biaxial bending design constant

The following two examples serve the purposes of testing the validity of the β-n charts and presenting the approximate procedure for the design of biaxially loaded columns according to the ACI.

For rectangular cross-sections with equal relative cover ratios and doubly symmetric reinforcement pattern, the relative uniaxial moment capacities are equal. Thus letting $m_{uy} = m_{uz} = m$ and rearranging Eq.(2):

$$\left(\frac{m_y}{m}\right)^{\alpha} + \left(\frac{m_z}{m}\right)^{\alpha} = 1.0 \qquad (4)$$

$$m = \left(m_y^{\alpha} + m_z^{\alpha}\right)^{1/\alpha}$$

Equation (4) can be used for design but involves trial and adjustment, because α is a function of β, which in turn can be determined only after the amount of reinforcement has been determined. Thus one starts with a trial section by assuming a value for β.

With an assumed starting β-value, the uniaxial bending moment capacity of the section is computed from Eq. (4). Next, the corresponding mechanical reinforcement ratio, ω, is read from the available uniaxial design charts [7]. In the third step the value of β corresponding to the most recently determined mechanical reinforcement ratio ω and the design value of the internal normal load is read from the β-n charts [10]. Steps 1, 2 and 3 are repeated until two consecutive values of ω are sufficiently close to each other. The procedure normally requires few iterations (not more than 3) and its application is demonstrated by the following two examples:

Example1

Given: - Geometry and material data
h/b = 400/400 mm
steel: S460
concrete: C30
-Design action effects
N = 870kN
M_y = 195kNm
M_z = 70knm

Required: Amount of Reinforcement

Solution:
Partial safety factors:
$\gamma_s = 1.15$, $\gamma_c = 1.5$
$f_{yd} = f_{yk}/\gamma_s = 460/1.15 = 400$Mpa
$f_{cd} = 0.85 f_{ck}/\gamma_c = 0.85*((0.8*30))/1.5$

$= 13.6$Mpa

$$n = \frac{N}{f_{cd}\,bh} = \frac{870*10^3}{13.60*400^2} = 0.40$$

$$m_y = \frac{M_y}{f_{cd}\,bh^2} = \frac{195*10^6}{13.60*400^3} = 0.2240$$

$$m_z = \frac{M_z}{f_{cd}\,hb^2} = \frac{70*10^6}{13.60*400^3} = 0.0804$$

Assuming four corner bars reinforcement pattern with cover ratio of 0.1 and assuming initially β = 0.60:

i. $\beta_1 = 0.600 \rightarrow \alpha_1 = 1.3569 \rightarrow$
$m_1 = 0.2639 \rightarrow \omega_1 = 0.370$

ii. $\beta_2 = 0.565 \rightarrow \alpha_2 = 1.2141 \rightarrow$
$m_2 = 0.2760 \rightarrow \omega_2 = 0.395$

iii. $\beta_3 = 0.560 \rightarrow \alpha_3 = 1.1955 \rightarrow$
$m_3 = 0.2779 \rightarrow \omega_3 = 0.400$

$\therefore\ \omega = 0.4 \rightarrow A_s = 2176\text{mm}^2$

The value of ω from Biaxial Chart No.1 [7] is exactly identical to this value.

Example2

Given: - Geometry and material data
~ Same as example 1
-Design action effects
$N = 1740$kN
$M_y = 140$kNm
$M_z = 50$kNm

Required: Amount of Reinforcement

Solution:

$$n = \frac{N}{f_{cd}\,bh} = \frac{1740*10^3}{13.60*400^2} = 0.80$$

$$m_y = \frac{M_y}{f_{cd}\,bh^2} = \frac{140*10^6}{13.60*400^3} = 0.1608$$

$$m_z = \frac{M_z}{f_{cd}\,hb^2} = \frac{50*10^6}{13.60*400^3} = 0.0574$$

Considering uniform distribution of reinforcement pattern with cover ratio of 0.1 and assuming initially $\beta = 0.65$:

i. $\beta_1 = 0.650 \rightarrow \alpha_1 = 1.6090 \rightarrow$
$m_1 = 0.1792 \rightarrow \omega_1 = 0.382$

ii. $\beta_2 = 0.638 \rightarrow \alpha_2 = 1.5423 \rightarrow$
$m_2 = 0.1814 \rightarrow \omega_2 = 0.390$

iii. $\beta_3 = 0.637 \rightarrow \alpha_3 = 1.5370 \rightarrow$
$m_3 = 0.1816 \rightarrow \omega_3 = 0.395$

$\therefore \omega = 0.395 \rightarrow A_s = 2149\text{mm}^2$

The exact value of ω from Biaxial Chart No.12 [7] is practically identical with this result.

