BIAXIAL CHARTS FOR RECTANGULAR REINFORCED COLUMNS IN ACCORDANCE WITH THE ETHIOPIAN BUILDING CODE STANDARD EBCS-2:PART1

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ABSTRACT

*The analysis of reinforced concrete sections are characterised by material non-linearity arising from the non-linear stress-strain relationships and the cracking of the cross-section. As a result, the systematic production of biaxial design charts necessitates the application of numerical methods that are based on iterations. The design charts may be conveniently represented as M_y-M_z diagrammes on planes of constant internal normal forces or as N-M diagrammes on planes of constant angles that relate the y- and z-components of the resultant moment **M**. The aim of this paper is to present an iterative procedure that has been successfully used to produce biaxial charts of the first type. The design charts are produced for biaxially loaded rectangular columns in accordance with the Ethiopian Building Code Standard, EBCS-2 : Part1 [1].*

BASIC ASSUMPTIONS

The analysis of a cross-section at the ultimate limit state for finding the coordinates of the points on the charts is based on the following assumptions.

1. Sections perpendicular to the axis of bending which are plane before bending remain plane after bending.

2. The strain in the reinforcement is equal to the strain in the concrete at the same level.

3. The stresses in the concrete and reinforcement are derived from the design stress-strain curves recommended by EBCS-2 [1], which are shown in Fig. 1 and Fig. 2, respectively.

4. Tensile strength of concrete is neglected.

5. Section strain distribution in the ultimate limit state are in accordance with EBCS-2 [1].

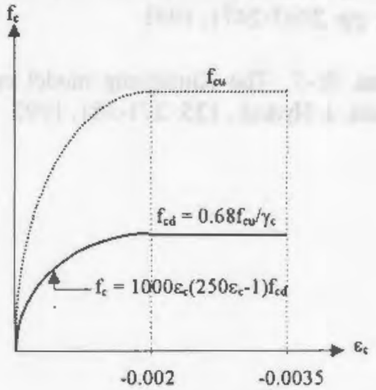

Figure 1 Parabolic-rectangular stress-strain diagram for concrete in compression

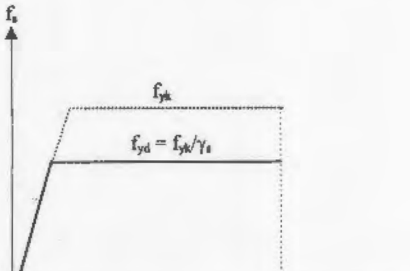

Figure 2 Design stress-strain diagram for reinforcement

STRAIN DISTRIBUTION IN SECTIONS UNDER BIAXIAL BENDING AND AXIAL LOAD

The strain at a point $\varepsilon(y,z)$ in a reinforced concrete section subjected to biaxial bending and axial load can be determined from Eq. 1:

$$\varepsilon(y,z) = \varepsilon_o + (1/r_y)\,y + (1/r_z)\,z \qquad (1)$$

where:

ε_o = Strain at the origin of the coordinate axes
$1/r_y$ = Curvature in y-direction
$1/r_z$ = Curvature in z-direction

Alternatively the strain distribution of the section can be determined by the direction angle of the resultant curvature α_k, which is a function of the component curvatures in the y- and z- directions (Eq. 2) and the strains at two characteristic fibers of the cross section, fibers (1) and (2) as shown in Fig. 3. These fibers denote in absolute value, the greatest compressive and tensile strain for sections in state II (cracked) or the greatest and smallest compressive strain for sections in state I (uncracked) respectively. For cracked sections the reinforcement steel with the greatest tensile strain is denoted as fiber (2).

$$\alpha_k = \arctan\,[(1/r_z)/(1/r_y)] \qquad (2)$$

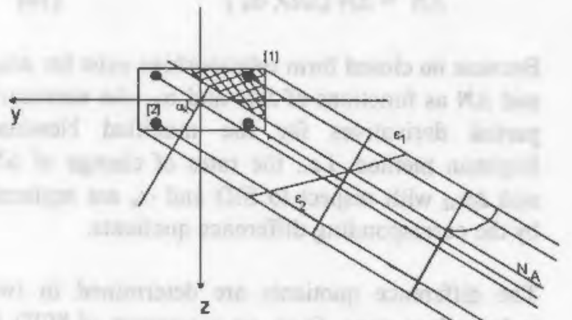

Figure 3 Cross section in state II, strain and stress distribution

In the ultimate limit state, the strain level in either of fibers (1) or (2), or both has reached its ultimate limit stipulated by EBCS-2 [1]. For the analytical treatment, it is expedient to describe the relationship of the strains ε_{1u} and ε_{2u} at the characteristic fibers by a variable *SID* [2], for strain identification number, which is chosen to vary between 0 and 33. Thus for the strain distribution in the ultimate limit state according to EBCS-2 [1], a change in *SID* equal to one, in the region $0 \le SID \le 5$, corresponds to a change in the ultimate strain of the characteristic fibers (1) and (2) by 0.3 mm/m and 0.4 mm/m respectively. Similar values for the ranges $6 \le SID \le 16$ and $16 \le SID \le 33$ are 1.0 mm/m and 0.79412 mm/m for the characteristic fibers (1) and (2) respectively. Table 1 shows the relationship between the strain identification number and the associated characteristic strain gradients across the section in the ultimate limit state. For a given section and reinforcing pattern, an arbitrary combination of the strain identification number, *SID* and the direction angle of the resultant curvature, α_k describes uniquely the section strain distribution in the ultimate limit state with the corresponding stress resultant representing the coordinates of a point (N, M_y, M_z) on the associated interaction surface.

