OPTIMAL DESIGN OF SOLAR WATER HEATING SYSTEMS

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ABSTRACT

Solar water heating systems are usually designed using simplified equation of annual efficiency of the heating system from solar radiation incident on the collector during the year and empirical values of annual efficiency. The performance of the preliminary design is predicted by using either f-chart method or by transient simulation of solar heating system. Often, optimization is done off-line after correlating the annual contribution of solar energy to the heating load and collector area using simulation results by analytical methods. In this work, a methodology and a computer programme using univariant search method is developed for optimal design of solar water heating system. The program uses the collector area and the storage tank volume as optimization parameters and the unit energy price of solar water heating system as objective function and the annual fraction of contribution of solar energy to the heating load is considered as constraint.

INTRODUCTION

Rising prices of conventional energy as well as environmental pollution and "the green house effect" due to combustion of fossil-fuels and biomass demand intensive exploitation of renewable energy resources. Among renewable energy resources, solar energy can be economically exploited for domestic and industrial water heating. Solar water heating systems are widely used in Israel and southern USA and they are being introduced in almost every country in the world.

Solar water heating systems with flat-plate collectors are classified into thermo-siphon and forced circulation systems.

1 Collector
2 Downcomer
3 Riser
4 Storage tank
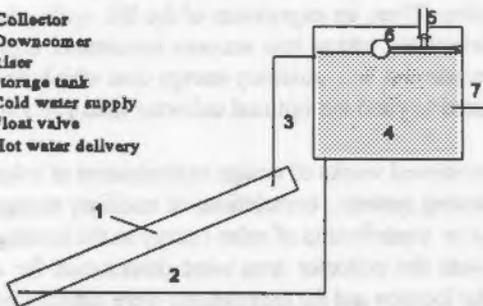5 Cold water supply
6 Float valve
7 Hot water delivery

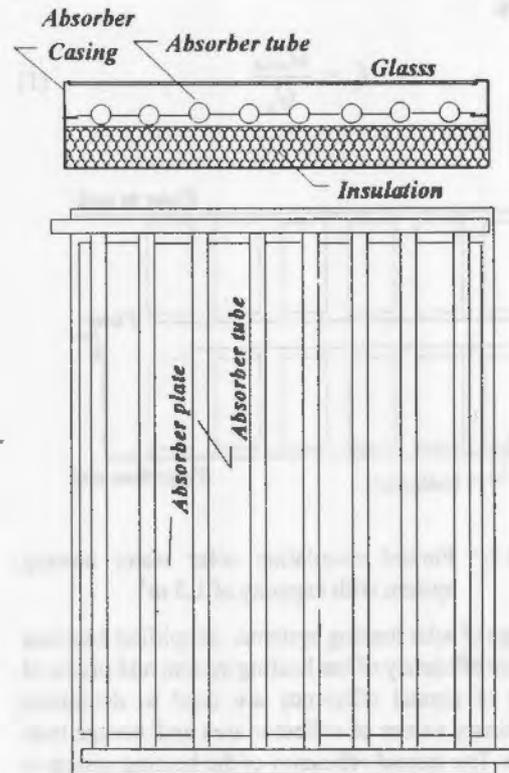Figure 1   Thermosiphon solar water heating system

Figure 2   Flat-plate solar collector

A thermo-siphon solar water heating system, which is given in Fig. 1, consists of one or two flat-plate collectors and a storage tank. A flat plate collector consists of a black absorber which is placed in a casing and covered with glass at the top and insulated at the bottom. Usually, the absorber is manufactured by joining tubes with a plate and connecting them to header tubes of larger diameter at the top and bottom as shown in Fig. 2. The cold water at the bottom of the storage tank flows to the bottom of the collector. As it is heated in the absorber, the density of water becomes lower and it rises through the absorber-tube and the riser-pipe to the top part of the storage tank. For large scale water heating, several collectors are required. Thus, it is economical to have a single storage tank from which the water can be distributed to collectors

by a pump. After the water passes through the collectors, it is collected by a single pipe which leads to the tank. A forced circulation solar water heating system of 1.5 m³ capacity is shown in Fig. 3.

A solar water heating system is designed so that solar energy covers a specified fraction of the heating load. The annual fraction of contribution of solar energy to the heating load is the ratio of solar energy used for water heating to the total energy requirement for water heating.

