DIGITAL COMPUTER CONTROL OF SERVOMOTOR ANGULAR POSITION

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ABSTRACT

The paper discusses the design and simulation methodology of digital control systems for the benefit of the interested practicing engineer. A lead-type digital controller for a 2nd order system and a lead-lag type digital controller for a 3rd order system are designed. The simulations show that the design methods are acceptable.

INTRODUCTION

The design methodology in this paper is entirely classical in the sense that a Bode plot of a transfer function gives information on phase margin, based on which is designed an analog controller. A corresponding digital controller is then found. The transfer function of the servomotor is simulated on the analog computer. The error in a unity-feedback situation with step-input is fed into a personal computer on which is programmed the digital controller. The PC also functions as analog-to-digital converter (ADC) and digital-to-analog converter (DAC).

Two cases are studied : the first deals with a terminal-voltage controlled d.c. servomotor for which a lead controller is designed. The second deals with a field-controlled servomotor for which a lag-lead digital controller is designed.

MODEL OF D.C. SERVOMOTOR

The operating equations are [2]

$$
\begin{aligned}
u &= R_a i_a(t) + L_a \frac{di_a}{dt} + E_a \\
E_a &= K_f \phi \omega \\
u_f &= R_f i_f + L_f \frac{di_f}{dt} \\
T &= K_T \phi i_a \\
J\frac{d^2\theta}{dt^2} &+ D\frac{d\theta}{dt} + T_L(t) = T(t) \\
\omega &= \frac{d\theta}{dt}
\end{aligned}
\tag{1}
$$

Various simplifications can be introduced depending on the mode of operation.

Terminal-Voltage Controlled Motor

Since field excitation is assumed constant, we have ϕ = constant so that $E_a = K_1\omega$ and $T = K_2 i_a$. Unless one is interested in studying load torque variations it is usual to set load torque to zero. With these assumptions introduced into the set of Eqs.1, it is straight-forward to get the Laplace-transformed transfer function

$$\frac{\theta(s)}{U(s)} = \frac{\dfrac{K_f}{DR_a + K_1K_2 + sDL_a}}{s\left(\dfrac{J(R_a + sL_a)}{DR_a + K_1K_2 + sDL_a}s + 1\right)} \tag{2}$$

Generally armature circuit inductance is small enough to be neglected, so that

$$\frac{\theta(s)}{u(s)} = \frac{k_m}{s(\tau_m s + 1)} \tag{3}$$

where

$k_m = \dfrac{K_2}{DR_a + K_1K_2}$ = motor gain constant

$\tau_m = \dfrac{JR_a}{DR_a + K_1K_2}$ = mechanical time constant

The model is second order.

Field-Current Controlled Motor

In this case we neglect R_a and L_a of the armature. Then, as before, for $T_L = 0$, we have

$$\frac{\theta(s)}{U_f(s)} = \frac{k_m}{s(\tau_m s + 1)(\tau_f s + 1)} \tag{4}$$

where $\tau_f = L_f/R_f$ = field time constant. The model is third order.

DESIGN OF A DIGITAL LEAD-CONTROLLER

For a common laboratory d.c. servomotor the following transfer function is typical:

$$\frac{\theta(s)}{U(s)} = \frac{1}{s(s+0.1)} \tag{5}$$

For a velocity error of 5% to a unit velocity input in a unity-feedback arrangement, we obtain a forward gain of 2 (Fig.1):

