UNBALANCE AND BALANCING OF THE 4 CYLINDER ENGINE A CRITICAL ANALYSIS OF LITERATURE

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ABSTRACT

The 4 cylinder 4 stroke engine is the most widely used IC engine type for cars, light and medium commercial vehicles and tractors as well as for small and medium stationary applications. These engines work with either the Otto (spark ignition) or Diesel (compression ignition) operation cycles. The large scale of application is the reason why this engine type has been considered at great length in the literature and why its problems of free mass forces balancing have played an important roll, especially when the engines are fast running. Unfortunately, the natural dynamic unbalance of this engine type and the measures to minimize its effects have neither been uniformly nor always correctly described and treated in the literature available at the Faculty of Technology of the Addis Ababa University.

Therefore a summary of the theoretical background and its simplifyed practical approach is given first. Important statements from the concerned literature are critically analyzed. Different possibilities for arranging balancing shafts in engines are shown. A simple model was developed and manufactured that can be advantageously used for demonstrations in class, to show the effects of reciprocating elements of different multi cylinder engine types.

INTRODUCTION

In IC engines one has to distinguish between internally and externally acting forces. The variable gas pressure inside the combustion chamber generates gas forces which are, for example, typical internal forces since the force vector polygons close inside the engine. But, since they built up the driving torque at the flywheel, they have an influence in its uniformity of motion, for which the effects on the engine block oscillation will be shown.

Mass forces (inertial forces) are due to the movements of the engine elements that are subjected to changes of the direction and magnitude of their accelerations. Such mass forces are first of all internal forces since they act like gas forces on bearings, guides and other force transmitting members of the engine. When the engine is seen as a whole, then some of these mass forces may not be counteracted completely inside the engine (not closing force polygons). They have then to be considered also as free mass forces acting on the exterior which usually lead to undesired engine vibrations being transmitted to supporting structures or the chassis in the case of motor vehicles.

To analyze these forces in the four cylinder engine, the motion characteristics of the single slider crank mechanism have to be studied first. This will be based on the model shown in Fig.1.

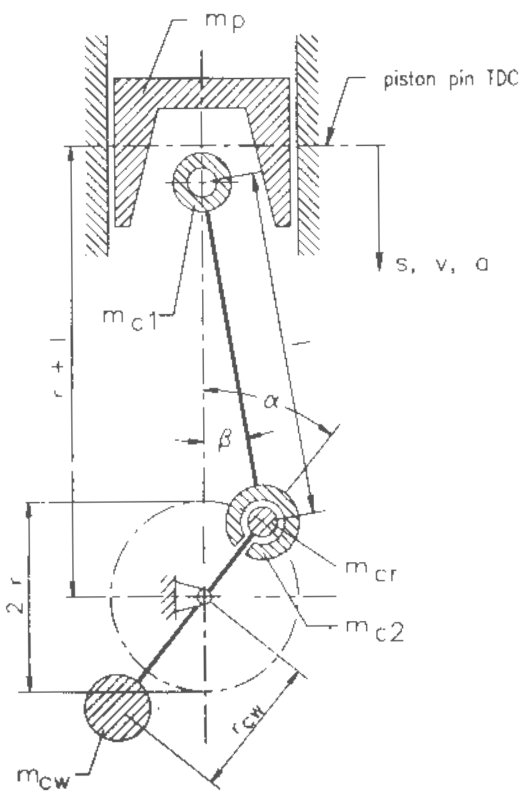


Figure 1 Piston Crank Mechanism

KINEMATICS OF THE SLIDER CRANK MECHANISM

Exact Equations

The piston displacement s is defined positive downward between zero and the entire stroke $2r$, starting at the top dead center (TDC), see Fig.1. This definition corresponds to the general pressure-volume diagram of the working cycle of engines and is therefore preferred to the definition of some authors [2,6,11,14,16] where the piston travel is defined between the $(l+r)$ position at TDC and $(l-r)$ at BDC.

Out of Fig. 1, the displacement of the piston is

$$s = r(1-\cos\alpha) + l(1-\cos\beta) \tag{1}$$

Furthermore itfollows from Fig.1, that

$$\sin\beta = \lambda\sin\alpha \tag{2}$$

where the con rod ratio λ is taken as

$$\lambda = \frac{r}{l} \tag{3}$$

With Eqs. (2) and (3), Eq. (1) can be rewritten as follows, where the displacement s is given in relation to the erank radius r, in order to obtain a unitless expression giving similar values for all sizes of engine

$$\frac{s}{r} = 1-\cos\alpha + \frac{1}{\lambda}\,[\,1-\sqrt{1-(\lambda s} \tag{4}$$