The proximity of the approximation according to the ACI to the more rigorous solution has been investigated in detail [10] and found out that the approximations gave practically the same results as the more rigorous solutions under wide variety of parameters. Fig. 4 shows some of the results of the investigations.

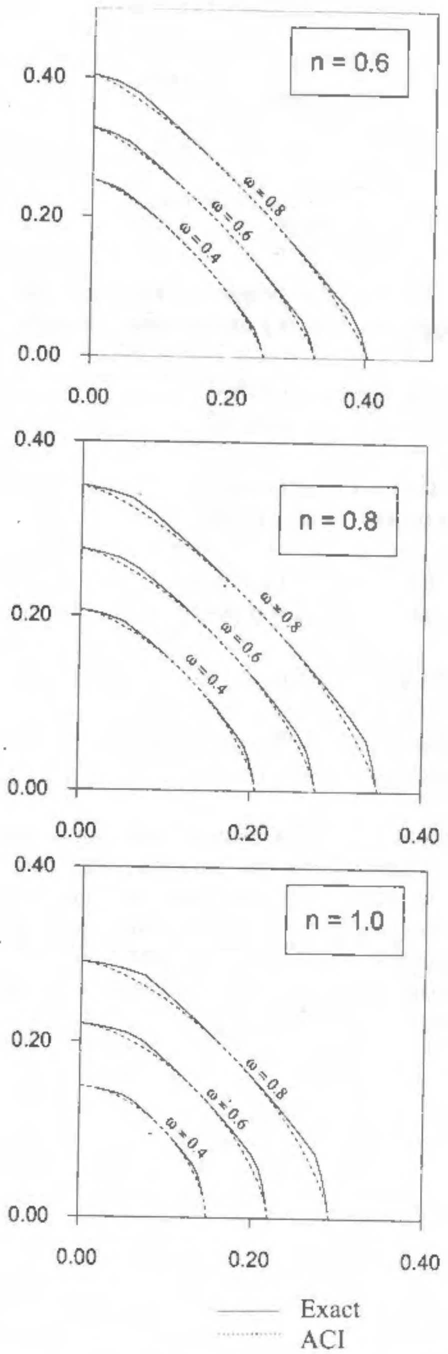


Figure 4 (a) Comparison of capacities of cross-sections with bars concentrated at the corners (cover ratio = 0.1)