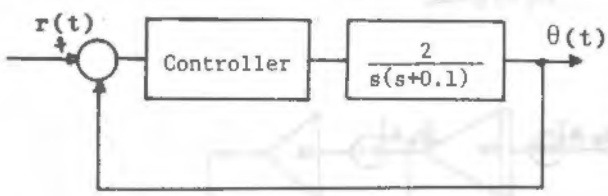


Figure 1 Angular position control

A Bode plot of the forward function shows the phase margin to be about zero degrees. Assume the compensator supplies about 55°. Standard calculations give a compensating function [1]

$$G_c(s) = \frac{s+0.443}{s+4.43} \tag{6}$$

Assume that a digital controller of the form

$$G_c(z) = \frac{k_d(z-z_o)}{z-z_p} \tag{7}$$

is prescribed [1,3]. We introduce the transformation

$$s = \frac{2}{T}\,\frac{z-1}{z+1} \tag{8}$$

to relate the frequency-variable in continous time (s) to the frequency-variable in discrete time (z). Eq.(8) is known variously as the bilinear or Tustin transformation and is based on the trapezoidal approximation in integration. Substitution of Eq.8 into Eq.6 and comparsion with Eq.7 gives the set

$$k_d = \frac{s_p(s_z+2/T)}{s_z(s_p+2/T)} \qquad z_o = \frac{2/T-s_z}{2/T+s_z} \qquad z_p = \frac{2/T-s_p}{2/T+s_p} \tag{9}$$

where $s_z = -0.443$, $s_p = -4.43$.

The sampling period T is selected by making a plot of the closed-loop freqnency response. In this case ω = 10 rad/sec can be taken as the cut-off frequency so that a sampling frequency of 50 rad/sec is sufficient, giving a sampling period of about 0.125 sec.

The digital controller, with input error $e(t)$ and output $s(t)$, feeds into an analog computer simulation of the servomotor (Fig. 2):

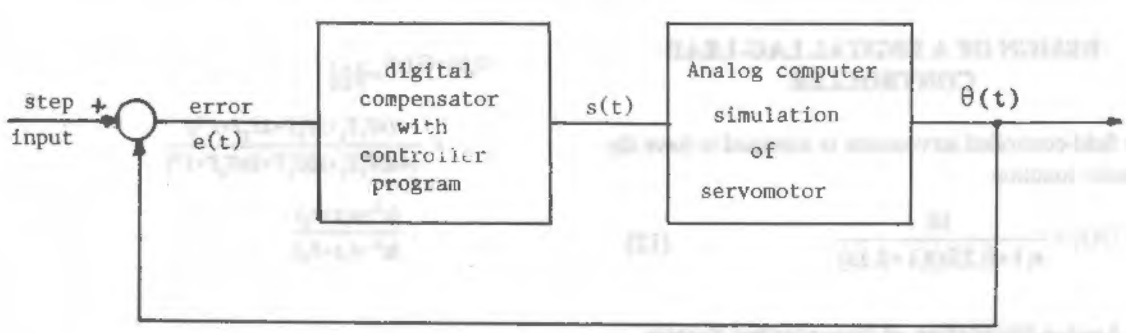


Figure 2 Digital control of servomotor

Thus,

$$\frac{S(z)}{E(z)} = \frac{k_d(z-z_o)}{z-z_p} \qquad (10)$$

that is

$$s(k) = z_p * s(k-1) + k_d * e(k) - k_d * z_o * e(k-1),\ k=1,2,.. \qquad (11)$$

This is the equation that is programmed on the digital computer.

Figure 4 shows the responses of the uncontrolled and controlled systems, and the control signal as functions of time.

Digital Controller Design Methodology

The Laplace transformed lag-lead controller has the form

$$G_c(s) = \frac{T_1 s+1}{\beta T_1 s+1} \cdot \frac{T_2 s+1}{\alpha T_2 s+1},\ \alpha<1,\ \beta>1 \qquad (13)$$