Table 1: Strain Identification Numbers Corresponding to Characteristic Strain Gradients in the Ultimate Limit State

SID	Ultimate Strains		Values according to EBCS-2	
Nr.	ε_{1u}	ε_{2u}	mm/m	mm/m
0	ε_{1cu}	ε_{2cu}	-2.0	-2.0
5	ε_{1cu}	0 (conc.)	-3.5	0.0
6	ε_{1cu}	0 (steel)	-3.5	0.0
16	ε_{1cu}	ε_{2su}	-3.5	+10.0
33	ε_{1su}	ε_{2su}	+10.0	+10.0

THE GOVERNING ULTIMATE LIMIT STATE OF A REINFORCED CONCRETE SECTION R_U ASSOCIATED WITH AN ARBITRARY INITIAL VECTOR R_I

The determination of the ultimate limit states of a reinforced concrete section by integration of the stress distribution corresponding to arbitrary combinations of the parameters *SID* and α_k is simple but not immediately useful for the purpose of the production of biaxial interaction diagrams, because these points do not normally lie on planes of constant normal forces. A useful solution strategy is to pursue the inverse problem of finding the governing ultimate limit strain state corresponding to a chosen normal load level N_i and an angle α_{Mi} that relates the moment components M_{yi} and M_{zi} of an initial vector R_i (N_i, M_{yi}, M_{zi}). However the solution necessitates the application of numerical methods based on iterations because of the non-linear response of reinforced concrete sections as a result of material non-linearity and cracking.

Figure 4 shows the interaction surface for a given section with the chosen initial vector $\mathbf{R}_i$ (N_i, M_{yi}, M_{zi}), the associated governing ultimate limit state $\mathbf{R}_u$ and the stress resultant corresponding to an approximated strain distribution in the ultimate limit state $\mathbf{R}_{ua}$ (N_{ua}, M_{yua}, M_{zua}). The angles α_{Mua} and α_{Mi} which are functions of the respective moment components of $\mathbf{R}_{ua}$ and $\mathbf{R}_i$ are given by:

$$\alpha_{Mua} = \arctan\,(M_{zua} / M_{yua}) \quad (3)$$

$$\alpha_{Mi} = \arctan\,(M_{zi} / M_{yi}) \quad (4)$$

The governing ultimate limit strain state corresponding to the initial vector $\mathbf{R}_i$ is found, when the normal components of the vectors $\mathbf{R}_{ua}$ and $\mathbf{R}_i$ and the direction angles relating their moment components coincide, i.e.:

$$\alpha_{Mua} = \alpha_{Mi} \quad (5)$$

$$\mathbf{N}_{ua} = \mathbf{N}_i \quad (6)$$

These quantities normally show discrepancies in the initial stage of the iteration, which must be continued until the differences given by Eqs. 7 and 8 are negligibly small.

$$\Delta\alpha_M = \alpha_{Mua} - \alpha_{Mi} \quad (7)$$

$$\Delta \mathbf{N} = \mathbf{N}_{ua} - \mathbf{N}_i \quad (8)$$

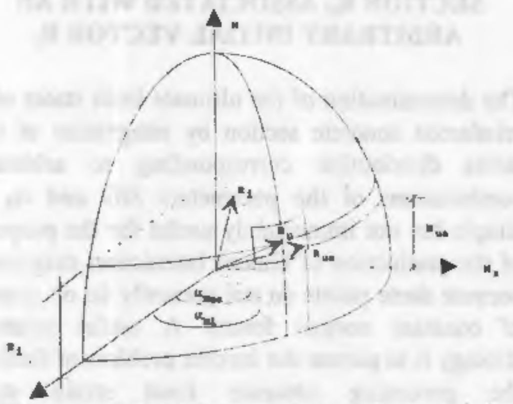

Figure 4 Interaction surface of a reinforced concrete section with an initial vector, $\mathbf{R}_i$, the associated governing ultimate limit state, $\mathbf{R}_u$ and the stress resultant corresponding to an approximated ultimate limit strain state, $\mathbf{R}_{ua}$.

ITERATIVE DETERMINATION OF THE GOVERNING ULTIMATE LIMIT STRAIN STATE

The iterative determination of the governing ultimate limit strain state associated with the chosen initial vector $\mathbf{R}_i$ is achieved through a stepwise improvement of an approximate strain distribution in the ultimate limit state, uniquely defined by a combination of the strain identification number, *SID* and the direction angle of the resultant curvature α_K. The corresponding stress resultant is designated by $\mathbf{R}_{ua}$ in Fig. 4.

The numerical procedure, which solves the problem successfully, i.e. the modified Newton-Raphson method, is based on the following considerations.

For a given section and material properties, it can be assumed that $\Delta\alpha_M$ and ΔN can be expressed as functions of *SID* and α_K as follows:

$$\Delta\alpha_M = \Delta\alpha_M\,(SID, \alpha_k) \quad (9)$$

$$\Delta N = \Delta N\,(SID, \alpha_k) \quad (10)$$

Because no closed form relationships exist for $\Delta\alpha_M$ and ΔN as functions of *SID* and α_K, the necessary partial derivatives for the modified Newton-Raphson method ,i.e., the rates of change of ΔN and $\Delta\alpha_M$ with respect to SID and α_k are replaced by the corresponding difference quotients.