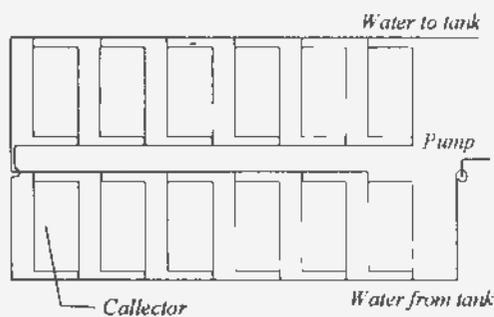$$f_a = \frac{Q_{sol,a}}{Q_a} \qquad (1)$$

Figure 3. Forced circulation solar water heating system with capacity of 1.5 m³

In design of solar heating systems, simplified equation of annual efficiency of the heating system and practical values of annual efficiency are used to determine preliminary values of collector area and storage tank volume. The annual efficiency of the heating system is given as

$$\eta_a = \frac{f_a Q_a}{A_c I_a} \qquad (2)$$

The preliminary design is then improved by using either f-charts method or computer simulation of the heating system.

F-charts are correlations of results of detailed parametric study of solar water heating in terms of easily determined dimensionless variables [1]. In design of solar water heating system, the f-charts are used to determine the monthly fraction of contribution of solar energy to the heating load from which the annual contribution of solar energy to the heating load is determined.

Compared with f-charts method, transient simulation of the heating system gives exact and reliable value of the annual fraction of contribution of solar energy to the heating load [2]. To predict the annual performance of the heating system, the computer program requires the following input data.

- Hourly values of solar radiation and ambient temperature
- Design parameters of collectors and storage tank
- Hot water delivery temperature
- Daily hot water demand and hourly consumption pattern

After carrying integration with time over a period of a year, the computer program gives the following results.

- Fraction of the heating load supplied by solar energy and efficiency of the system on monthly and annual basis
- Hourly storage, absorber and glass temperatures for selected days of the year

The computer simulation helps to improve the design through parametric study.

When this design refinement goes up to finding the design with maximum life cycle saving, it evolves from a design problem to a design optimization problem. Different researchers have followed different paths in generating optimal sizes of solar water heating systems. After an expression of life cycle savings had been formulated and a quadratic relationship had been established between the life cycle savings and the collector area, tables and graphs for design optimization were generated [3]. A mathematical correlation between the fraction of auxiliary energy demand and collector area was established for a specified storage volume and hot water delivery temperature for a particular location using computer simulation. Then, an expression of the life cycle cost was formulated taking into account investment cost, maintenance cost and auxiliary energy cost which was minimized to yield the optimal collector area [4] [5].

In the reviewed works of design optimization of solar water heating systems, correlations of auxiliary energy demand or contribution of solar energy to the heating load with the collector area were determined for a particular location and the expressions were substituted in cost functions to minimize the life cycle costs or

savings. Though computer simulations were used, their application was limited to generation of performance data for off-line optimization. Thus, systematic parameter variation employing mathematical programming techniques or search methods of optimization were not used to obtain the optimum collector area and storage tank volume in these works.

This work deals with the development of a computer system for optimal design solar water heating system with forced circulation.

ECONOMIC ANALYSIS OF SOLAR WATER HEATERS

A solar water heater is designed for supplying a specified quantity of hot water per day at a specified temperature. Thus, the annual heating load of water heating becomes

$$Q_a = 365 \, \rho_w \, V_w c_w (T_{hot} - T_{cold}) \qquad (3)$$

While a fraction of this heating load is covered by solar energy, the rest is met by auxiliary energy supply system. Therefore, the annual contribution of solar energy to the heating load becomes:

$$Q_{a,solar} = f_a \, Q_a \qquad (4)$$

The annual contribution of solar energy to the heating load is obtained by computer simulation of the heating system from the input of climatic data and collector and storage tank design parameters.