Let the Z-transformed controller have the form

$$G_c(z) = \frac{K_d(z^2+a_1 z+a_2)}{z^2+b_1 z+b_2} \qquad (14)$$

Now, substitution of the transformation Eq.8 into Eq.13 gives,

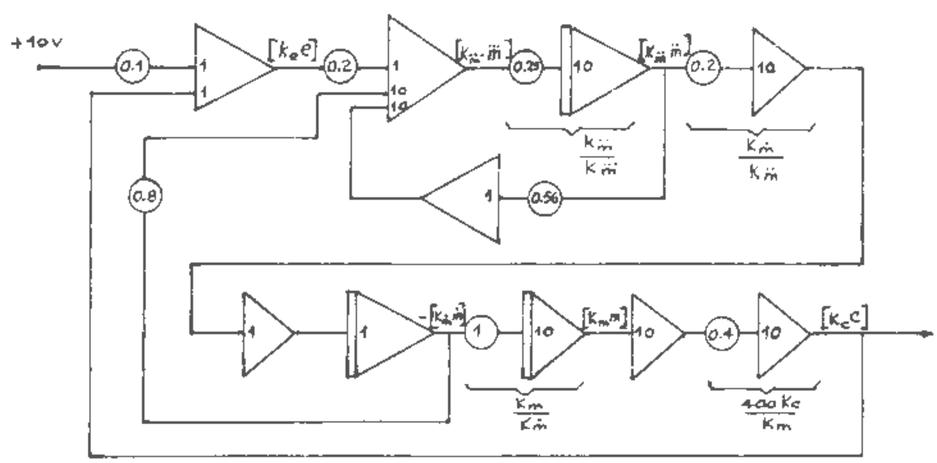


Figure 3 Magnitude-scaled analog simulation of the field-controlled servomotor

DESIGN OF A DIGITAL LAG-LEAD CONTROLLER

The field-controlled servomotor is assumed to have the transfer function

$$G(s) = \frac{10}{s(1+0.25s)(1+0.1s)} \qquad (12)$$

Analog Simulation of Uncontrolled System

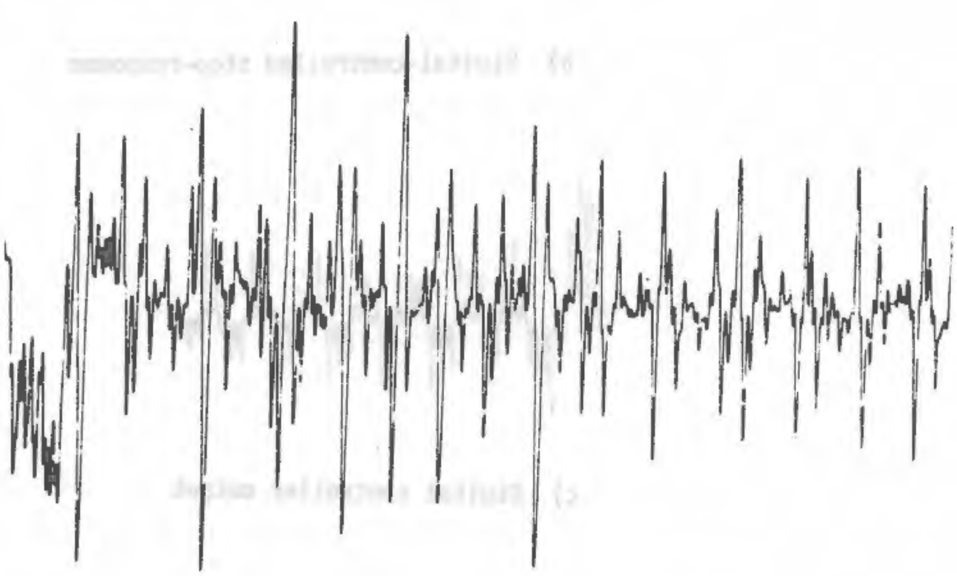
The parameters in $G(s)$ are such that direct simulation of Eq.12 was not possible without magnitude scaling. Fig. 3 shows the complete scaled simulation diagram [1].

$$G_c(z) = G_c(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

$$= \frac{(4T_1T_2+2T_1T+2T_2T+T^2)}{(4\alpha\beta T_1T_2+2\beta T_1T+2\alpha T_2T+T^2)} \cdot \frac{(z^2+a_1z+a_2)}{(z^2+b_1z+b_2)} \qquad (15)$$

