ASSESSMENT OF IMPORTANT SEISMIC PROVISIONS OF EBCS 8 – 1995 FROM STRUCTURAL DYNAMICS PERSPECTIVE

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ABSTRACT

In this paper, an attempt is made to assess some of the important seismic provisions and specifications of the 1995 Ethiopian Building Code Standard (EBCS 8) from the perspective of the theory of structural dynamics and principles of earthquake engineering. For this purpose, a concise but thorough theoretical background is provided on the vibration of structural systems. Only horizontal earthquake excitation of the systems is treated, as this causes the most important effects on building structures. Many concepts that are unknown to the average structural designer, but specified by the Code for strict adherence, are clarified and critically assessed. The design spectra of the Code are compared with early research results that served as the basis for the preparation of design spectra of many international codes. Existing discrepancies are shown and illustrated using an example problem. Important conclusions are drawn and some recommendations made. Comparison of the provisions of the Code with those of other international codes is not the intention of the paper. This is postponed to a future work.

Keywords: *single-degree-of-freedom-system (SDOF), multi-degree-of-freedom-system (MDOF), generalized-single-degree-of-freedom-system (GSDOF), response spectrum, design spectrum, time-history analysis (THA), response-spectrum analysis (RSA), tripartite plot, Rayleigh quotient, story drift.*

INTRODUCTION

Provisions for earthquake resistant design of structures in Ethiopia have been made available since the time of release of the three-volume Ethiopian Standard Code of Practice (ESCP) in 1983. The seismic provisions in this code occupied only few pages in the small volume for loading, ESCP 1, and were limited to pseudo-static analysis only [1]. The revised edition of the Code published in 1995 devoted an independent volume, EBCS 8, for earthquake design [2]. In addition to the provisions for pseudo-static analysis, this recent code specified also design spectra for dynamic analysis among many other important provisions.

The relations provided in both the 1983 and 1995 versions for pseudo-static analysis are indeed relatively simple to use. However, much is unknown to the average user about their theoretical background. Such a situation creates a filling of uncertainty and lack of confidence on the use of computed results.

The 1995 edition of the Code further requires dynamic analysis for certain classes of structures making it relatively demanding for many structural designers with limited or no exposure to structural dynamics and earthquake engineering.

With this in mind, this paper attempts to assess some of the provisions of the appropriate volume, EBCS 8, of the recent code from structural dynamics perspective. With the majority of designers in view, the author found it necessary to provide as much background material as possible before directly embarking into the assessment of the selected code requirements and specifications. This is accomplished by presenting the governing equations and solutions of earthquake excited discrete-mass systems ranging from SDOF through GSDOF to MDOF. A background material on the preparation of response and design spectra is also provided.

This is then employed to view some important provisions and requirements of the Code. The application of the design spectra of EBCS 8 are illustrated using an example problem, and a comparison is made with design spectra proposed in early research works [4,7]. Important conclusions are drawn and recommendations are made on the basis of this review.

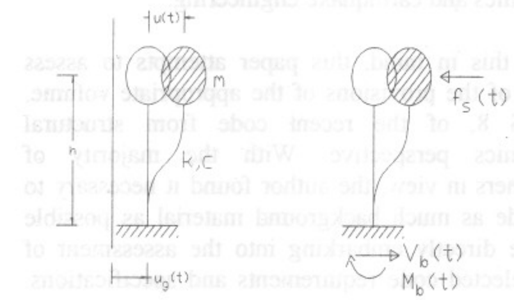
Comparison of the provisions of EBCS 8 with those of other international codes is not the intention of this paper. This is postponed as the subject of a future work.

It is hoped that the presented work will serve a purpose as enlightenment on key concepts of structural dynamics pertinent to seismic provisions. Taking into account the inevitability of regularly updating provisions of codes, the work is also believed to have indicated important aspects of our seismic code provisions that need revisiting.

THEORETICAL BACKGROUND

Earthquake Excitation of Single-Degree-of-Freedom Systems (SDOF)

With the aim of introducing some of the fundamental concepts underlying the construction of response and design spectra specified by design codes and treated later in this paper, reference is made to the SDOF system shown in Fig. 1(a).



(a) (b)

Figure 1 An SDOF system subjected to a horizontal ground motion component:
(a) the displacement response;
(b) the equivalent static force, base shear and overturning moment.

The governing differential equation of motion for the relative deformation, $u(t)$, of the linear SDOF system shown with mass m, stiffness k, and damping coefficient c subjected to the ground motion of $u_g(t)$ is given by

$$\ddot{u}(t) + 2\xi\omega_n \dot{u}(t) + \omega_n^{\ 2} u(t) = -\ddot{u}_g(t) \qquad (1)$$

in which $\omega_n = \sqrt{k/m}$ is the natural frequency and $\xi = c/(2m\omega_n)$ is the damping ratio of the system. The dots on top of u and u_g indicate derivatives with respect to time - the single dot meaning velocity and the double dot acceleration.

Time History of Responses

Because of the jaggedness of the time variation of records of earthquake ground acceleration, $\ddot{u}_g(t)$, which are available in digitized form at short intervals of time (normally 0.02 or 0.01s), Eq. (1) is normally solved by direct time stepping methods.

Once the time history of the relative deformation u(t) is known, the time history of the equivalent static force, $f_s(t)$, indicated in Fig. 1(b) is obtained as

$$f_s(t) = ku(t) = m\omega_n^{\ 2} u(t) = mA(t) \qquad (2)$$

In Eq. (2), $A(t) = \omega_n^{\ 2} u(t)$ is referred to as the pseudo-acceleration response and is neither equal to the relative acceleration $\ddot{u}(t)$ nor to the total acceleration $\ddot{u}^t(t)$.