The derivation of the piston displacement over time gives the piston velocity. Since uniform motion of the crankshaft is usually assumed, the internal derivation of the crank angle over time is taken as the angular crank velocity ω

$$\omega = \frac{d(\alpha)}{d\,t} \tag{5}$$

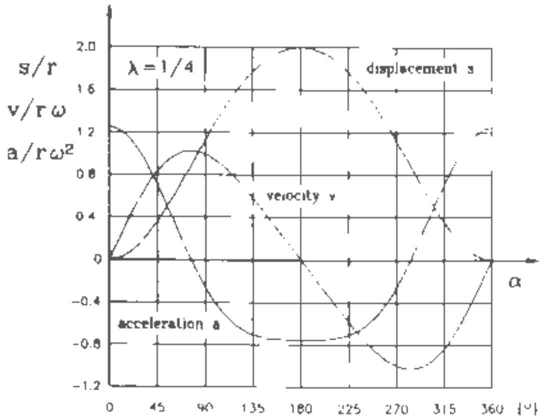
With it the unitless equation for the velocity is obtained

$$\frac{v}{r\,\omega} = \sin\alpha + \frac{\lambda\,\sin2\alpha}{2\sqrt{1-(\lambda\sin\alpha)^2}} \tag{6}$$

Another derivation over time leads finally to the unitless equation of the piston acceleration, Eq.(7):

$$\frac{a}{r\omega^2} = \cos\alpha + \frac{\lambda\cos2\alpha}{\sqrt{1-(\lambda\sin\alpha)^2}} + \frac{\lambda^3\,(\sin2\alpha)^2}{4\sqrt{[1-(\lambda\sin\alpha)^2]^3}} \tag{7}$$

Undoubtedly, with today's computation facilities, there wouldn't be any problem, either for engineers or students, to calculate the piston displacement, velocity and acceleration over crank angle according to these equations. This could be useful especially for those cases where the calculation of forces (as inner forces) would be required for strength analyses or dynamic bearing calculations, for example.



Firuge 2 Piston Displacement, Velocity and Acceleration

Figure 2 shows the development of the piston motion parameters throughout one revolution of the crank. The con rod ratio was here taken as a general value somewhere in the middle of the normal range of application which is about $1/3 < \lambda < 1/5$. Since all characteristics are symmetrical from TDC to BDC and back to TDC, all following graphs will consider the downward stroke of the piston only.

Approximated Equations

Despite modern computation facilities, today it is still common practis to use approximated equations for engine balancing considerations (and for vibration problems too) since then the dynamic nature of the forces can be much better studied.

One portion from Eq. (1), namely $\cos\beta$, can be replaced and expanded in a succession as follows, Eq. (8):

$$\begin{aligned}\cos\beta &= \sqrt{1\ \ \lambda^2\sin^2\alpha}\\ &= 1-\frac{1}{2}\lambda^2\sin^2\alpha-\frac{1}{8}\lambda^4\sin^4\alpha-\frac{1}{16}\lambda^6\sin^6\alpha-\dots\end{aligned} \tag{8}$$

The powers of sinα are replaced according to the rules of trigonometry, Eqs. (9) .

$$\begin{aligned}\sin^2\alpha &= \frac{1}{2}-\frac{1}{2}\cos2\alpha\\ \sin^4\alpha &= \frac{3}{8}-\frac{1}{2}\cos2\alpha+\frac{1}{8}\cos4\alpha\\ \sin^6\alpha &= \frac{5}{16}-\frac{15}{32}\cos2\alpha+\frac{3}{16}\cos4\alpha-\frac{1}{32}\cos6\alpha\end{aligned} \tag{9}$$

Finally (see also Low [9] and Taylor [16]), the serial development of the piston displacement is given by

$$\frac{x}{r} = 1 - \cos\alpha - (a_0 + a_2\cos 2\alpha + a_4\cos 4\alpha + a_6\cos 6\alpha + ...) \quad (10)$$

with a_0 up to a_6 being factors that depend on the con rod ratio only where higher powers of λ than 5 were neglected a priori:

$$a_0 = -(\frac{\lambda}{4} + \frac{3}{64}\lambda^3 + \frac{5}{256}\lambda^5 + ...)$$
$$a_2 = \frac{\lambda}{4} + \frac{1}{16}\lambda^3 + \frac{15}{512}\lambda^5 + ...$$
$$a_4 = -(\frac{1}{64}\lambda^3 + \frac{3}{256}\lambda^5 + ...) \quad (11)$$
$$a_6 = \frac{1}{512}\lambda^5 + ...$$

This serial development of the displacement-crank angle relation according Eq. (10) together with the expressions for the factors, Eqs.(11), show clearly that the piston has got a simple harmonic motion together with higher, even harmonics which influence is strongly decreasing with their order. This nature of multiple harmonics cannot be understood by the exact equation, Eq.(3), nor by Fig.1, but by the serial development only. It will be shown later that these harmonics play an important roll in engine balancing and for torsional oscillation damping of the block. Thus, the question should be carefully answered, as to which harmonics are to be considered for engine balancing.

The derivation of Eq. (10) over time gives the piston velocity again as a succession of harmonics, Eq. (12),

$$\frac{v}{r\omega} = \sin\alpha + 2a_2\sin 2\alpha + 4a_4\sin 4\alpha + 6a_6\sin 6\alpha + ... \quad (12)$$

and another derivation leads to the approximation of the piston acceleration, Eq. (13).

$$\frac{a}{r\omega^2} = \cos\alpha + 4a_2\cos 2\alpha + 16a_4\cos 4\alpha + 36a_6\cos 6\alpha + ... \quad (13)$$

Both equations are again given as unitless and dimensionless similarity expressions and can be generally used in this form since they depend on the con rod ratio and crank angle only.