The difference quotients are determined in two independent steps. First, an increment of δSID is given to the strain gradient while α_K is maintained constant. The corresponding stress resultant $\mathbf{R}_{ua}$(*SID*+δ*SID*, α_K) is determined by integrating the stress distribution using the idealised stress-strain relationships for concrete (Fig. 1) and steel (Fig. 2). The associated angle α_{Mua} (*SID*+δ*SID*, α_K), and the differences $\Delta\alpha_M$ (*SID*+δ*SID*, α_K) and ΔN (*SID*+δ*SID*, α_K) are then calculated from Eqs. 3, 7, and 8 respectively giving the difference quotients of Eqs. 11 and 12.

$$\frac{\delta(\Delta\alpha_M)}{\delta SID} = \frac{\Delta\alpha_M(SID+\delta SID,\alpha_k) - \Delta\alpha_M(SID,\alpha_k)}{\delta SID} \quad (11)$$

$$\frac{\delta(\Delta N)}{\delta SID} = \frac{\Delta N(SID+\delta SID,\alpha_k) - \Delta N(SID,\alpha_k)}{\delta SID} \quad (12)$$

Secondly, an increment in the direction angle of the resultant curvature α_K is given to the same initial strain distribution in the section while the strain gradient, i.e., *SID* is maintained constant. Using similar procedures the stress resultant $\mathbf{R}_{ua}(SID, \alpha_K + \delta\alpha_K)$ and the differences $\Delta\alpha_M (SID, \alpha_K + \delta\alpha_K)$ and $\Delta N (SID, \alpha_K + \delta\alpha_K)$ are determined. The rates of change $\delta(\Delta\alpha_M)/\delta\alpha_K$ and $\delta(\Delta N)/\delta\alpha_K$ are then calculated from:

$$\frac{\delta(\Delta\alpha_M)}{\delta\alpha_k} = \frac{\Delta\alpha_M(SID,\alpha_k+\delta\alpha_k)-\Delta\alpha_M(SID,\alpha_k)}{\delta\alpha_k} \quad (13)$$

$$\frac{\delta(\Delta N)}{\delta\alpha_k} = \frac{\Delta N(SID,\alpha_k+\delta\alpha_k)-\Delta N(SID,\alpha_k)}{\delta\alpha_k} \quad (14)$$

The system of equations for the determinations of the "Newton-improvements" $dSID$ and $d\alpha_K$ have the following matrix form:

$$\begin{bmatrix} \frac{\delta(\Delta N)}{\delta SID} & \frac{\delta(\Delta N)}{\delta\alpha_k} \\ \frac{\delta(\Delta\alpha_M)}{\delta SID} & \frac{\delta(\Delta\alpha_M)}{\delta\alpha_k} \end{bmatrix} \begin{bmatrix} dSID \\ d\alpha_k \end{bmatrix} = \begin{bmatrix} -\Delta N \\ -\Delta\alpha_M \end{bmatrix} \quad (15)$$

Direct addition of these improvements to the approximate values without additional restrictions would frequently lead to convergence problems. For example strain identification numbers outside the valid range or alternating improvements $d\alpha_K$ and $dSID$ could occur. To avoid such problems, different modifications of the Newton-Raphson method are possible [2]. The method applied in this paper consists in applying constant limits on the "Newton-improvements" while maintaining the originally calculated "Newton-direction". The calculated improvements $dSID$ and $d\alpha_K$ are limited to about 1/30 of the corresponding ranges for *SID* and α_K. Inorder to keep the originally calculated "Newton-direction", the limited "Newton-improvements" $\underline{dSID}$ and $\underline{d\alpha_K}$ are further modified to the values given by Eqs16 and 17.

for $\quad |\underline{dSID}/dSID| \le |\underline{d\alpha_k}/d\alpha_k|$:

$$\underline{d\alpha_k} = (\underline{dSID}/dSID).\ d\alpha_k \quad (16)$$

for $\quad |\underline{d\alpha_k}/d\alpha_k| \le |\underline{dSID}/dSID|$:

$$\underline{dSID} = (\underline{d\alpha_k}/d\alpha_k).\ dSID \quad (17)$$

At the end of the current iteration step it will be checked whether the iteration is to be continued or not. This will be decided by the magnitudes of the most recently calculated "Newton-improvements" $dSID$ and $d\alpha_K$. For the case that $|dSID| \le 10^{-4}$ and $|d\alpha_K| \le 10^{-4}$, the parameters of the governing ultimate limit strain state, *SID* and α_K, have been determined accurately enough, allowing the iteration to be stopped. Otherwise $\mathbf{R}_{ua}$ will be updated and the iteration continued until the convergence criteria are satisfied.

A converged solution of the iteration scheme yields the governing ultimate limit state $\mathbf{R}_u$ of a given section associated with the initial vector $\mathbf{R}_i$. The moment components of $\mathbf{R}_u$ (N_u, M_{yu}, M_{zu}) represent one point on the interaction diagram (M_y - M_z diagram) drawn on a plane of constant normal force $\mathbf{N}_i$ = constant. More points on the interaction diagram are achieved by systematically varying the direction angle α_{mi} of the resultant moment $\mathbf{M}_i$ (M_{yi}, M_{zi}). Similar curves for other values of mechanical reinforcement ratio ω and subsequently, other levels of normal forces are obtained by repeating the procedure for systematically varied ω and $\mathbf{N}_i$.