The time history of the base shear, $V_b(t)$, and the overturning moment, $M_b(t)$, can be determined from static analysis of the system subjected to the equivalent static force (see Fig. 1(b)). Thus,

$$V_b(t) = \frac{A(t)}{g}W \qquad (3)$$
$$M_b(t) = \frac{A(t)}{g}Wh$$

in which W is the weight of the structure, h is the height to the location of the mass from the base and g is the gravitational acceleration.

Response Spectra

The design of structures is generally based on peak responses instead of their time history. Convenient tools to summarize the peak responses of all possible SDOF systems to a given ground motion are response spectra, in which the peak values of response quantities are plotted against the natural period or frequency for selected values of the damping ratio.

Accordingly, the deformation response spectrum, D, for an SDOF system of known period, T_n, and damping ratio, ξ, can be expressed as

$$D = u_0(T, \xi) = \max |u(t, T, \xi)| \quad (4)$$

The pseudo-acceleration response spectrum, A, follows from Eq. (2) as

$$A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D \quad (5)$$

Similarly, the pseudo-velocity response spectrum, V, is defined as $V = \omega_n D = (2\pi/T_n)D$, where $T_n = 2\pi/\omega_n$ is the natural period of vibration of the system. It is worth noting that the three spectra are interdependent and convey the same information.

Because of their fundamental use in the design of structures, the preparation of plots of normalized pseudo-acceleration spectra or base-shear coefficients, A/g, of Eq. (6) against T_n for representative damping ratios right after recording of ground motions has become a routine exercise. Such plots for El Centro ground motion for damping ratios of 0, 2, 5, 10, and 20% are shown in Figure 2 to a normal scale as an example.

Design Spectra

The design spectrum against a future earthquake for a given region should naturally be a statistical representative of all past earthquakes in the region recorded under similar conditions. It must also be composed of a set of smooth curves, as it is difficult to predict the jaggedness of the spectra of a future earthquake ground motion (see Fig. 2).

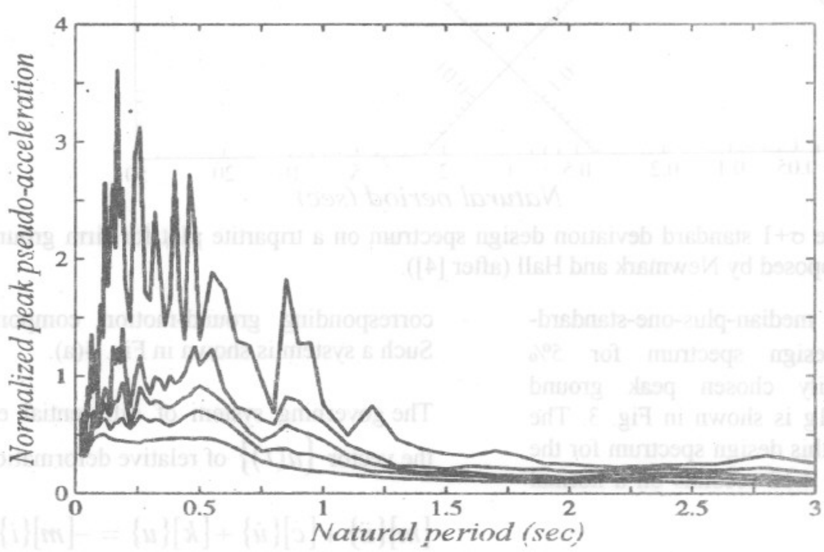


Figure 2 The normalized pseudo-acceleration, or base shear coefficient, spectra for El Centro ground motion plotted to a normal scale for ξ=0, 2, 5, 10 and 20% (after [4]).

Comparison of Eq. (5) with the first of Eq. (3) indicates that the peak value, V_{bo}, of the base shear, or simply the base shear spectrum can be expressed as

$$V_{bo} = f_{so} = \frac{A}{g} W \quad (6)$$

The dimensionless ratio, A/g, in Eq. (6) is referred to as the normalized pseudo-acceleration response spectrum and may be interpreted as the base-shear coefficient.

If no ground motion is recorded in a region, it has been a normal practice to base the design spectrum on ground motions recorded elsewhere under similar conditions. This is, for example, the case with the design spectra specified by EBCS 8 for Ethiopia, which are presumably based on the ground motions of the shallow earthquakes recorded in the western part of the USA and southern Europe.

The most popular procedure for the construction of design spectra is the one developed by N. Newmark and W. Hall based on statistical analysis of ensembles of ground motions recorded on firm ground. They proposed smooth normalized pseudo-acceleration design spectra composed of a series of straight lines on a tripartite logarithmic plot [4].