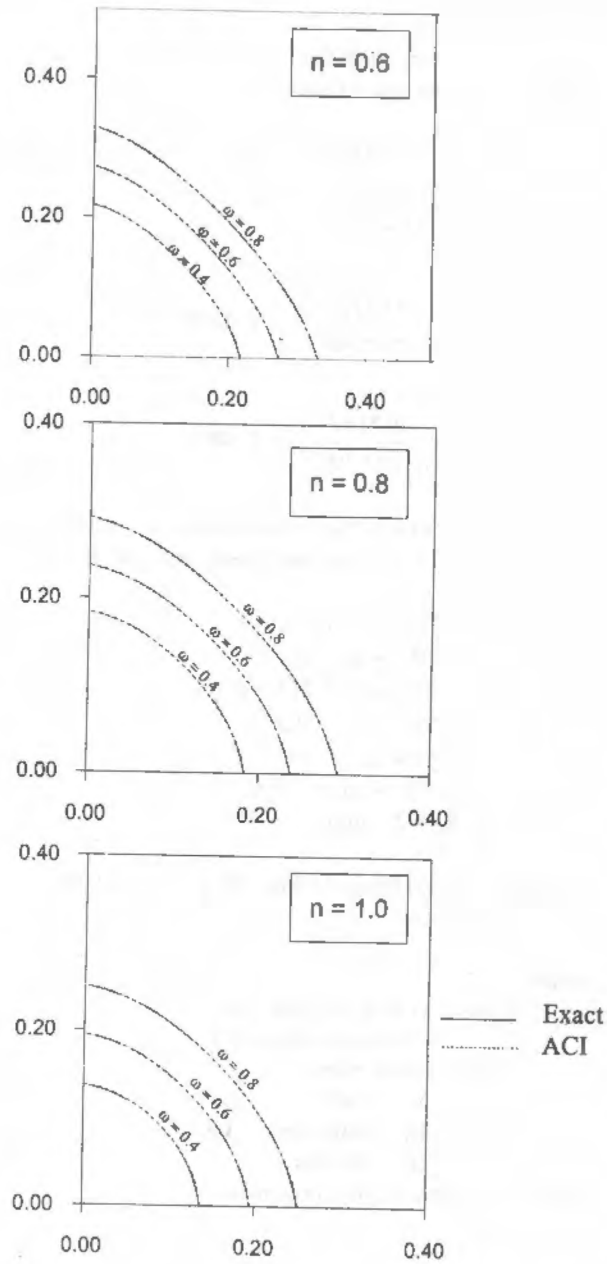


Figure 4 (b) Comparison of capacities of cross-sections with uniform distribution of bars along the four faces (cover ration = 0.1)

C) Approximation According To CP110

BS8110 recommends the use of the approximate procedure suggested in CP110 which, like the ACI uses simplified expression for the load contours. The exponent α in the interaction Eq. (2) is however approximated quite simply using the expression given by Eq. (5).

$$\alpha = \frac{1}{3}\left(2 + 5\frac{N}{N_o}\right) \quad (5)$$

In the above equation, N_o represents the capacity of the cross-section under axial force. Neglecting the concrete displaced by the reinforcement, it can be expressed as:

$$N_o = A_c f_{cd} + A_s f_{yd} = A_c f_{cd}(1+\omega) \quad (6)$$

Thus,

$$\alpha = \frac{1}{3}\left(2 + \frac{5n}{1+\omega}\right) \quad (7)$$

Where: $n = \dfrac{N}{f_{cd} A_c}$

$\omega = \dfrac{A_s f_{yd}}{A_c f_{cd}}$

The proximity of the approximation according to CP110 to the more rigorous solution has also been investigated in detail [10]. The method is found to give results close enough to the more rigorous solution only for sections with uniformly distributed reinforcement on all four faces under moderate levels of normal forces (n = 0.6, 0.8).

For lower normal load levels ($n \leq 0.4$) on the other hand, the discrepancy between the approximate and rigorous solutions is quite high. Considerable differences are also observed [10] at larger levels of normal force ($n \geq 0.8$) for cross-sections with concentrated reinforcements at the corners. Moreover the approximation in such cases lies on the unsafe side.

The deviation tends to decrease as the arrangement of bars tends to be more and more uniformly distributed. Other factors like amount of reinforcement and concrete cover are also found to have significant effects. Some of the results of the investigation are reproduced and shown in Fig.5.

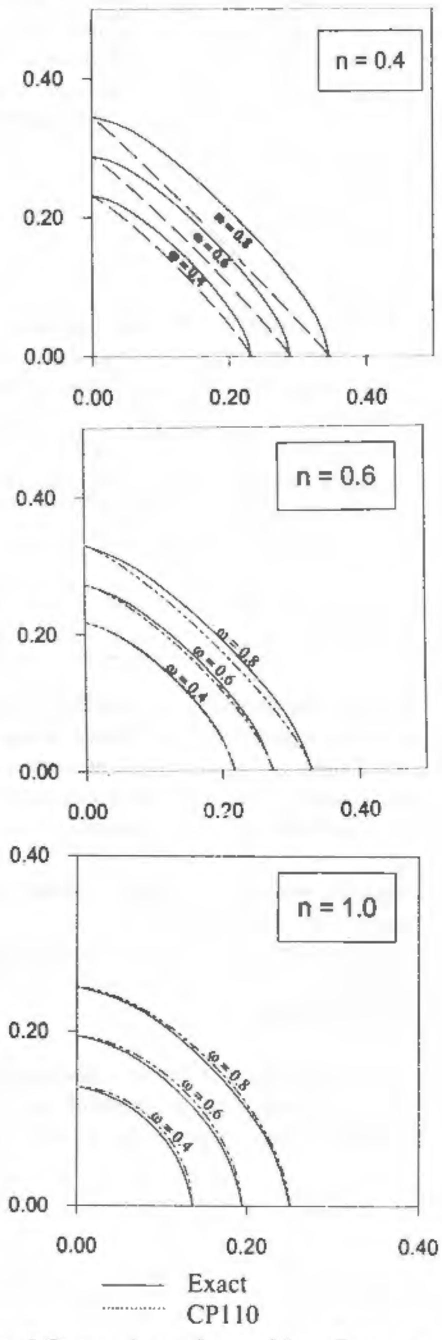


Figure 5 Comparison of capacities of cross-sections with uniform distribution of bars along the four faces (cover ration = 0.1)