A computer program [2] originally developed for the design and analysis of arbitrarily shaped reinforced concrete sections has been further developed by incorporating the iteration scheme described above to meet the purpose of producing the biaxial interaction diagrams in EBCS-2:Part 2 [3]. The charts are prepared for five reinforcement patterns with two cover ratios each (d'/h = 0.1, 0.2) and relative normal forces ν varying between 0.0 and 1.4, in intervals of 0.2. The limitation to only two cover ratios or the choice of bigger load intervals in lieu of smaller ones is solely in the interest of brevity. Figures 5 to 9 show typical examples of the charts constructed. A full set of the design charts are available in EBCS-2:Part 2 [3].

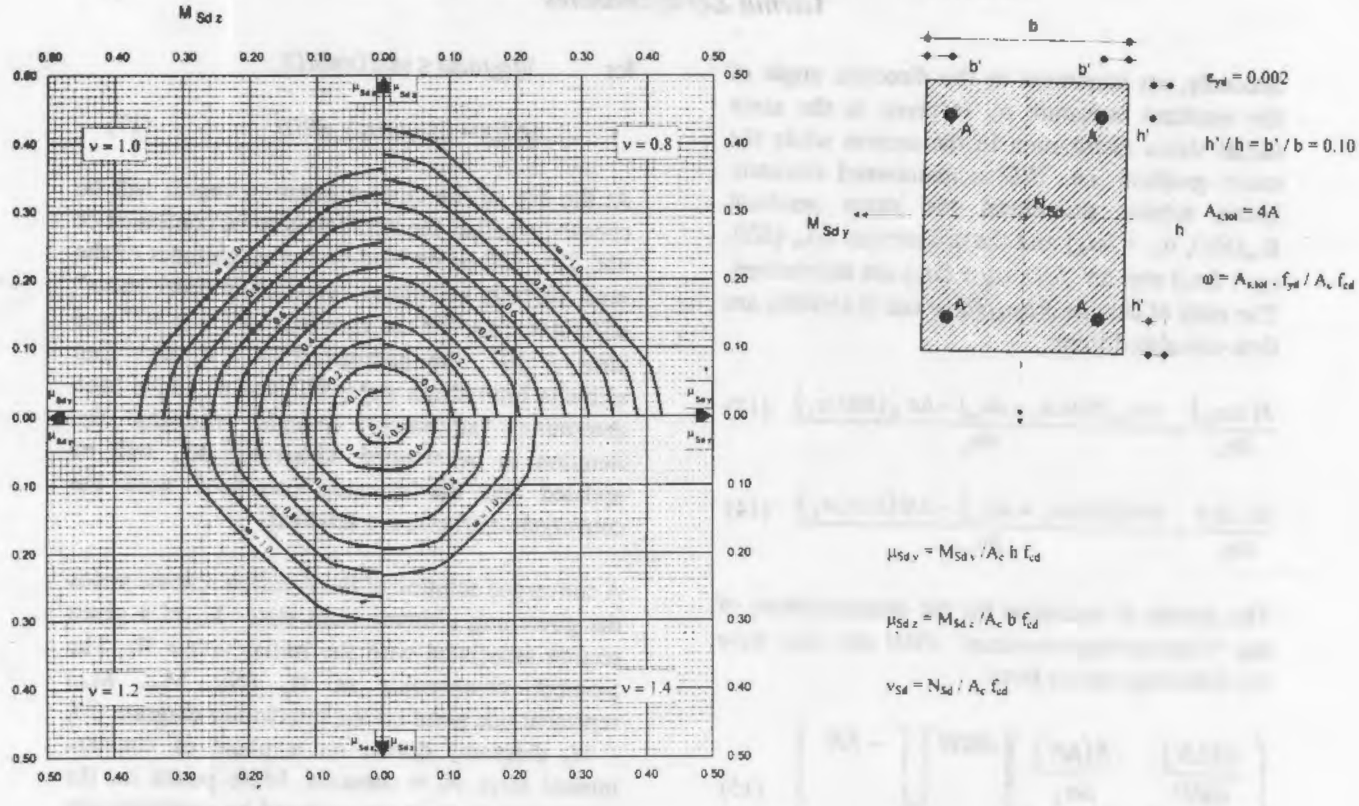

Figure 5 Biaxial chart No. 1

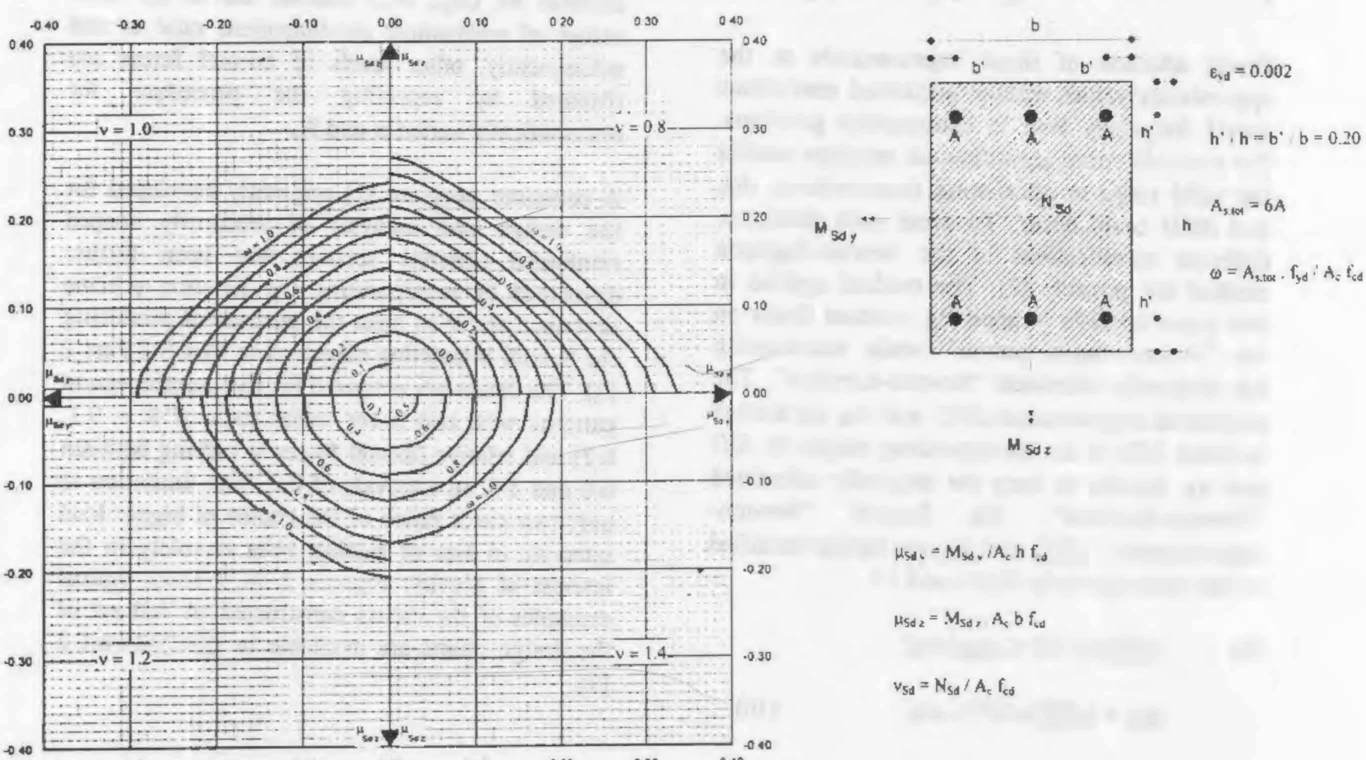

Figure 6 Biaxial chart No. 2

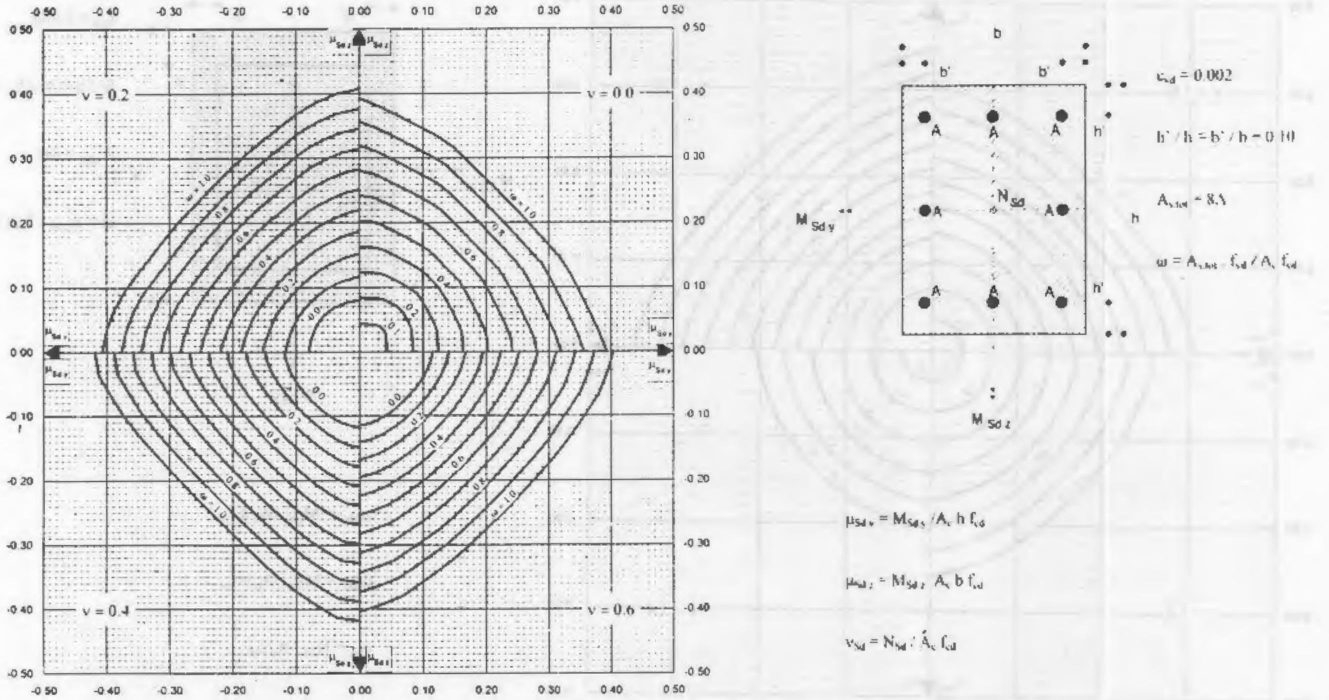

Figure 7 Biaxial chart No. 3