Figures 3, 4 and 5 show the relative errors of the approximated equations of piston displacement Eq. (10), velocity (Eq.12) and acceleration Eq. (13) with respect to the exact expressions developed before. These errors were calculated for a different number of harmonics which were considered in the approximated equations. Error level 2 means that only the harmonics up to the second were included (error level 4: up to the fourth; error level 6: up to the sixth harmonics). Many authors dealing with engine dynamics and balancing take usually only the first and second harmonics into account and

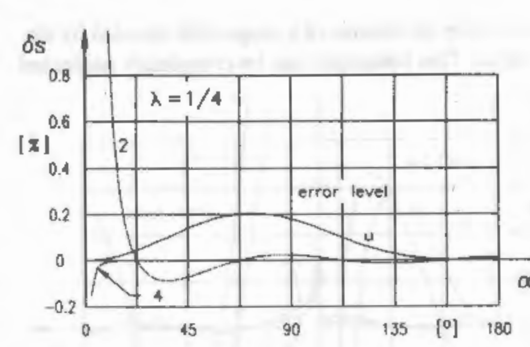


Figure 3 Piston Displacement Error

neglect higher powers of lambda in addition. This is considered in Figs. 3, 4 and 5 by the error level "*u*", standing for "usual approach". The equations of the three magnitudes for this case then look finally as follows:

$$\frac{s}{r} = 1 - \cos\alpha + \frac{\lambda}{4}(1 - \cos 2\alpha) \quad (14)$$

$$\frac{v}{r\omega} = \sin\alpha + \frac{\lambda}{2}\sin 2\alpha \quad (15)$$

$$\frac{a}{r\omega^2} = \cos\alpha + \lambda\cos 2\alpha \quad (16)$$

Figures 3, 4 and 5 show that the usual approach is best for the displacement and worst for the acceleration. But for a large range of the crank angle, the acceleration error is still less than ± 0.6 % at this con rod ratio. In the middle of the stroke where the acceleration itself is passing through zero, its error at any error level has got a singularity which is due to the general definition of any

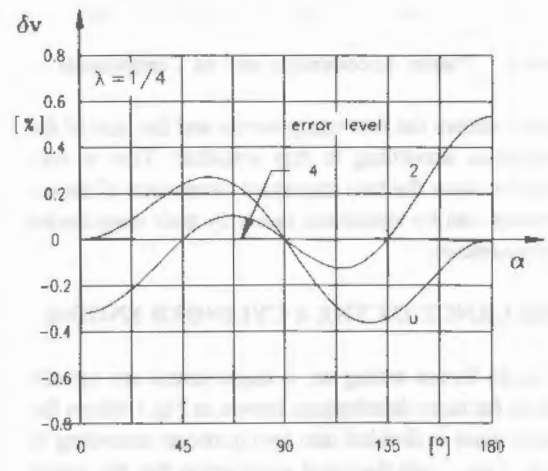


Figure 4 Piston Velocity Error

relative error (deviation of a magnitude divided by its true value). This behaviour can be completely neglected.

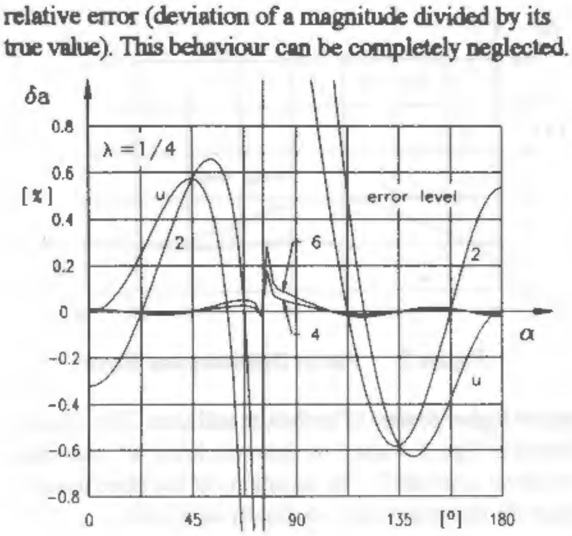


Figure 5 Piston Acceleration Error

In the following discussion, the usual approach to acceleration according Eq. (16) will be considered exclusively.

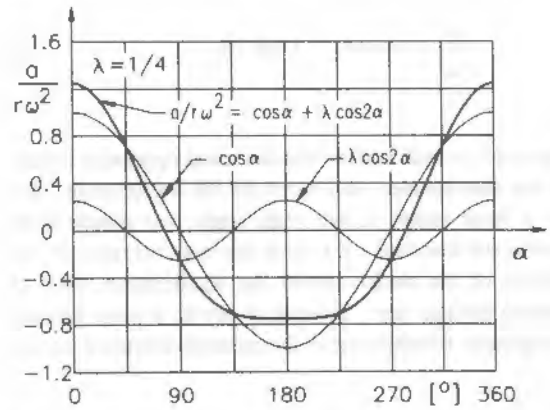


Figure 6 Piston Acceleration and its Components

Figure 6 shows the two components and the sum of the acceleration according to this equation. This is very instructive since the two important harmonics of the acceleration can be visualized easily by their magnitudes and frequencies.