In conducting economic analysis of energy conversion systems, life cycle costs and savings are determined. The life cycle costs include investment capital and present worth of operating cost during the useful life of the plant. The operating cost of solar water heating system is mainly the maintenance cost. The life cycle cost of a solar water heating system is determined from the investment cost and the present value of maintenance costs distributed over n years as follows.

$$LCC = C_i + C_m \frac{1+i}{1-i} \left(1 - \frac{1}{(1+i)^n}\right) \qquad (5)$$

The life cycle costs have to be recovered from the savings of energy costs during the life of the plant. Considering, discounting of a sum available in future and inflation rate of energy price, the present worth of

life cycle energy cost savings becomes:

$$LCS = p_e f_a \, Q_a \frac{1+i}{i-i_e} \left(1 - (\frac{1+i_e}{1+i})^n\right) \qquad (6)$$

The unit price of solar energy for water heating is determined from Eqs. 5 and 6

$$p_e = \frac{C_i}{f_a Q_a} \; \frac{1 + \frac{C_m}{C_i} \frac{1+i}{i-1} (1 - (\frac{1}{1+i})^n)}{\frac{1+i}{i-i_e} (1 - (\frac{1+i_e}{1+i})^n)} \qquad (7)$$

If solar energy has to be economically viable, the unit cost of water heating system must be less than that of other alternatives. As the optimal design of solar water heating has the task of minimizing costs of hot water generation, the unit price of solar energy for water heating is made in this work the objective function of the optimization problem.

PARAMETRIC OPTIMIZATION

For optimizing a solar water heating with specified hot water demand at a specified temperature, the collector area and the storage tank volume is varied until a minimum value of the unit price of solar energy for water heating is obtained while observing the minimum value fraction of solar energy contribution to the heating load.

The objective function cannot be minimized from Eq. 7 due to the difficulty of finding a general expression for the annual fraction of solar energy contribution to the heating load as function of the collector area and the storage tank volume. The annual fraction of solar energy contribution to the heating load depends on climatic data, hot water delivery temperature, daily hot water demand and hourly water consumption pattern as well as on design parameters of the collector and the storage tank. Moreover, the collector area and the storage tank volume can be varied only in discrete steps. In such type of problems, search methods were successfully utilized for systematic parameter variation to find the optimum [6].

Search methods usually start with varying parameters one after the other to obtain either a decrease or an increase in the objective function. Search algorithms must have a mechanism for checking convergence and differentiating between local and global optimum.

Search methods differ from each other in the method of feasible path selection and in the speed of convergence. Some of classical multi-variable search methods are alternate search, univariant search and lattice search [7].

In alternate search method, variables are increased or decreased alternatively until a minimum is obtained. Then, the procedure is continued with reduced step size.

Lattice search starts at a point and checks number of points in the different direction of a grid. The feasible point from the previous cycle will become a center for conducting another search. The procedure continues until no point becomes feasible than the central point.

Univariant search starts by carrying out exhaustive one dimensional search and then the cycle repeats itself with changed variable until an increase or a decrease in the objective function cannot be obtained. The univariant search method is best suited when the objective function is strongly dependant on a single parameter.
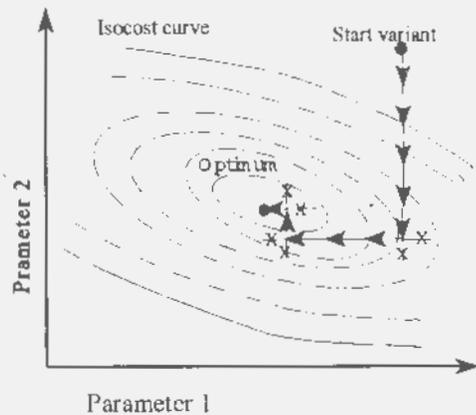

Figure 4   Univariant search method

In the given problem, it was noted that the unit energy price of solar heating is strongly dependant on the collector area. Therefore, the univariant method was selected for optimizing the design of solar water heating system.

OPTIMIZATION PROGRAM

A computer program for the optimization of the heating system was developed using Borland Pascal for MS DOS operating system. The program was finally ported to MS Windows using Visual Basic programming language. This program consists of modules for data input, parameter variation, transient simulation of solar water heating systems, evaluation of the objective function and numerical and graphical output of the results. The program determines the optimal size of solar water heating system or the number of collectors (collector area) and the storage tank volume. Here, it is assumed that the collectors are already optimally designed and they are titled and oriented optimally.

The optimization procedure is carried as follows:

- Number of collectors and storage volume of the start variant are given to the program.

- The optimization program begins the parameter variation cycle of the number of collectors at first and storage tank volume next.

- At each time of parameter variation, the program calls the simulation module to predict the annual performance of the heating system using representative days of every month.