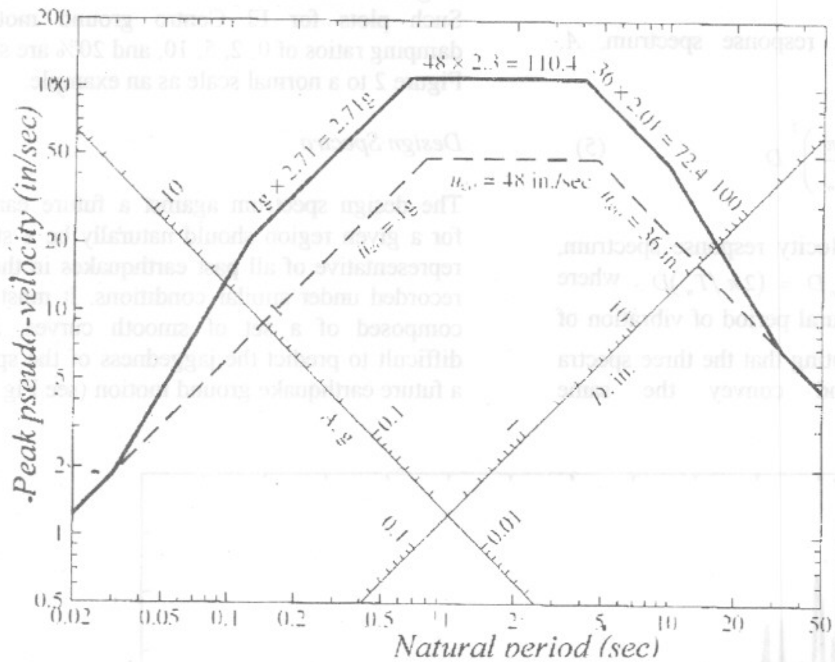


Figure 3 The σ+1 standard deviation design spectrum on a tripartite plot for firm groun proposed by Newmark and Hall (after [4]).

The Newmark-Hall median-plus-one-standard-deviation ($\sigma + 1$) design spectrum for 5% damping and arbitrarily chosen peak ground acceleration (PGA) of 1g is shown in Fig. 3. The following equation fits this design spectrum for the period range of zero to 4.12 seconds on a normal plot:

$$\frac{A}{g} = \begin{cases} 1; & T \le 0.03s \\ 11.7T^{0.704}; & 0.03 < T \le 0.125s \\ 2.71; & 0.125 < T \le 0.66s \\ 1.8/T; & 0.66 < T \le 4.12s \end{cases} \quad (7)$$

A plot of Eq. (7) is presented in Fig. 5 together with the design spectra specified by EBCS 8 for pseudo-static and dynamic analysis.

Earthquake Excitation of Multi-Degree-of-Freedom Systems (MDOF)

In this section, a brief account is made of the vibration of an MDOF-system subjected to a horizontal component of an earthquake ground motion. For symmetric-plan multi-story buildings, the analysis can generally be performed on the basis of plane frames in each of the two mutually perpendicular horizontal directions subjected to the corresponding ground-motion component, $u_g(t)$. Such a system is shown in Fig. 4(a).

The governing system of differential equations for the vector $\{u(t)\}$ of relative deformation is

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = -[m]\{i\}\ddot{u}_g(t) \quad (8)$$

in which $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrices of the system, respectively, and $\{i\}$ is the influence vector whose elements represent the displacement or rotation in each degree of freedom due to a unit magnitude of $u_g(t)$.

Forced Vibration – Time History Analysis (THA)

The equation of the forced vibration established in Eq. (8) can be solved by transformation of coordinates from geometric to modal using the substitution [4,6]:

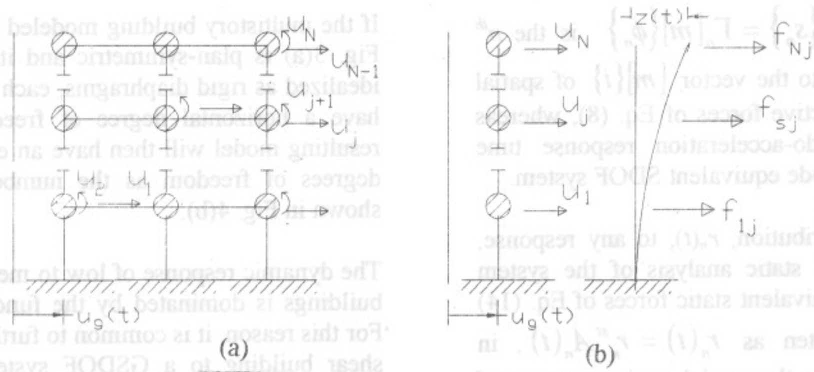


Figure 4 An MDOF system subjected to a horizontal ground motion component: (a) moment frame; (b) shear building.

$$\{u(t)\} = \sum_{r=1}^{N} \{\phi_r\} q_r(t) = [\Phi]\{q(t)\} \quad (9)$$

In Eq. (9), $[\Phi]$ is the modal matrix with the natural modes as its columns and $\{q(t)\}$ is the vector of modal coordinates yet to be determined. Introducing this equation in Eq. (8), pre-multiplying the resulting equation with $[\Phi]^T$ and utilizing the orthogonal nature of the natural modes, one obtains the following system of N uncoupled differential equations for proportionally damped systems:

$$M_n\ddot{q}_n + C_n\dot{q}_n + K_n q_n = -L_n\ddot{u}_g(t); \quad n=1,2,\dots,N. \quad (10)$$

In Eq. (10) M_n, C_n, K_n and L_n are known as the generalized mass, damping, stiffness and loading coefficient, respectively, that can easily be shown to be given by

$$M_n = \{\phi_n\}^T[m]\{\phi_n\}, \quad K_n = \{\phi_n\}^T[k]\{\phi_n\},$$
$$C_n = \{\phi_n\}^T[c]\{\phi_n\} \text{ and } L_n = \{\phi_n\}^T[m]\{i\}.$$

Dividing Eq. (10) by M_n throughout results in

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n\ddot{u}_g(t); \quad n=1,2,\dots N \quad (11)$$

where ω_n is the natural frequency and ξ_n is the damping ratio of the n^{th} mode, while $\Gamma_n = L_n/M_n$

Eq (11) is analogous to Eq. (1), except the coefficient, Γ_n, on the right-hand side, and is solved in a similar manner. If we replace $u(t)$ in Eq. (1) by $D_n(t)$, the solution of Eq. (11) can thus be written as

$$q_n(t) = \Gamma_n D_n(t) \quad (12)$$

Once the modal coordinates are known as such, the displacement response vector is obtained using Eq. (9). Since for most structures the total response is dominated by the first few modes, the summation is usually limited to these modes only so that

$$\{u(t)\} = \sum_{n=1}^{N_d}\{\phi_n\}\Gamma_n D_n(t), \quad (13)$$

where N_d is the number of significant modes considered. A technique of accounting for the static contribution of the remaining higher modes is developed in Reference [6].