UNBALANCE OF THE 4 CYLINDER ENGINE

The mass forces acting on a single crank are usually based on the mass distributions shown in Fig.1 where the con rod mass is divided into two portions according to $m_c = m_{c1} + m_{c2}$, with the usual assumption that the center of gravity of the con rod is maintained. The effect of eccentric masses of the crank shaft which are thought to be concentrated at the crank pin, together with the con rod mass m_{c2}, can be counteracted by a counterweight according to

$$m_{cw} r_{cw} = r[m_{cr} + m_{c2} + \gamma(m_p + m_{c1})] \quad (17)$$

The last term in Eq.(17) may be considered particularly in fast running one cylinder engines (important for motor cycle engines) when the reciprocating masses $m_{rec} = (m_p + m_{c1})$ are to be balanced to a certain degree $\gamma < 1$ by additional mass pieces on the counterweights (usually: $0.2 < \gamma < 0.5$). However, this measure creates an additional horizontal unbalance which would not exist for $\gamma = 0$ but may be accepted for those cases rather than having too big unbalanced vertical forces.

When the entire 4 cylinder engine is treated, the counterweights for rotating force balancing are usually attached to the shaft as shown in Fig.7, to optimize bearing forces. But, the degree of rotating force balancing does not play any roll for the entire engine since they are counteracted anyway by the usual 0/180/180/0 degree crank arrangement of this in-line engine. Therefore, in the following sections, the action of the reciprocating masses is seen only.

The acceleration of reciprocating masses produces inertial forces in direction of the cylinder axis at any position of the crank only. This is due to the basic definition of motion and acceleration, see Fig.1. Especially, no rotating forces of the reciprocating masses do exist and therefore also no sinus components (horizontal components) of inertial forces. (The known lateral piston tilting with excessive clearance of the piston skirt which may lead to strong strokes against the cylinder liner is not considered here.)

The inertia forces of the reciprocating masses of a single cylinder section based on the usual approach

$$\begin{aligned} F_M &= m_{rec} r\omega^2(\cos\alpha + \lambda\cos 2\alpha) \\ &= F_{MI} + F_{MII} \end{aligned} \quad (18)$$

are given by Eq. (18) where F_{MI} is the primary (or first order) force oscillating with the single angular frequency of the crankshaft, whereas F_{MII} is the secondary (second order) force going with the double frequency. Because of their different frequencies, it is convenient to consider them separately.

The sum of the primary forces F_{MI} of the four cylinder engine is zero meaning they are completely balanced with this crank arrangement Fig. 7. To analyze the free moments of the engine acting in its vertical plane, the reference plane perpendicular to the shaft should be always taken through the center of the crankshaft (which is correct for any engine), since an engine with its flexible supporting elementsis to be considered a free

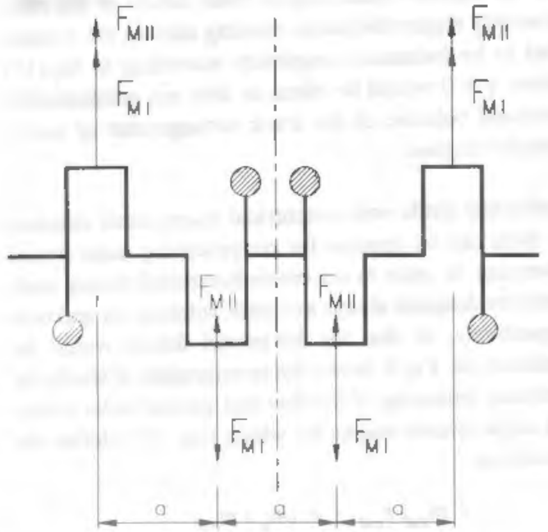


Figure 7 First and Second Order Forces at the Four Cylinder Crank Shaft

body which oscillates round its center of gravity. It is obvious that the first order moments of cylinders 1 and 4 as well as of cylinders 2 and 3 cancel out each other.

The sum of the secondary forces gives 4 F_{MII} as the remaining (unbalanced) shaking force that is acting vertically in the central plane. Even in this case there is no remaining couple: the secondary moments round the central plane are completely balanced.

Some authors show this nature in a table since for them "it is convenient to construct a table" [11] where the varying components of the forces and moments are listed for all cylinders and finally added. This is always done

Table 1: Force and Moment Components

cyl.no.	1	2	3	4	sum
α	0	180	180	0	
cosα	1	-1	-1	1	0
cos2α	1	1	1	1	4
center dist	-3/2a	-1/2a	1/2a	3/2a	
dist*cosα	-3/2a	1/2a	-1/2a	3/2a	0
dist*cos2α	-3/2a	-1/2a	1/2a	3/2a	0

for the first cylinder in its TDC position and for the remaining ones according to the crank angle difference with the first. Such table would look in its simplest form as shown in Table 1, showing that only the sum of cos2α = 4 remains: the variable component of the second order forces F_{MII}.