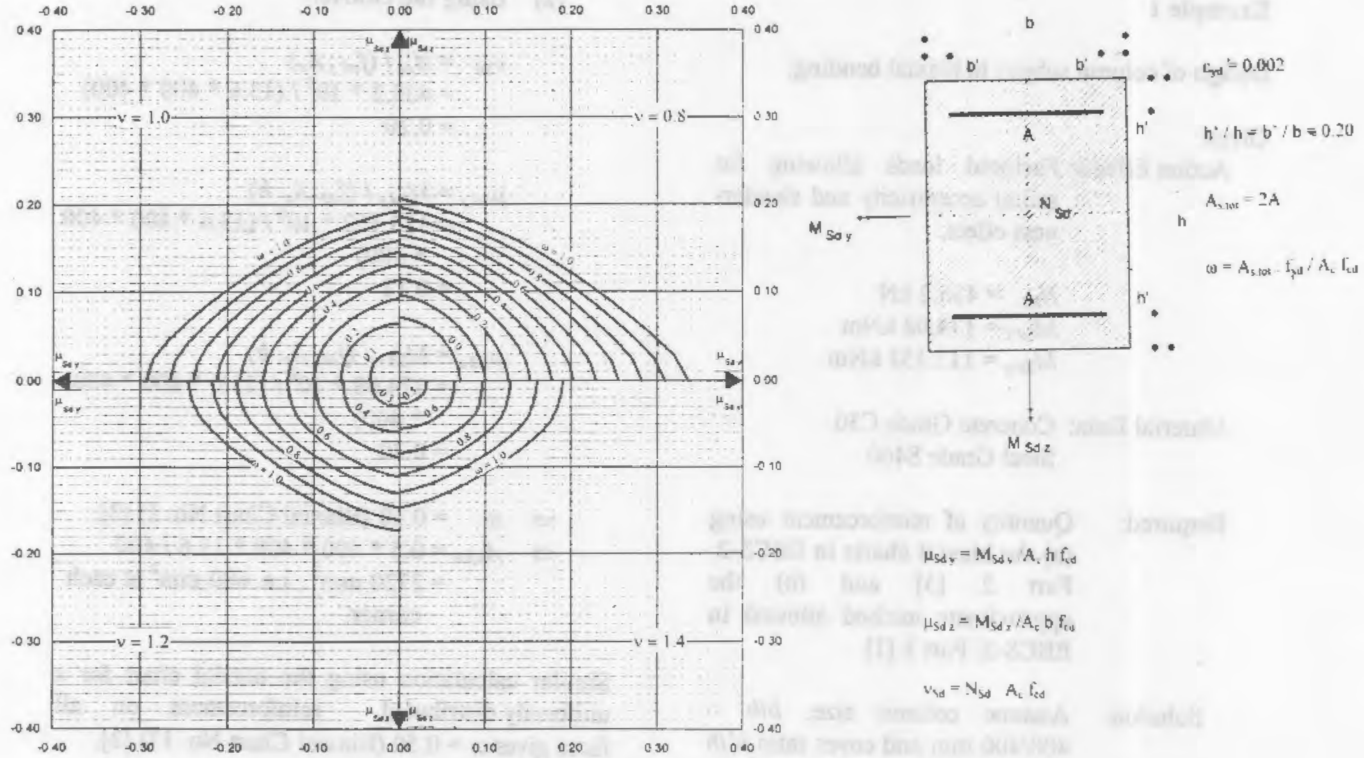

Figure 8 Biaxial chart No. 4

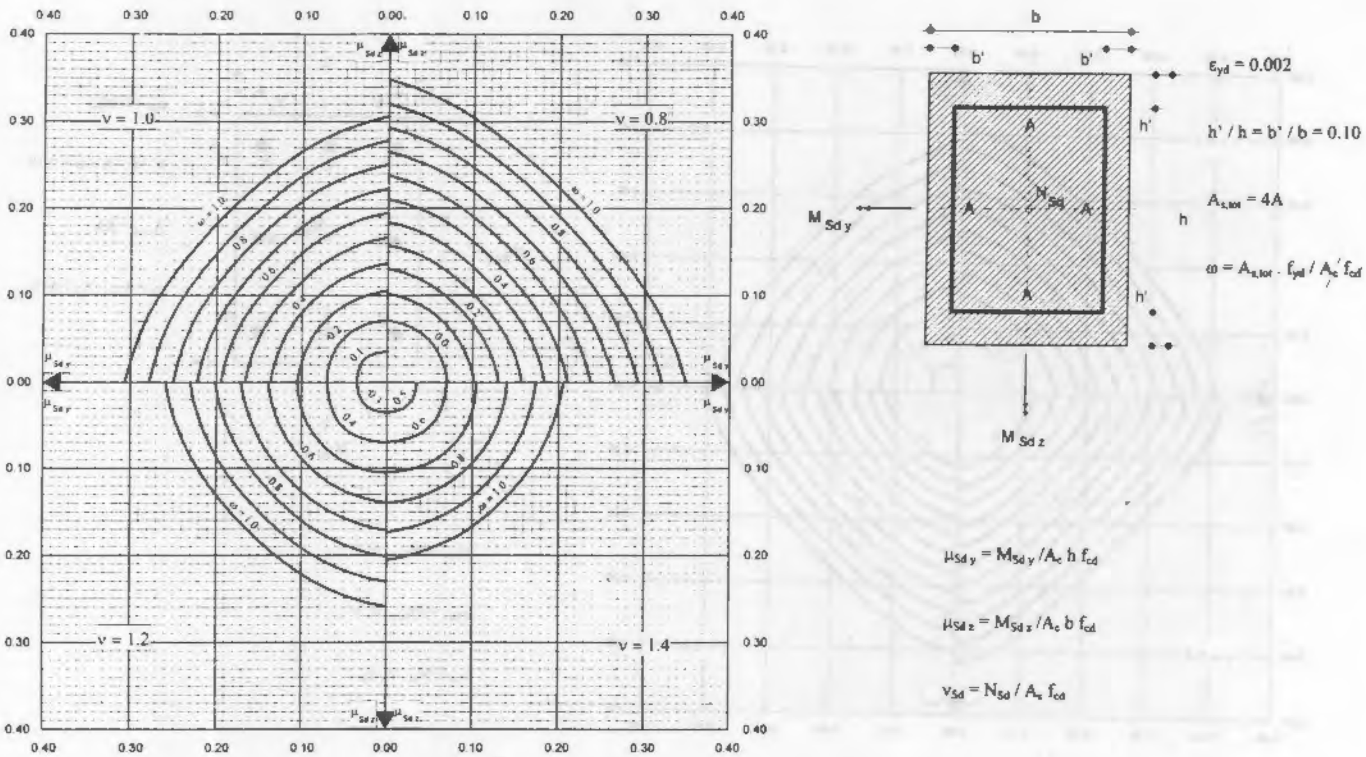

Figure 9 Biaxial chart No. 5

Example 1

Design of column subject to biaxial bending:

Given:

Action Effects: Factored loads allowing for initial eccentricity and slenderness effect.

N_{Sd} = 435.2 kN
$M_{Sd\,z}$ = 174.08 kNm
$M_{Sd\,y}$ = 113.152 kNm

Material Data: Concrete Grade C30
Steel Grade S460

Required: Quantity of reinforcement using (a) the biaxial charts in EBCS-2: Part 2 [3] and (b) the approximate method allowed in EBCS-2: Part 1 [1]

Solution: Assume column size, b/h = 400/400 mm and cover ratio $h'/h = b'/b = 0.1$

(a) Using the charts:

$\nu_{Sd} = N_{sd} / (f_{cd} \, . \, A_c)$
$= 435.2 * 10^3 / (13.6 * 400 * 400)$
$= 0.20$

$\mu_{Sd\,y} = M_{Sd\,y} / (f_{cd} . A_c . h)$
$= 113.152 * 10^6 / (13.6 * 400 * 400 * 400)$
$= 0.13$

$\mu_{Sd\,z} = M_{Sd\,z} / (f_{cd} . A_c . b)$