Any other instantaneous response is generally obtained by static analysis of the system subjected to the vector of equivalent static forces, which is obtained analogous to Eq. (2) using Eq. (13) as

$$\{f_{sn}(t)\} = \omega_n^{\,2}[m]\{u_n(t)\} = (\Gamma_n[m]\{\phi_n\})\omega_n^{\,2}D_n(t) \\ = \{s_n\}A_n(t) \quad (14)$$

where the vector $\{s_n\} = \Gamma_n[m]\{\phi_n\}$ is the n^{th} mode contribution to the vector $[m]\{i\}$ of spatial distribution of effective forces of Eq. (8), whereas $A_n(t)$ is the pseudo-acceleration response time history of the nth mode equivalent SDOF system.

The n^{th} mode contribution, $r_n(t)$, to any response, $r(t)$, obtained from static analysis of the system subjected to the equivalent static forces of Eq. (14) may now be written as $r_n(t) = r_n^{st}A_n(t)$, in which r_n^{st} represents the modal static response of the system subjected to the time-independent modal vector of forces $\{s_n\}$ of Eq. (14). The total response is then given by

$$r(t) = \sum_{n=1}^{N_d} r_n(t) = \sum_{n=1}^{N_d} r_n^{st} A_n(t) \quad (15)$$

The modal pseudo-acceleration time history, $A_n(t)$, is obtained from dynamic analysis of N_d different equivalent SDOF systems.

Forced Vibration – Response Spectrum Analysis (RSA)

As indicated earlier in connection with SDOF systems, the peak responses are what are needed for design purposes. The peak value, r_{no}, of any modal response, r_n, follows as

$$r_{n0} = r_n^{st} A_n \quad (16)$$

where $A_n = A_n(\xi_n, T_n)$ is the n^{th}-mode pseudo-acceleration spectrum. This value is directly read from available design spectra like those shown in Figure (5) by entering with the natural period, T_n, and damping ratio, ξ_n, of the n^{th} mode.

The total peak response, r_0, due to all modes considered in the RSA is obtained by employing available modal combination rules.

Earthquake Excitation of Generalized-Single-Degree-of-Freedom Systems (GSDOF)

If the multistory building modeled by the frame in Fig. 3(a) is plan-symmetric and its floors can be idealized as rigid diaphragms, each floor mass will have a horizontal degree of freedom only. The resulting model will then have an equal number of degrees of freedom as the number of floors as shown in Fig. 4(b).

The dynamic response of low to medium-rise shear buildings is dominated by the fundamental mode. For this reason, it is common to further simplify the shear building to a GSDOF system. A GSDOF system is assumed to vibrate according to a single assumed deflected shape only. This shape is mostly the approximated fundamental mode $\{\phi_1\}$. On the basis of this mode, the vector of displacement response can be expressed as

$$\{u(t)\} \approx \{\phi_1\}z(t) \quad (17)$$

where $z(t)$ is known as the generalized coordinate and is yet to be found.

The equation of motion of the system can be easily formulated using the principle of virtual work and is

$$\tilde{m}\ddot{z} + \tilde{c}\dot{z} + \tilde{k}z = -\tilde{L}\ddot{u}_g(t), \quad (18a)$$

where $\tilde{m} = \{\phi_1\}^T[m]\{\phi_1\}$, $\tilde{k} = \{\Delta\phi_1\}^T[k]\{\Delta\phi_1\}$, $\tilde{c} = \{\phi_1\}^T[c]\{\phi_1\}$ and $\tilde{L} = \{\phi_1\}^T[m]\{I\}$ are the generalized mass, stiffness, damping and load factor, respectively, for the assumed shape. The influence vector $\{I\}$ has the value of unity for all its elements, while the vector $\{\Delta\phi_1\}$ is the vector of story drift. Dividing Eq. (18a) by $\tilde{m}$ throughout yields

$$\ddot{z}(t) + 2\xi_1\omega_1\dot{z}(t) + \omega_1^{\,2}z(t) = -\tilde{\Gamma}\ddot{u}_g(t), \quad (18b)$$

in which $\omega_1 = \sqrt{\tilde{k}/\tilde{m}}$ is the approximated fundamental frequency, ξ_1 is the corresponding damping ratio and $\tilde{\Gamma} = \tilde{L}/\tilde{m}$.

Since both the stiffness and mass matrices for a shear building are diagonalized, the expression for the fundamental frequency takes the following form:

$$\omega_1^2 = \frac{\tilde{k}}{\tilde{m}} = \left(\sum_{j=1}^{N} k_j (\Delta\phi_1)_j^2\right) \Big/ \left(\sum_{j=1}^{N} m_j (\phi_1)_j^2\right) = \left(\frac{2\pi}{T_1}\right)^2 \quad (19a)$$

Noting that the summation in the numerator of Eq. (19a) is the total internal work done and should equal the total external work done due to the developed inertia forces at the masses, the expression for the fundamental period follows easily as

$$T_1 = 2\pi \sqrt{\left(\sum_{j=1}^{N} W_j (\phi_1)_j^2\right) \Big/ \left(g \sum_{j=1}^{N} f_j (\phi_1)_j\right)} \quad (19b)$$

The relation in Eq. (19b) is called *Rayleigh Quotient*, in which W_j is the weight at the j^{th} floor and f_j is the corresponding inertia force induced during the free vibration. As long as this equation is used for the estimation of the fundamental period, any reasonable set of forces, f_j, other than the inertia forces may be used that brings about the assumed deformed shape, $\{\phi_1\}$, as the fundamental mode.