TREATMENT IN THE CONCERNED LITERATURE

As far as the first and second order forces as well as the first order moments are concerned, there are no contradicting statements in the studied sources.

Some problems apparently happen for the sum of the second order moments which is zero, see above. Different to this, sources [2, 5, 6, 11, 14] indicate M_{II}/F_{MII} = 6a or as "not balanced" [4, 11, 13]. These wrong results have been determined based on an unfavourable selection of the reference plane in the first cylinder.

Only Phelan corrected his first findings afterward in verbal statements [13] mentioning "that the moment is not a couple, but, rather, is the result of the resultant inertia force acting at a distance z from cylinder 1" which means they are acting in the center of the engine thus not producing a shaking moment.

Without giving clear statements, similar conclusions may be taken from equations of Holowenko [6], Martin [11] and Shigley [14]. It is really question-able whether, for example, any student can come to correct conclusions by himself after having found in tables of published textbooks [2, 5, 6, 11, 14] that the conditions for balancing are not fulfilled or after having read: "Secondary moments unbalanced" [11]. Even when the final statement in [5] is correct, the expression can't be accepted "that, if the plane midway between the two middle cylinders is chosen to be the reference plane, then the secondary couples are in balance,...". This would indicate that the appearance or non appearance of couples depended on the chosen reference plane rather than from the correct theory.

It is also interesting that Shigley has given up complicated equation developments and the component summation in the form of tables still applied in [14], and has come to correct results in his second textbook (together with Uicker) [15].

Another problem occurs in the literature [2, 4, 5, 6, 11, 14] with respect to the expansion of the equa-tions of the inertia forces. Instead of giving the summation of the inertia forces (now separating primary and secondary forces) according Eqs. (19),

$$F_I = m_{rec} r \omega^2 \sum_{i=1}^{4} \cos\alpha_i$$
$$F_{II} = m_{rec} r \omega^2 \lambda \sum_{i=1}^{4} \cos 2\alpha_i \quad (19)$$

($\omega * t$) is used in these sources as the crank angle of the first cylinder and α_i as the difference crank angle with respect to the first crank. Then the summations were expanded according to trigonometry and became like Eq.(20), here only shown for the primary force. The constant factors outside the summations are not considered in this equation:

$$\sum_{i=1}^{4} \cos(\omega t + \alpha_i) = \sum_{i=1}^{4} (\cos\omega t \cos\alpha_i - \sin\omega t \sin\alpha_i) \quad (20)$$

Although this mathematical expansion is principally correct for different variables, the composed crank angle definition of the sources can't be accepted here at all since, with it and the expansion, some sinus components of inertia forces of the reciprocating masses and those of the moments respectively seem to exist. But this is not true considering the basic definitions which were shown before.

The consequences of this equation expansion procedures are obvious: Sinus components appear in the force summation tables [2,4,5,6,11,14] that are not zero for certain engines [2,4,6] and some authors even show horizontal vectors (at $\alpha = 90$ °) of reciprocating inertial forces in graphs for in-line engines [4,11], for example the four cylinder four stroke in- line engine with 90 degree crank arrangement.

Table 2 (see next page) gives a summary of impor-tant results of the different sources encountered. For simplicity and uniformity, inertia forces and moments of all sources are shown according to the above given definitions and they are related to the concerned inertia force of one cylinder, in order to obtain them dimensionless and unitless. It is obvious, that the critical definition of the reference plane, incorrect second order moments, complicated force summation tables, the "complicated" equations (based on the inadmissible equation expansions) and sinus components (horizontal vectors) of inertial forces coincide in almost all of these sources.

Some textbooks for IC engines, motor vehicle engineering and general mechanical engineering hand-books have been analyzed. They don't usually have these problems, for example [1,3,7,8,9,10,12,16].

BALANCING OF MASS FORCES AND MOMENTS

For the separate balancing of mass forces of the reciprocating engine elements, rotating masses are considered to be balanced completely according to Eq.(17) where $\gamma = 0$ would be taken or they are automatically balanced because of the crank arrangement of multi-cylinder engines.

Additional shafts with eccentrical mass pieces attached to them can be applied for reciprocating mass forces balancing. In order to not create horizontal forces, such shafts are designed always as a pair, rotating cw and ccw respectively, so that the horizontal forces would be canceled out. Fig.8 shows the arrangement of shafts for complete balancing of the first and second order forces of a single cylinder engine for which Eqs. (21) define the conditions.

$$2\, m_{cwI}\, r_{cwI} = r\, (m_p + m_{cI})$$
$$2\, m_{cwII}\, r_{cwII} = \frac{1}{4}\, r\, \lambda\, (m_p + m_{cI}) \quad (21)$$