$= 174.08 * 10^6 / (13.6 * 400 * 400 * 400)$
$= 0.20$

⇒ ω = 0.50 (Biaxial Chart No. 1) [3].
⇒ $A_{s,tot} = 0.5 * 400 * 400 * 13.6 / 400$
$= 2720 \text{ mm}^2$, i.e. 680 mm^2 at each corner.

Similar calculation using the biaxial chart for a uniformly distributed reinforcement on all faces gives ω = 0.59 (Biaxial Chart No. 17) [3].

(b) Using the approximate method:

$e_z = M_{Sdy} / N_{Sd}$
$= 113.152 * 10^6 / (435.2 * 10^3)$
$= 260$ mm

$e_y = M_{Sdz} / N_{Sd}$
$= 174.08 * 10^6 / (435.2 * 10^3)$
$= 400$ mm

$k = 260/400 = 0.65 > 0.2$

$e_{eq} = e_{tot} (1 + k.\alpha)$
$= 400 (1 + 0.65 * 0.8)$
$= 608$ mm

$\Rightarrow$ $M_{eq} = 435.2 * 0.608 = 264.6$ kNm
$\mu = 264.6 * 10^6 / (13.6 * 400 * 400 * 400)$
$= 0.304$

$\Rightarrow$ $\omega = 0.56$ (Uniaxial Chart No. 2) [3]
$\Rightarrow$ $A_{s,tot} = 3046.4$ mm^2 , i.e 761.6 mm^2 at each corner

The result shows an increase in reinforcement by 12%.