Coming now back to Eq. (18b), its solution is analogous to Eq. (12). Thus, the generalized coordinate is obtained as

$$z(t) = \tilde{\Gamma} D_1(t) \quad (20)$$

Once the generalized coordinate, $z(t)$, is so obtained, the time history of the displacement vector is determined using Eq. (17).

The vector of peak displacement response follows from Eq. (17) as

$$\{u_0\} = \{\phi_1\} \tilde{\Gamma} D_1 \quad (21)$$

Analogously, the vector of peak equivalent static forces becomes

$$\{f_{s0}\} = \tilde{\Gamma} [m] \{\phi_1\} A_1 \quad (22)$$

In Eq. (21) and (22), D_1 and A_1 are the peak displacement and peak pseudo-acceleration, respectively, obtained from design spectra by entering with the natural fundamental period T_1 and the corresponding damping ratio ξ_1

The peak values of all responses then follow from static analysis of the system subjected to the forces of Eq. (22). The peak value of the base shear in particular becomes

$$V_{b0} = \sum_{j=1}^{N} f_{j0} = \tilde{\Gamma} A_1 \sum_{j=1}^{N} m_j (\phi_1)_j = \tilde{L} \tilde{\Gamma} A_1 \quad (23)$$

Introducing Eq. (23) in Eq. (22), the equivalent static force at the i^{th} floor becomes

$$(f_{s0})_i = \frac{W_i \phi_i}{\sum_{j=1}^{N} W_j \phi_j} V_{b0} \quad (24)$$

It will be shown later on that Eq. (24) is the basis for formulas provided by codes for the distribution of the base shear among the floors in the method of pseudo-static analysis.

STRUCTURAL DYNAMICS IN EBCS 8, 1995

Base Shear

The appropriate volume of the 1995 Ethiopian Building Code Standard - Design of Structures for Earthquake Resistance (EBCS 8) - specifies the base shear as

$$F_b = S_d W = \alpha \beta \gamma W = \alpha_0 I \beta \gamma W \quad (25)$$

where, according to the Code, $S_d(T_1)$ is the ordinate of the *design spectrum* at period T_1; α_0 is the bedrock acceleration ratio for the site that depends on the seismic zone; I is the importance factor; β is the design response factor for the site; γ is the behavior factor that accounts for the energy dissipation capacity of the system; W is the seismic dead load of the structure.

The bedrock acceleration ratio, α_0, for the site assumes one of the five values of 0.10, 0.07, 0.05, 0.03, or 0 depending on whether the site is located in Zone 4, 3, 2, 1, or 0, respectively. To the importance factor, I, is assigned one of the values of 1.4, 1.2, 1.0, or 0.8 depending on the function of the building and the degree of ensuing hazard if damaged by earthquake.

The design response factor for the site, β, is determined from

$$\beta(T_1) = \frac{1.2S}{T_1^{2/3}} \leq 2.5 \qquad (26)$$

where S is referred to as the site coefficient for soil characteristics that assumes one of the values 1.0, 1.2, or 1.5 for the subsoil class A, B, or C, respectively, defined in the Code, the largest being for deep soft deposits.

T_1 in Eq. (26) is the fundamental period of vibration of the structure for the translational motion in the direction considered. The Code allows for the use of approximate expressions based on methods of structural dynamics for the determination of T_1 citing *Rayleigh method* as an example. Instead of providing such an expression that has a rational basis in structural dynamics as shown earlier (see Eq. (19b)), it gives only empirical relations that depend on the dimensions of the building, the building material type, and the lateral force resisting system.

It is, however, more appropriate to calculate the fundamental period based on an approximate shape for the fundamental mode. For shear buildings, a linear deformed shape is recommended [5]. Adopting this recommendation, the expression in Eq. (19b) becomes

$$T_1 = 2\pi\sqrt{\left(b\sum_{j=1}^{N} W_j (\phi_1)_j^2\right) \Big/ \left(g\sum_{j=1}^{N} f_j (\phi_1)_j\right)} \qquad (27)$$

in which b is the slope of the assumed linear fundamental mode. While this expression is given by many codes, EBCS 8 does not explicitly provide it.

Values of the behavior factor, γ, are specified by the Code depending on material type, ductility class, geometry, and structural system only, though the factor also exhibits some variation with respect to the natural period [4,7].

For an ordinary building (I=1) to be constructed at a site of PGA of 1g and subsoil class A (S=1), the elastic seismic coefficient, $S_e(T_1)$, corresponding to $S_d(T_1)$ of Eq. (25) for $\gamma = 1$, becomes

$$S_e(T_1) = \frac{1.20}{T_1^{2/3}} \leq 2.50 \qquad (28)$$

A plot of Eq. (28) against T_1 to a normal scale is shown in Figure 5 for the period range of 0 to 4 seconds together with the $\sigma + 1$ design spectra of Eq. (7) proposed by Newmark and Hall.