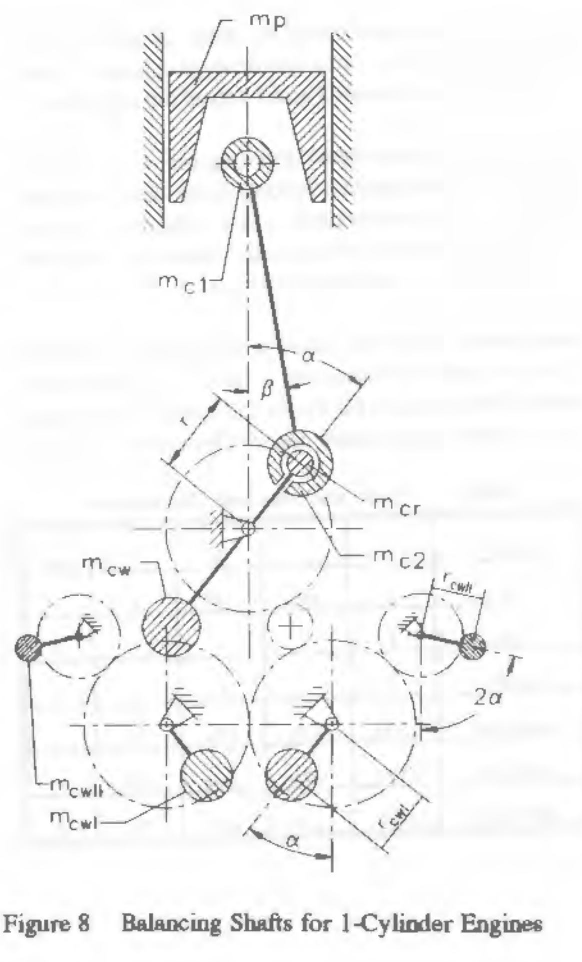


Figure 8 Balancing Shafts for 1-Cylinder Engines

Table 2: Listing of Important Statements from the Literature

source	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
refer.plan	m	1.c	m	1.c	1.c	1.c	m	m	1.b	m	1.c	m	1.c	1.c	-	
F_I/F_{MI}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{II}/F_{MII}	4	4	4	not	4	4	4	4	not	4	4	4	not	4	4	4
M_I/F_{MI}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$M_{II}F_{MII}$	0	6a	0	not	6a*	6a	0	0	0	0	6a	0	6a*	6a	0	0
summ.table	-	w	-	-	w	w	-	-	-	-	w	-	-	w	-	
vector rep.	y	-	y	w	-	-	y	-	y	y	w	y	-	-	w	y
force equ.	s	cpl	s	cpl	cpl	cpl	s	-	s	s	cpl	s	s	cpl	-	s
double α	y	y	y	y	y	y	y	y	y	y	y	y	y	y n	y	y
high.harm.	y	-	y	y	y	y	y	y	y	-		-	y	-	-	y
balancer	2ω	-	-	-	2ω		2ω	2ω	-	-	y	2ω	2ω	-	-	-
tang.force diagram	y	y	y	y	y		y	y	-	-		-	-	-	-	y
tang.force balancer	y	-		-	-		y	-	-	-		-	-		-	

m = middle plane, 1.c – first cylinder, 1.b = first bearing, not = not balanced,
w = wrong, y – yes, s = simple, cpl = complicated,
* = in verbal statements : zero, 2ω = 2 ω (Lanchester) balancer

The balancing shafts are located in the crankcaseusually below the crank mechanism and are driven by an additional gear drive of which the pitch circles are shown schematically. Two shafts with their mass pieces m_{cwI}, rotating cw and ccw with the single engine speed ω, generate vertical forces which counteract the primary force. Similarly, the mass pieces c_{wII} rotating with 2ω generate the necessary secondary balancing forces.

The author has carried out a number of tests and research on a special IFA one cylinder test Diesel engine which was equipped with the shown four balancing shafts. Such great efforts in the design and manufacture of balancing systems are reasonable for those cases of 1 cylinder test engines only where smooth and quiet operation is essential.

A series production stationary one cylinder Diesel engine of Yanmar, model TD 185, is equipped with the shown two single ω balancing shafts. This fast running horizontal engine is relatively heavy when compared with fast running motor cycle engines. It was found to run without strong vibrations on the test bench of the Faculty of Technology, AAU, where it is used for tests and lab exercises.

Because of the basic secondary force unbalance of four cylinder engines, a 2ω balancer would be required. Fig.9 shows schematically a gear drive and a chain drive (or tooth belt drive) as possible solu-tions for driving the two balancing shafts cw and ccw respectively. The two mass pieces have to make up the unbalance of the 4 cylinder engine, Eq. (22):

$$m_{cwII}\ r_{cwII} = \frac{1}{2}\ r\ \lambda\ (m_p + m_{cl}) \tag{22}$$

The 2ω balancing system of four cylinder engines is sometimes referred to as "the Lanchester" harmonic balancer and some applications have been mentioned [1,8,11,12]. Some sources show a crossed helical gear set to drive the laterally arranged balancing shafts directly from the central webs of the crank shaft [5,11,12]. The parallel balancing shaft arrange-ment (like Fig.9) was also named "the Meadows" balancer [12].