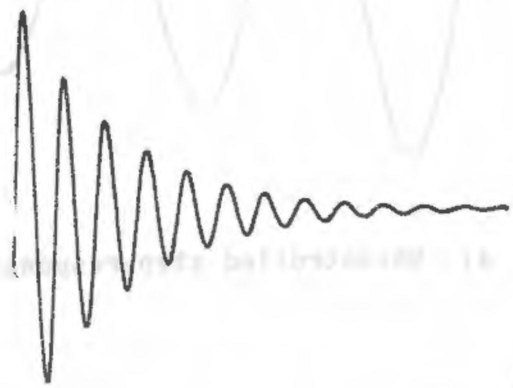
a) Uncontrolled step-response

b) Digital-controlled step-response



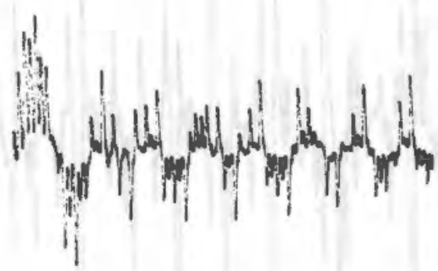
c) Digital controller output

Figure 4 Voltage-controlled system



a) Uncontrolled step-response

b) Digital-controlled step-response



c) Digital controller output

Figure 5 Field-controlled system

Comparison with Eq.14 shows that

$$K_d = \frac{4T_1T_2+2T_1T+2T_2T+T^2}{4\alpha\beta T_1T_2+2\beta T_1T+2\alpha T_2T+T^2}$$

$$a_1 = \frac{2T^2\ \ 8T_1T_2}{4T_1T_2+2T_1T+2T_2T+T^2}$$

$$a_2 = \frac{4T_1T_2-2T_1T-2T_2T+T^2}{4T_1T_2+2T_1T+2T_2T+T^2} \qquad (16)$$

$$b_1 \quad \frac{2T^2-8\alpha\beta T_1T_2}{4\alpha\beta T_1T_2+2\beta T_1T+2\alpha T_2T+T^2}$$

$$b_2 - \frac{4\alpha\beta T_1T_2-2\beta T_1T-2\alpha T_2T+T^2}{4\alpha\beta T_1T_2+2\beta T_1T+2\alpha T_2T+T^2}$$

For the present system with the transfer function given in Eq.12, standard design procedure [1] leads to the values

$$T_1 = 2.97\ ;\ \beta T_1 = 4.48$$
$$T_2 = 0.297\ ;\ \alpha T_2 = 0.0436$$

The simulation arrangement is again as in Fig. 2, with

$$\frac{s(z)}{e(z)} = \frac{k_d(z^2+a_1z+a_2)}{z^2+b_1z+b_2} \qquad (17)$$

from which follows the discrete equation

$$\begin{aligned} s(k) - k_d e(k) &+ a_1 k_d e(k-1) \\ &+ k_d a_2 e(k-2) - b_1 s(k\ \ 1) \\ &- b_2 s(k-2), k - 1, 2, \ldots \end{aligned} \qquad (18)$$

This is the equation that is programmed on the digital computer. Figure 5 shows the various uncontrolled and controlled responses.

CONCLUSION

The paper is a tutorial presentation of the joint use of analog and digital computers for the simulation of a system for which a digital controller is designed by classical frequency-response techniques. The responses show that the design techniques used are satisfactory.

The analog computer is a convenient replacement of the actual device to be controlled in the sense that control of the actual system will generally require appropriate power sources, detectors, actuators, etc., thus complicating the basic problem. It is of course a disadvantage if an analog computer is not available. In this case a totally digital simulation can be done. The use of the analog computer for the simulation of a given system is the nearest thing to actual system realization. Moreover, it is convenient for application of the ADC as continuous-time signals are always available. The DAC may give output values in the range 0 to +5 volts whereas the input signal is in the range -5 to +5 volts. Hence, signal restoration may be necessary.

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