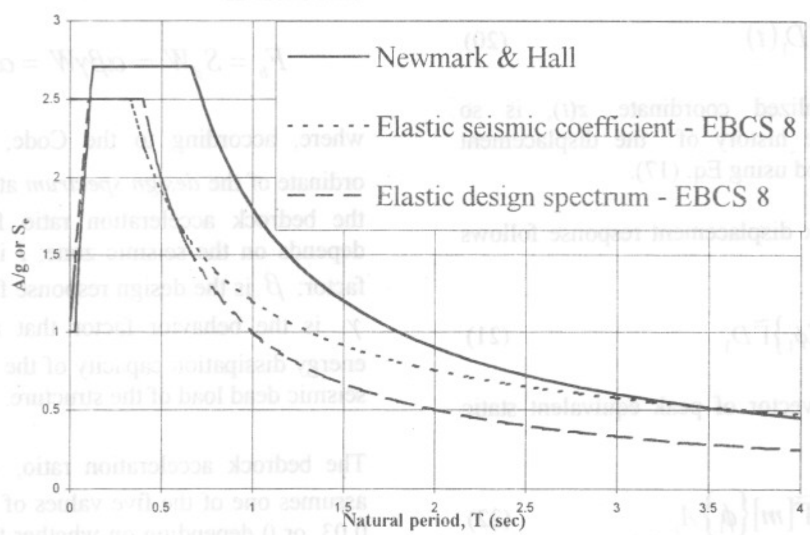


Figure 5 Comparison of the EBCS 8 elastic design spectrum for dynamic analysis and the elastic seismic coefficient for pseudo-static analysis with the Newmark and Hall σ+1 design spectrum for firm ground.

The similarity in trend of the two curves indicates that the expression in Eq. (28) for the elastic seismic coefficient specified by EBCS 8 has its roots in the theory of structural dynamics. This is presumably the reason why the Code refers to S_d as the *design spectrum* (see the statement under Eq. (25)). It is, however, important to observe the significant difference in the ordinates of the two curves in the important period range indicating that EBCS 8 spectrum for pseudo-static analysis is much lower than the Newmark-Hall design spectrum. This observation is especially important in view of the fact that the pseudo-static analysis does not properly account for the contribution of the higher modes.

Lateral Force Distribution

The Code specifies that the base shear force obtained according to Eq. (25) be distributed over the height of the building according to

$$F_j = (F_b - F_t)\left(W_j h_j \Big/ \sum_{i=1}^{N} W_i h_i\right) \quad (29)$$

where a top force, F_t, is to be added to F_N obtained from Eq. (29). This top force is given by

$$F_t = 0.07 T_1 F_b \quad (30)$$

Eq. (29) is practically the same as Eq. (24) with the exception of the top force that is deducted from the base shear with the intention of accounting for the influence of the higher modes that increase the story shears at the top stories for long-period (flexible or tall) buildings. It is, however, important to note that, unlike other codes [3], EBCS 8 specifies the deduction of F_t invariably for all structures.

Design Spectra

For use in dynamic analysis, EBCS 8 also specifies the "elastic response spectra" (presumably to mean the pseudo-acceleration design spectra)[1] in a graphical form, which, for $I = 1$, $S = 1$, and a site of PGA of 1g can mathematically be expressed as

$$\frac{A}{g} = \begin{cases} 1 + 15T; & 0 \le T \le 0.10\,s \\ 2.5; & 0.10 < T \le 0.40\,s \\ 1/T; & T > 0.40\,s \end{cases} \quad (31)$$

The plot of Eq. (31) is also shown in Fig. 5 together with the plots of C_e and the $\sigma + 1$ design spectrum of Newmark and Hall. The similarity in trend of all the three curves is once again evident. However, it is important to observe that the EBCS 8 design spectrum for dynamic analysis yields once again consistently less magnitudes of base shear than the Newmark and Hall spectra. It also gives less base shear than the one provided for pseudo-static analysis for T_n>0.6 sec. Since the dynamic analysis considers the contribution of modes other than the fundamental one, there seems to be a certain degree of compromise within the two EBCS 8 spectra. However, the justification is not apparent for specifying both curves significantly lower than the Newmark and Hall design spectra, which are based on an exhaustive statistical study of past earthquakes and are the basis for similar provisions of many international codes. This issue is left as the subject of a future work, in which similar provisions of other international codes will be compared.

Example:

The use of the EBCS 8 and the Newmark-Hall design spectra in both the pseudo-static and dynamic analyses is illustrated on a four-story reinforced concrete building schematically represented in Fig. 6(a). The building is an ordinary office building with rigid diaphragms to be constructed at a site of firm ground in Zone 4 in Ethiopia. It is required to determine the height-wise distribution of peak equivalent static forces, the elastic base shear and base overturning moment. Take m=140x10^{3}kg, k=12x10^{3}kN/m and h=3.0m.

Solution:

(1) Pseudo-static Analysis According to EBCS 8

(a) Period Computation – Empirical

The natural period is computed using Eq. (2.3) of the Code to get T_1=0.48sec. For firm ground, S=1 is used. The design response factor is computed using

[1] The concept of elastic *response spectrum*, even if it is used to mean elastic pseudo-acceleration spectrum, is quite different from the concept of elastic pseudo-acceleration *design spectrum* commonly specified in design codes as explained in the previous section. The latter is a smoothened statistical average of ensembles of the former and doesn't represent the response of structures to any particular earthquake.

Eq. (26) to get β=1.96. In addition, for the given conditions, $\alpha_0 = 0.1$ and $\gamma = I = 1$ are taken. The base shear is then computed using Eq. (25) to obtain F_b=1098 kN.

using the empirical relation provided by EBCS 8. This indicates that the EBCS 8 formula leads to an uneconomical design of the lateral force resisting system, at least in this example.