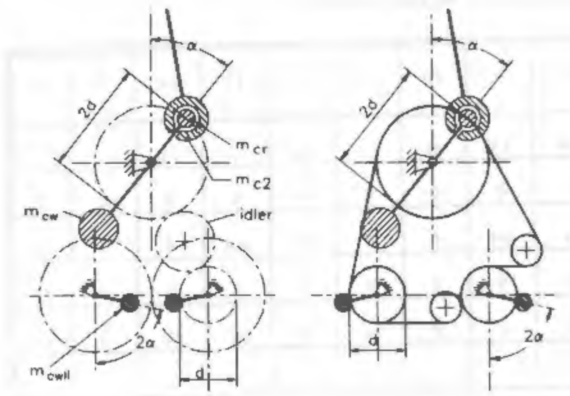


Figure 9 2ω Gear and Chain Drives of Four Cylinder Engine Balancing Shafts

TORQUE BALANCING AT THE ENGINE BLOCK

Besides the vertical vibrations, the vertically oscillating mass forces together with the gas forces acting on the piston produce, in addition, a periodically alternating torque on the crankshaft with any engine type. This torque can be smoothed over for driving by a big enough inertia momentum of the flywheel and the arrangement of many cylinders operating one after another. On the other hand, the action of the reacting torque may produce unacceptable torsional oscillations of the engine block as well as those of the frame and body of cars.

The arrangement of the above shown balancing shafts parallel to the crankshaft is well suitable for modern four cylinder in-line engines to match additionally the excitation of these torsional oscillations on the engine block round its longitudinal axis. For the representation of the oscillating torque, it is common use to calculate the tangential forces acting at the crank pin and to show them in a diagram over the crank angle.

Figure 10 shows the tangential forces diagrams of two 4-stroke direct injection Diesel engines of a size they may be applied in cars. For the one cylinder engine at 2500 rpm, the compression and expansion gas forces before and after $\alpha = 360°$ dominate the behaviour of the graph. Their superposition for all 4 cylinders according to the firing order shows a somewhat harmonic development of the tangential forces already for 2500 rpm which is due to both the gas and oscillating mass forces of all cylinders. At 4000 rpm, strong maxima and minima appear at about α = 45, 135, 225, 315,... degree crank angle.

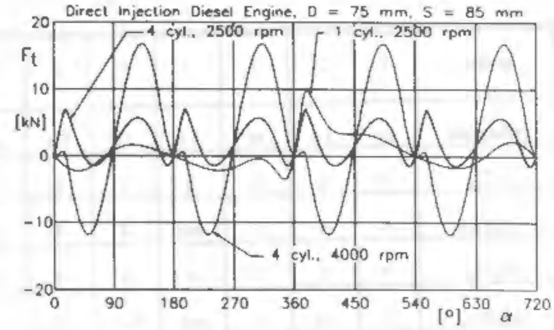


Figure 10 Tangential Forces Diagram

To smooth over the oscillating counter torque, a vertical offset of the above described 2ω balancing shafts can be applied according to Fig.11.

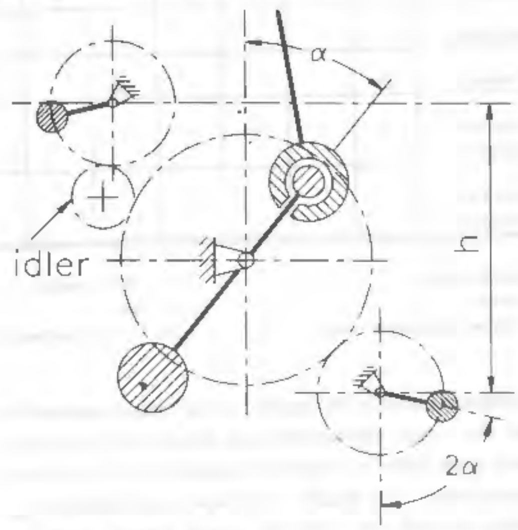


Figure 11 Height Offset of 2ω Balancing Shafts

The offset and the position of the balancing shafts as well as the gear drive are shown schematically only. The offset *h* itself is decisive for the degree of balancing of the torque oscillations but the force balancing would be maintained according to Eq. (22). The shaft offset for complete balancing of the oscillating torque due to the alternating mass forces can be derived out of Eq.(18) which was applied firstly for a one cylinder section. The oscillating tangential force is then given by Eq. (23) :

$$F_{tM} = F_M \frac{\sin(\alpha+\beta)}{\cos\beta} = m_{rec} r\omega^2(\cos\alpha + \lambda\cos 2\alpha)\frac{\sin(\alpha+\beta)}{\cos\beta} \quad (23)$$

The oscillating moment follows simply by multiplying Eq. (23) with the crank radius *r*. As they were already derived from Fig.10, the tangential forces according to

Eq. (23) of all cylinders of the usual 4 cylinder crank arrangement are maximum and minimum for $\alpha = \pm 45 + n * 180°$.