APPROXIMATION BASED ON EQUIVALENT UNIAXIAL BENDING ACCORDING TO EBCS-2

The Ethiopian building code, EBCS-2 [1] allows an approximate method of design in which a biaxially loaded rectangular reinforced concrete column can be designed for the given normal force and uniaxial bending moment computed using the equivalent eccentricity of load given by Eq. (8):

$$e_{eq} = e_{tot}\left(1 + k\gamma\right) \qquad (8)$$

in which:

- e_{tot} denotes the total eccentricity allowing for initial imperfections and second order effects in the direction of the larger relative eccentricity.
- k denotes the relative eccentricity ratio.
- γ is a factor which depends on the relative normal force as shown in Table 1.

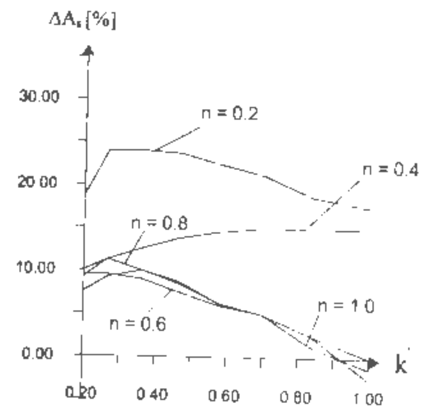
Table 1:

n	0.0	0.2	0.4	0.6	0.8	≥1.0
γ	0.6	0.8	0.9	0.7	0.6	0.5

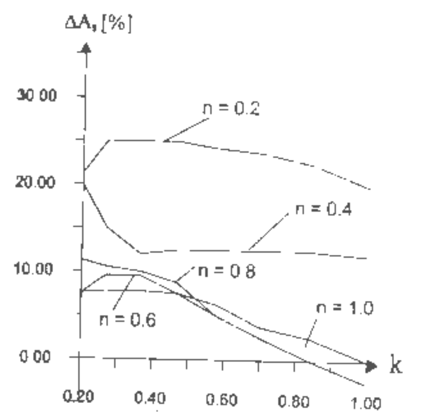
For this approximate method, one-fourth of the total reinforcement must either be distributed along each face of the column or concentrated at each corner. Thus the application of this method is limited to only these two arrangements of reinforcement.

The investigation was made through comparison of steel requirements of column cross-sections under different combinations of normal force and biaxial bending on the one hand and equivalent uniaxial bending on the other hand.

The percentage difference in the required amount of reinforcement, ΔAs, has been determined and plotted against the relative eccentricity ratio, k, as shown in Fig. 6.



h'/h = b'/b = 0.15



h'/h = b'/b = 0.20

Figure 6(a) Comparison of amount of reinforcement with bars concentrated at each corner