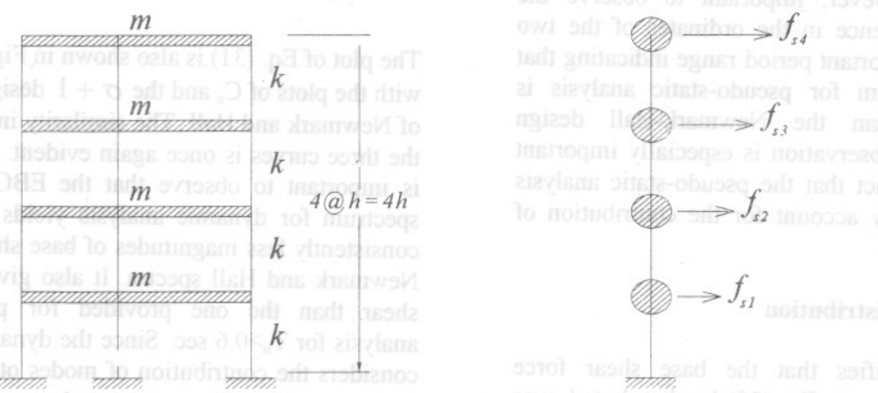


Figure 6 (a) A four-story building with rigid diaphragms; and (b) peak equivalent static forces

The base shear is distributed according to Eqs. (29) and (30). These are tabulated in Table 3 together with the corresponding results of the other methods. The base overturning moment is found to be M_b= 9992 kN-m.

(b) Period Computation – Rayleigh

We compute now the natural period using Rayleigh Quotient (Eq. (19b)) assuming a linear deformed shape with unit deformation at the top-most floor. This gives a fundamental mode of $\{\phi_1\}^T = \{1, 0.75, 0.50, 0.25\}$. Using the set of lateral forces computed in (a) above as f_j, the natural period as computed using Eq. (19b) becomes T_1=3.56sec. This is significantly larger than the value of T_1=0.48sec computed in (a). With this value of the period, the design response factor becomes β=0.51 and the base shear F_b= 286kN. The lateral distribution of this shear force is also shown in Table 3.

It is also common to use the weight of each floor applied horizontally for the forces f_j in Rayleigh quotient. This results in T_1=1.74sec, β=0.83, and F_b= 465kN.

These values of the base shear computed on the basis of Rayleigh Quotient for the natural period are only 26% and 42% of the one computed earlier

(2) Dynamic Analysis

(a) Building Modeled as GSDOF System with Linear Deformed Shape

Noting that $\Delta\phi_j = 0.25$, the natural period is computed using Eq. (19a) to obtain T_1=1.88sec. The pseudo-acceleration spectrum is determined from Eqs. (7) and (31) according to Newmark & Hall and EBCS 8, respectively. For the period just obtained, these equations simplify to $A_1 = 1.8\alpha_0 g/T_1$ and $A_1 = \alpha_0 g/T_1$ and yield, with the above value of the period, 0.523m/sec^2 and 0.942m/sec^2, respectively, for the peak pseudo-acceleration.

The generalized parameters are obtained as $\widetilde{L} = 350x10^3 kg-m$, $\widetilde{m} = 262.5x10^3 kgm^2$ and $\widetilde{\Gamma} = 1/3m^{-1}$. Then, the base shear is computed using Eq. (23) to get 439kN and 244kN according to Newmark & Hall and EBCS 8, respectively. The distribution of the peak values of the equivalent static forces is shown in Table 3.

(b) RSA Considering All Modes

The mass and stiffness matrices of the system are

$$[m]=m\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \text{ and } [k]=k\begin{bmatrix}2&-1&0&0\\-1&2&-1&0\\0&-1&2&-1\\0&0&-1&1\end{bmatrix}$$

With these matrices, the natural periods are computed as T_1=1.963sec; T_2=0.679sec; T_3=0.443sec; and T_4=0.361sec.

The natural modes normalized with respect to the displacement of the top-most floor become

$$\{\phi_1\}=\begin{Bmatrix}1\\0.88\\0.65\\0.35\end{Bmatrix};\quad \{\phi_2\}=\begin{Bmatrix}1\\0\\-1\\-1\end{Bmatrix};$$

$$\{\phi_3\}=\begin{Bmatrix}1\\-1.35\\-0.53\\1.51\end{Bmatrix};\quad \{\phi_4\}=\begin{Bmatrix}1\\-2.53\\2.87\\-1.88\end{Bmatrix}$$

The generalized parameters in Eq. (10) are computed and tabulated in Table 1 together with the pseudo-acceleration spectral values, A_n.

Any modal response is then obtained by statical analysis of the system subjected to the modal peak equivalent static forces. The peak values of the equivalent static forces are shown in Table 2 together with the base shear and base overturning moment for each mode.

The combined base shear using the combination rule of the square-root-of-the-sum-of-the-squares (SRSS) is found to be 260.2kN and 467.3kN according to EBCS 8 and Newmark & Hall design spectra, respectively. The combined base overturning moment becomes also 2170kNm and 3906kNm, respectively. It is worth noting that these values are very close to the corresponding values contributed by the fundamental mode only (see Table 2).

Table 1: Values of modal generalized parameters in the RSA.