- The main module checks whether the annual fraction of solar energy to the heating load is above the minimum value. If it is not, it increases number of collectors to satisfy this constraint.

- The main module carries parameter variation and checks whether there is a decrease in the unit water heating price. If there is a progress it continues with the parameter variation. Otherwise, it begins variation of the next parameter.

- The search is carried on with the given stepping until no further improvement in the criterion function is gained. The procedure of optimization then continues with reduced stepping as before. After a cycle of step reduction, the optimal design parameters are obtained.

The flow chart of the computer program for optimization of solar water heating system is given in Fig. 5.
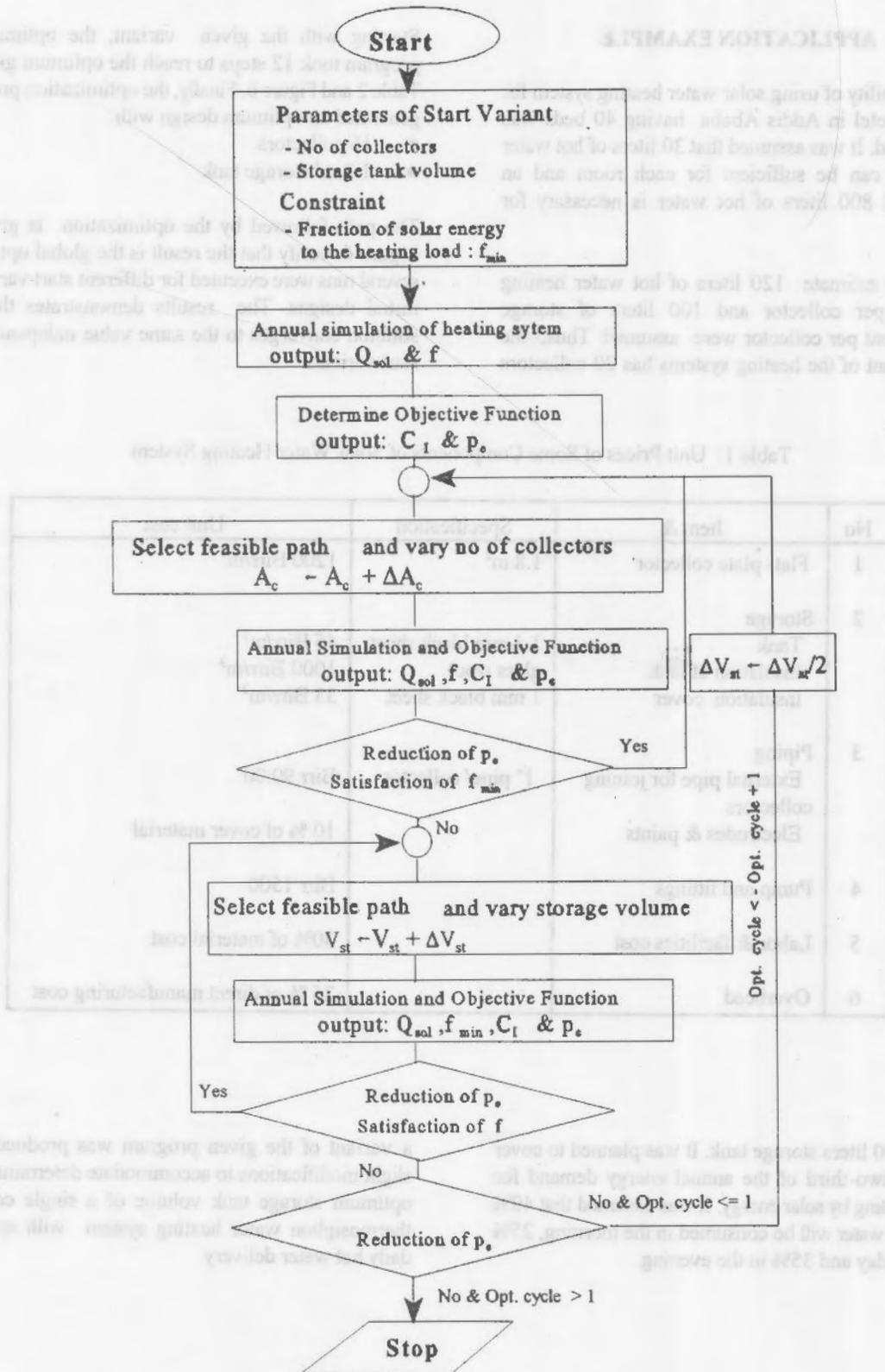
```
                          ┌───────────┐
                          │   Start   │
                          └───────────┘
                               │
        ┌──────────────────────────────────────────┐
        │      Parameters of Start Variant          │
        │      - No of collectors                   │
        │      - Storage tank volume                │
        │      Constraint                           │
        │      - Fraction of solar energy           │
        │        to the heating load : f_min        │
        └──────────────────────────────────────────┘
                               │
        ┌──────────────────────────────────────────┐
        │      Annual simulation of heating sytem   │
        │          output:  Q_sol  &  f             │
        └──────────────────────────────────────────┘
                               │
        ┌──────────────────────────────────────────┐
        │      Determine Objective Function         │
        │          output:  C_I  &  p_e             │
        └──────────────────────────────────────────┘
                               │
```