Similar calculation using the uniaxial chart with steel reinforcement uniformly distributed on all faces yieldes $\omega = 0.71$ (Uniaxial Chart No. 7) [3], showing an increase in reinforcement by 20.3%.

Example 2

One way to check the validity and correctness of the biaxial charts is by testing the solutions for the case of uniaxial bending analytically.

Given: The section in Fig. 10 with $\omega = 0.4$ and cover ratios $b'/b = h'/h = 0.2$. Concrete grade is C 30 and steel grade is S 460.

Required: Design values of the ultimate uniaxial moment capacities of the section corresponding to N_{Sd} = 1360 kN, i.e. $\nu_{Sd} = 0.8$ and comparison of the same with the chart values.

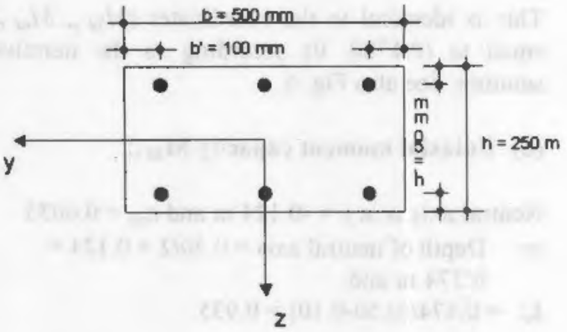

Figure 10 Reinforced concrete section for Example 2

Solution: **(a) Uniaxial moment capacity M_{Sdy}:**

Neutral axis is at $z = 0.0656$ m and $\varepsilon_{cm} = 0.0035$

$\Rightarrow$ Depth of neutral axis = 0.25/2 + 0.0656 = 0.1906 m and $k_x = 0.1906/(0.25\text{-}0.05) = 0.953$