	Mode	1	2	3	4
M_n		2.32m	3m	5.38m	19.17m
L_n		2.88m	-m	0.63m	-0.54m
Γ_n		1.241	-0.333	0.117	-0.028
A_n(m/sec^2)	EBCS 8	0.500	1.445	2.214	2.500
	Newmark & Hall	0.90	2.60	2.71	2.71
$\Gamma_n A_n m$(kN)	EBCS 8	86.87	-67.37	36.27	-9.80
	Newmark & Hall	156.37	-121.21	44.39	-10.62

Table 2: Values of modal lateral forces, base shear and base overturning moment

	Mode	1	2	3	4
Peak eq. static forces (kN)					
4th Floor	EBCS 8	86.87	-67.37	36.27	-9.8
	Newm. & Hall	156.37	-121.21	44.39	-10.6
3rd Floor	EBCS 8	76.45	0	-48.96	24.79
	Newm. & Hall	137.60	0	-59.32	26.86
2nd Floor	EBCS 8	56.47	67.37	-19.22	-28.1
	Newm. & Hall	101.65	121.21	-23.52	-30.45
1st Floor	EBCS 8	30.40	67.37	54.77	18.42
	Newm. & Hall	54.72	121.21	67.03	19.96
V_b (kN)	EBCS 8	250.2	67.37	22.86	5.31
	Newm. & Hall	450.4	121.20	28.00	5.75
M_b (kNm)	EBCS 8	2160.5	-202.1	43.6	-7.83
	Newm. & Hall	3888.9	-363.63	53.4	-8.28

A comparison of the contribution of the fundamental mode to the total peak equivalent lateral forces, the base shear and base overturning moment with the corresponding values obtained using the different methods is given in Table 3.

This example illustrates that the elastic design spectrum for firm ground provided by EBCS 8 for dynamic analysis results in consistently smaller values of lateral forces, base shear and overturning moment than those obtained using the corresponding Newmark & Hall spectrum. This is also evident from Fig. 5.

Table 3: A comparison of peak equivalent forces, base shear and base overturning moment

Floor	Pseudo-static		Linear GSDOF		1st Mode of RSA	
	Empirical	Rayleigh	EBCS 8	Newmark & Hall	EBCS 8	Newm. & Hall
4th	461.3	114.4	97.6	175.6	86.87	156.37
3rd	318.3	85.8	73.2	131.7	76.45	137.60
2nd	212.2	57.2	48.8	87.8	56.47	101.65
1st	106.2	28.6	24.4	43.9	30.40	54.72
V_b (kN)	1098	286	244	439	250.2	450.4
M_b (kNm)	9992	2488.2	2122.8	3819.3	2160.5	3888.9

It is also clear from Figure 5 that the plot of the seismic coefficient, S_e, specified for pseudo-static analysis is much lower than the other two curves for periods greater than about 0.6sec. However, the base shear of 1098kN obtained using the pseudo-static analysis is excessively greater than the values of 260.2kN and 467.3kN computed using full dynamic RSA with the participation of all modes. This paradoxical result is a direct consequence of the use of the empirical relation provided by the code for the estimation of the natural period.

It has been shown, on the basis of the example, that a better prediction of the base shear and overturning moment is achieved by employing Rayleigh's Quotient of Eq. (19b), which, unfortunately, is not provided by the Code. It is also shown that the estimation of the base shear can further be improved by using the lateral forces obtained on the basis of the empirical relation of the Code for the forces, f_i, in Eq. (19b).

CONCLUSIONS AND RECOMMENDATIONS

On the basis of the theoretical analysis and the illustrative example solved, the following conclusions and recommendations are made:

CONCLUSIONS

1. The empirical relations provided by EBCS 8 for the estimation of the natural fundamental period can result in highly erroneous values of the base shear.

2. The EBCS 8 spectra evidently deviate from the Newmark & Hall σ+1 spectrum, despite the fact that the latter is based on a thorough statistical analysis of carefully selected earthquake ground motions recorded on firm ground under similar conditions. In view of the fact that this spectrum is the basis for provisions of many international codes, no explanation is apparent why the EBCS 8 spectra are so much smaller than the Newmark & Hall spectra.

3. The top force given by Eq. (30) is supposed to be always deducted from the base shear irrespective of the magnitude of the period. It is, however, important to note that this reduction is needed only for long-period (flexible or high-rise) structures to account for the effect of the higher modes.

4. In comparison with the Newmark & Hall σ+1 spectrum, also the EBCS 8 design spectum for dynamic analysis gives smaller design forces for short-period (stiff) structures.

RECOMMENDATIONS

1. EBCS 8 allows indeed the use of Rayleigh method for the estimation of the natural fundamental period but does not provide the expression while it gives empirical relations for this purpose. It is rather recommended that a convenient form of Rayleigh Quotient like

Eq. (27) be incorporated in the Code and the empirical relations used only for the purpose of estimating the lateral forces, f_j, needed in this equation.

2. The fact that the EBCS 8 design spectra for both dynamic and static analysis lie significantly below the Newmark & Hall σ+1 spectrum need reconsideration and explanation.

3. The expression for the top force to be reduced from the base shear should be different for different period ranges. For short-period structures, for example, no reduction should be made (F_t=0). The specifications of the Uniform Building Code (UBC 1994 or UBC 1997) may be adopted.

4. A comparison of the design spectra of EBCS 8, 1995 with the corresponding spectra of the recent versions of international codes like UBC 1997 and the International Building Code (IBC 2000) should be encouraged, so that the status of the former is identified.

5. This paper dealt with EBCS 8 design spectra for firm ground. Further studies on the design spectra of the Code for other soil categories should be encouraged.

6. Consideration of other aspects like inelastic response and soil-structure interaction in both the pseudo-static and dynamic analysis would make the code specifications more realistic and complete.

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