The desired countermoment obtained by an offset of the 2ω balancing shafts, Fig.11, is defined by Eq. (24), where Eq. (22) (for 4 cylinder engines) was introduced:

$$M_c = m_{cwII}\, r_{cwII}\, (2\omega)^2 h \ \sin 2\alpha = 2\lambda m_{rec} r \omega^2 h \ \sin 2\alpha \qquad (24)$$

It gives maxima and minima for the same crank angles as above shown. Taking the oscillating moment for all four cylinders according to Eq. (23) and equating it with Eq. (24) gives finally a general expression for the shaft offset for complete moment balancing (considering the mass forces only):

$$h = 2\frac{r}{\lambda}(\cos\alpha + \lambda\cos 2\alpha)\frac{\sin(\alpha + \beta)}{\cos\beta \ \sin 2\alpha} \qquad (25)$$

For $\alpha = 45$ and $\lambda = 0.25$, this gives h = 4.72 *r*.

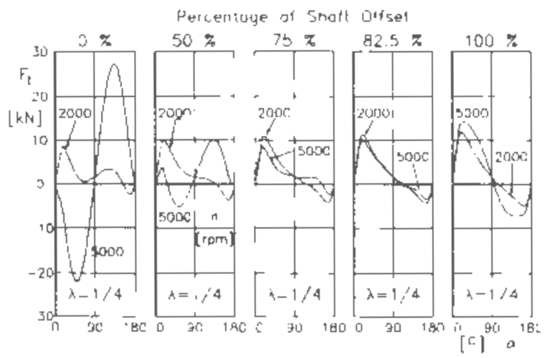


Figure 12 **Torsional Moment Balancing**

The effect of the shaft offset is well visible in Fig.12. It shows the total tangential forces of a 4 cylinder engine (including the gas forces and the counter moment) in all cases for one stroke only. Especially for high speed there is a great influence from zero shaft offset (= 0 % torque balancing) up to the offset according Eq. (25) (= 100 %). In the same way as a uniform driving torque is always desirable, the engine block should be turned by a constant counter torque only with as small as pos-sible alterations. This means, a positive and uniform torque would be the best option. For the assumed case of Fig.12, it is obvious that the optimum is therefore near 75 % of the theoretical shaft offset which is here an effect of the tangential gas forces, since very small positive and negative amplitudes appear only, especially at higher speeds.

IC ENGINE BALANCING MODEL

To visualize the effects of not balanced crank mechanisms of IC engines in the class room, a simple two shaft balancing model was developed and manufactured in the workshop of the Faculty of Technology (Final Project of Yohannes Assefa, AAU, 1993).

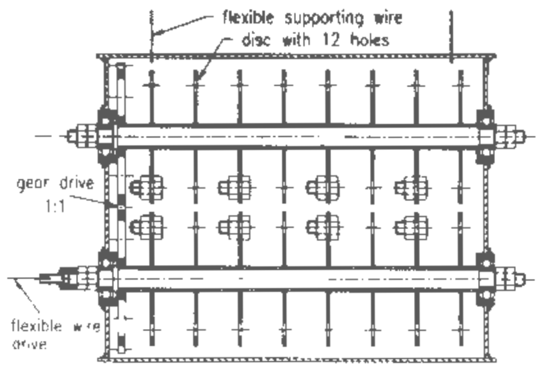


Figure 13 Balancing Modell

The model shown in Fig.13 is vertically supported on an external frame by flexible steel wires (the external frame is not shown) and horizontally adjusted to this frame by small expansion springs. The cw and ccw rotating shafts (center distance 90 mm) are driven 1:1 by a hand drilling machine through a flexible shaft and a gear drive.

The idea of this model is that in the same way as balancing shafts of IC engines generate vertical counter forces, be it for the speed ω or 2ω, such a shaft arrangement is also able to generate and show the effects of the different forces of not balanced engines if the original crank mechanisms and balancing shafts are not provided. The shaft arrangement one above the other means simply that it would correspond to a horizontal (laying) engine.

The reciprocating masses of engines are represented by interchangeable bolts attached to the discs which in turn represent the cranks. Because of the eight discs all having twelve bolt holes, 1 up to 8 cylinder in line engines and different V engines can be modelled. When the model is slowly driven it oscillates at low frequency according to the case chosen by the arrangement of the mass pieces. There was no intention to simulate the magnitudes of the masses and the oscillation frequencies of real engines. The indicated mass pieces arrangement of Fig.13 simu-lates the effect of the second order mass forces of the four cylinder four stroke engine

SUMMARY AND CONCLUSIONS

The theoretically exact equations, their development in successions and the very common approximations of piston displacement, velocity and acceleration are given and the behaviour shown in graphs. Based on it, the natural unbalance of 4-cylinder in-line engines with the normal crank arrangement is derived. Some inaccuracies and incorrect details found in several textbooks are shown and their problems were critically analyzed.

Since mass balancing of engines in general and some cases of applications are well known, the layout of different balancing systems is shown. An additional torque balancing can be achieved by a proper vertical offset of the 2ω balancing shafts. For demonstrations in the class room, a simple balancing model was developed and represented.

Out of the analysis of the available literature emerges the need and responsibility of the university lecturer to study well the theory when working out the teaching material of his field.

All shown graphs were calculated by Pascal programs generating automatically script files for AutoCAD which afterward automatically traced the graphs that were finally converted to WordPerfect graphic files [17]. Drawings and sketches were elaborated with AutoCAD and then also converted.

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