$\alpha_c = (3\varepsilon_{cm} - 2)\, k_x / 3*\varepsilon_{cm}$
$= (3*3.5 - 2) * 0.953 / (3*3.5)$
$= 0.7715$

$\beta_c = \{(\varepsilon_{cm}(3*\varepsilon_{cm} - 4) + 2) / (2*\varepsilon_{cm} (3*\varepsilon_{cm} - 2))\}\, k_x$
$= 0.3964$

Check the satisfaction of force equilibrium:

$N_{Sd} = \{(\omega/2)(\sigma_1/f_{yd}) + \alpha_c - (\omega/2)(\sigma_2/f_{yd})\}\, f_{cd}bd$

$\varepsilon_{s1} = 2.582$ %o and $\varepsilon_{s2} = 0.1726$ %o

$\Rightarrow$ $\sigma_{s1} = 400$ N/mm^2 and $\sigma_{s2} = 34.5226$ N/mm^2. Also $\omega = (0.25/0.20) * 0.4 = 0.5$ when related to $b*d$ instead of $b*h$.

$\Rightarrow$ $N_{sd} = \{(0.5/2)(400/400 - 34.5226/400) + 0.7715\}\, f_{cd}bd$
$= 1.0 * f_{cd}bd = 0.8\, f_{cd}bh$

$\Rightarrow$ $\nu_{Sd} = N_{Sd}/ f_{cd}bh = 0.8$

Determine M_{Sdy}:

$M_{Sdy} = \{\alpha_c(h/2 - \beta_c d_h) + (\omega/2)(\sigma_{s2}/f_{yd})(h/2 - d') + (\omega/2)(\sigma_{s1}/f_{yd})(h/2 - d')\}\, f_{cd}bd_h$
$= 0.0556\, f_{cd}bd_h = 0.0445\, f_{cd}bh$

$\mu_{Sdy} = M_{Sdy} / f_{cd}bh^2 = 0.1780$

This is identical to the coordinates ($M_{Sd\,y}$, $M_{sd\,z}$) equal to (0.1780, 0) according to the iterative solution. See also Fig. 6.

(b) Uniaxial moment capacity $M_{Sd\,z}$:

Neutral axis is at y = -0.124 m and ε_{cm} = 0.0035

$\Rightarrow$ Depth of neutral axis = 0.50/2 + 0.124 = 0.374 m and

k_x = 0.374/(0.50-0.10) = 0.935

$\alpha_c = (3\varepsilon_{cm} - 2)\, k_x / 3*\varepsilon_{cm}$
$= (3*3.5 - 2) * 0.935 / (3*3.5)$
$= 0.7569$

$\beta_c = \{(\varepsilon_{cm}(3*\varepsilon_{cm} - 4) + 2) / (2*\varepsilon_{cm}(3*\varepsilon_{cm} - 2))\}\, k_x$
$= 0.3889$

Check the satisfaction of force equilibrium:

$N_{Sd} = \{(\omega/3)(\sigma_1/f_{yd}) + (\omega/3)(\sigma_3/f_{yd}) + \alpha_c - (\omega/2)(\sigma_2/f_{yd})\}\, f_{cd}bd$

ε_{s1} = 2.564 ‰, ε_{s3} = 1.1604 ‰ and ε_{s2} = 0.2433 ‰ $\Rightarrow \sigma_{s1}$ = 400 N/mm^2, σ_{s3} = 232.0856 N/mm^2, and σ_{s2} = 48.6631 N/mm^2.

$\Rightarrow N_{Sd} = \{(0.5/3)(400/400 + 232.0856/400 - 48.6631/400) + 0.7569\}\, f_{cd}bd$
$= 1.0 * f_{cd}bd = 0.8\, f_{cd}bh$

$\Rightarrow \nu_{Sd} = N_{Sd}/f_{cd}bh = 0.8$

Determine $M_{Sd\,z}$:

$M_{Sd\,z} = \{\alpha_c(b/2 - \beta_c d_b) + (\omega/3)(\sigma_{s2}/f_{yd})(b/2 - b') + (\omega/3)(\sigma_{s1}/f_{yd})(b/2 - b')\}\, f_{cd}hd_b$
$= 0.0995\, f_{cd}hd_b = 0.0796 f_{cd}hb$

$\mu_{Sd\,z} = M_{Sd\,z}/f_{cd}hb^2 = 0.1592$

This is identical to the coordinates ($M_{Sd\,y}$, $M_{sd\,z}$) equal to (0, 0.1592) according to the iterative solution. See also Fig. 6.

CONCLUSIONS

1. An iterative numerical procedure suitable for the systematic production of biaxial charts for rectangular reinforced concrete sections has been developed. The procedure converges to the required solution reliably as verified by the preparation of the interaction diagrams [3] for biaxially loaded columns with various reinforcement patterns and two cover ratios.

2. The typical example solved (Example 2) demonstrates the validity and correctness of the design charts, through checking their values in the limiting case of uniaxial bending for which analytical solutions for the stress resultants are available, provided that the governing ultimate limit strain state has been determined.

3. A rigorous solution for the problem of biaxially loaded columns such as the one presented in the paper, allows the evaluation of the different approximate methods recommended by building codes through assessment of the extent to which the use of these methods may lie on the conservative or the unconservative side.

4. Based on the results of Example 1, it can be concluded that the use of the charts in lieu of the approximate method recommended by EBCS-2 [1] can lead to a snbstantial saving in reinforcing steel. This is particularly the case where moment resisting frames such as the grid frames are chosen as the lateral load resisting system, because for such frames all the columns have to be designed for biaxial loads [4].

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