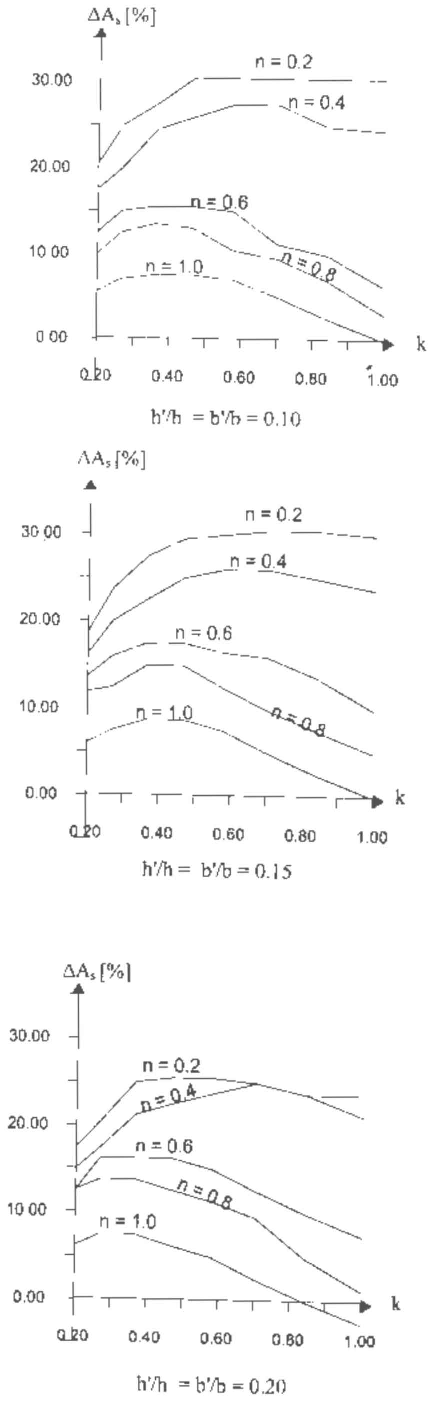


Figure 6 (b) Comparison of amount of reinforcement with bars uniformly distributed along the four sides of the cross-section.

The curves relating the change in the amount of reinforcement to the relative eccentricity ratio at moderate to large normal force levels ($n \geq 0.6$) show that the percentage difference in the amount of reinforcement decreases significantly as the relative eccentricity ratio approaches unity. Furthermore it can be observed that, the approximate method tends to become unsafe for the four-bar reinforcement pattern at higher levels of normal forces ($n \geq 0.8$) as the relative eccentricity ratio approaches 1.0. Both trends seem to be more pronounced as the concrete cover is increased.

On the other hand, for arrangement of bars with uniform distribution, the approximate method is found to give mostly conservative solutions as shown in Fig. 6 (b). The percentage difference in the amount of reinforcement (ΔA_s) decreases with increase in the normal force level except for $n = 0.4$. This may be attributed to the relatively high value of the coefficient γ in table 1, for $n = 0.4$. For reinforcement arrangement with uniformly distributed on all faces, the concrete cover does not seem to have significant effects. However it is observed that the percentage difference in the required amount of reinforcement decreases with increase in concrete cover at lower normal force levels. The approximate method with uniform distribution of bars tends to lie on the unsafe side at normal load levels greater than 1 ($n \geq 1.0$), as the relative eccentricity ratio approaches unity. It gives safer values, however, when compared to the four bar arrangement.

CONCLUSIONS & RECOMMENDATIONS

Based on the results of the investigations, the following conclusions and recommendations are made.

- The approximate expression for biaxial moment capacity of rectangular cross-sections under constant normal force in terms of interaction equation according to ACI [5] is found to represent a very good approximation and has close proximity to the more rigorous solution for cross-sections with symmetrical reinforcement patterns.

- In comparison, the approximation for biaxial moment capacity of cross-sections by the approximate interaction equation according to CP110 [2,3] gave results close to the more rigorous solutions only at moderate load levels ($n = 0.6, 0.8$) and for reinforcement pattern with uniform distribution on all faces. Moreover the approximation lies on the unsafe side, especially at larger normal force levels.

- The approximate method of design for biaxially loaded rectangular reinforced concrete columns according to EBCS-2 [1] in which the biaxial moments are converted in to equivalent uniaxial bending moment is found to give mostly conservative results. In very few cases slightly unsafe results with a maximum difference of –2.5% in the required amount of reinforcement are observed. In the majority of the cases investigated, the amount of reinforcement obtained using the approximate method is greater than that of the more rigorous solution, with an average value for the percentage difference ΔA_s of 10% and 16%, for reinforcing bars concentrated at the corners and uniformly distributed on all four sides of the cross-section respectively. Extreme values as high as 25% and 32.5% are observed for the respective arrangement of reinforcement. Such high deviations are attributed to relatively higher values of the coefficient γ in table 1, especially for $n = 0.2$ and 0.4. It is thus recommended that the values of γ as modified in table 2 be used in lieu of the α- values recommended by EBCS-2 [1].

Table 2:

N	0.0	0.2	0.4	0.6	0.8	≥1.0
γ	0.60	0.65	0.75	0.70	0.60	0.50

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