Select feasible path   and vary no of collectors
$$A_c \leftarrow A_c + \Delta A_c$$

Annual Simulation and Objective Function
output: Q_{sol}, f, C_I & p_e

$$\Delta V_{st} \leftarrow \Delta V_{st}/2$$

Reduction of p_e
Satisfaction of f_{min} Yes

No

Select feasible path   and vary storage volume
$$V_{st} \leftarrow V_{st} + \Delta V_{st}$$

Annual Simulation and Objective Function
output: Q_{sol}, f_{min}, C_I & p_e

Opt. cycle < - Opt. cycle + 1

Opt. cycle < - Opt. cycle

Reduction of p_e
Satisfaction of f Yes

No

No & Opt. cycle <= 1

Reduction of p_e

No & Opt. cycle > 1

┌───────────┐
│   Stop   │
└───────────┘

Figure 5   Flow chart of univariant optimization method of solar water heating systems

APPLICATION EXAMPLE

The possibility of using solar water heating system for a small hotel in Addis Ababa having 40 beds was investigated. It was assumed that 30 liters of hot water at 60 °C can be sufficient for each room and an additional 800 liters of hot water is necessary for washing.

As initial estimate 120 liters of hot water heating capacity per collector and 100 liters of storage requirement per collector were assumed. Thus, the start variant of the heating systems has 20 collectors

Starting with the given variant, the optimization program took 12 steps to reach the optimum given in Table 2 and Figure 6 Finally, the optimization program generated an optimum design with:
- 16 collectors
- 1.2 m³ storage tank

The path followed by the optimization is given in Figure 6. verify that the result is the global optimum, several runs were executed for different start variant or initial designs. The results demonstrates that the solution converges to the same value independent of start variants

Table 1: Unit Prices of Some Components of Solar Water Heating System

No	Item &	Specification	Unit cost
1	Flat-plate collector	1.8 m²	1200 Birr/m²
2	Storage Tank Insulation of tank Insulation cover	1.5 mm black sheet glass wool 1 mm black sheet	45 Birr/m² 1000 Birr/m³ 33 Birr/m²
3	Piping External pipe for joining collectors Electrodes & paints	1" pipe/ collector	Birr 90.00 10 % of cover material
4	Pump and fittings		Birr 1500
5	Labor & facilities cost		30% of material cost
6	Overhead		25 % of direct manufacturing cost

and a 2000 liters storage tank. It was planned to cover at least two-third of the annual energy demand for water heating by solar energy. It was assumed that 40% of the hot water will be consumed in the morning, 25% after midday and 35% in the evening.

a variant of the given program was produced with slight modifications to accommodate determination of optimum storage tank volume of a single collector thermosiphon water heating system with specified daily hot water delivery.

Table 2: Output of Optimizations Run for Designing a 2 m³ Water Heating System

v_n m³/collector	V_{st} m³	n	p_e Birr/kwh	F	η	C_1 Birr
0.120	2.400	20	0.3567	0.7465	0.4960	
0.120	2.280	19	0.3501	0.7237	0.5027	58298
0.120	2.160	18	0.3431	0.6987	0.5099	55368
0.120	2.040	17	0.3360	0.6718	0.5176	52437
0.120	1.920	16	0.3299	0.6470	0.5246	49503
0.110	1.760	16	0.3276	0.6502	0.5211	49339
0.100	1.600	16	0.3256	0.6529	0.5172	49169
0.090	1.440	16	0.3234	0.6565	0.5121	48994
0.080	1.280	16	0.3230	0.6578	0.5061	48812
0.070	1.120	16	0.3231	0.6595	0.4980	48621
0.075	1.200	16	0.3229	0.6589	0.5023	48718
0.070	1.120	16	0.3231	0.6595	0.4980	48621
0.080	1.280	16	0.3230	0.6578	0.5061	48812

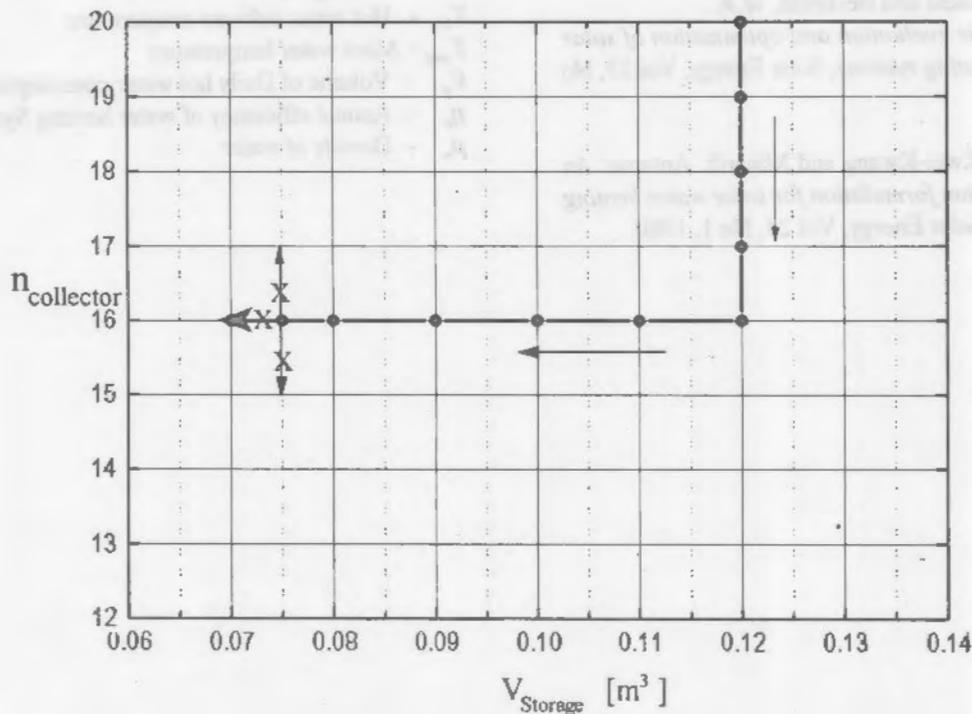Figure 6   Variation of no of collectors with storage volume per collector during optimization of a solar water heating system for supplying 2 m³ of hot water per day at 60°C at annual fraction of contribution of solar energy to the heating load of 66 %.

CONCLUSION

In this work, the development and computer implementation of a method for optimal design of solar water heating systems are discussed. Then, an example on its application was presented. The work has demonstrated that search methods can be used to find the optimum design of solar water heating system.

The given computer program can be used for optimal design of solar water heating system for hotels, hospitals and tanneries depending on their hot water demand and consumption pattern saving substantial amount of investment cost.

To achieve the stated objective, the computer program was made user-friendly by adding to the original programme a professional user interface.

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NOMENCLATURES

A_c - Collector area

LCC - Life cycle costs of Solar water heating system

LCS - Life cycle energy savings

C_i - Investment cost of solar water heating system

C_m - Annual maintenance cost

c_w - Specific heat of water heating

f_a - Annual fraction of contribution of solar energy to heating load

I - Interest rate

i_e - Inflation rate of energy price

I_a - Annual incident solar radiation per unit area

n - Useful life of solar water heating system

p_e - Unit energy price for water heating

Q_a - Annual heating load (Annual end energy demand for water heating)

$Q_{a,sol}$ - Annual contribution of solar energy to the heating load

T_{hot} - Hot water delivery temperature

T_{cold} - Main water temperature

V_w - Volume of Daily hot water consumption

η_a - Annual efficiency of water heating System

ρ_w - Density of water