ANALYSIS OF CONCRETE SLABS SUBJECT TO CONCENTRATED LOADS

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ABSTRACT

A simplified procedure for the calculation of the additional reinforcement which is required under concentrated loads acting on concrete slabs is provided. The introduced method can also be used for the treatment of line loads. Conditions for the application of this procedure are stated and tables for necessary design values have been prepared. Recommendations for the arrangement of the additionally required reinforcement are given. A numerical example shows the application of the procedure.

INTRODUCTION

Many reinforced concrete slabs are not only subject to uniformly distributed loads but also to heavy concentrated loads, even moveable ones. Several methods for the more exact analysis of such slabs exist [1], [2], [3], [4]. To ease the design work of the engineer many specifications allow the use of simplified procedures [5], [6], [7], [8]. Here, only a certain width of a slab, the so called effective width b_e (see Fig. 1), can be assumed to support these concentrated loads. After calculating this effective width, the moment on a 1 m strip can be easily determined by $1/b_e$ parts of the concentrated load.

In the following article such a simplified procedure, as offered in [7] and [8], for the treatment of concentrated loads acting on *solid, reinforced concrete, one-way slabs* shall be presented.

For two-way slabs the effective width in the main bearing direction may be determined according to this procedure. To provide for the lateral distribution of the concentrated loads in the other direction the additional internal forces are to be considered at least approximately.

The introduced procedure can not be used if the slab consists of several prefabricated concrete strips.

EFFECTIVE WIDTH b_e FOR REINFORCED CONCRETE SLABS

The actual load area $(a_x \times a_y)$ of the concentrated load can be uniformly increased under an angle of 45^0 up to the middle surface of the slab, see Fig.2. Additional layers on top of the slab may be considered if they are able to distribute the load. The measurements of this increased load area can be determined as

$$t_x = a_x + 2(s + h/2) \qquad (1a)$$

$$t_y = a_y + 2(s + h/2) \qquad (1b)$$

where t_x, t_y are the measurements of the increased load area
a_x, a_y are the measurements of the actual load area
s is the thickness of a load distributing layer
h is the thickness of the slab

In determining the effective widths it must be differentiated whether the analysis is made for bending moments or for shear forces. The corresponding formulae are given below.

for the moment:

$$b_{e,M} = t + k_M\, l \leq actual\ b_e \qquad (2)$$

for the shear:

$$b_{e,V} = t + k_V\, l \leq actual\ b_e \qquad (3)$$

where k_M and k_V are values which depend on the statical system of the slab and on the location of load. If the conditions of Table 1 are fulfilled, values for k_M and k_V can be taken from table 2a and 2b.

l is the span of the slab

Moment and shear per m width can be determined as:

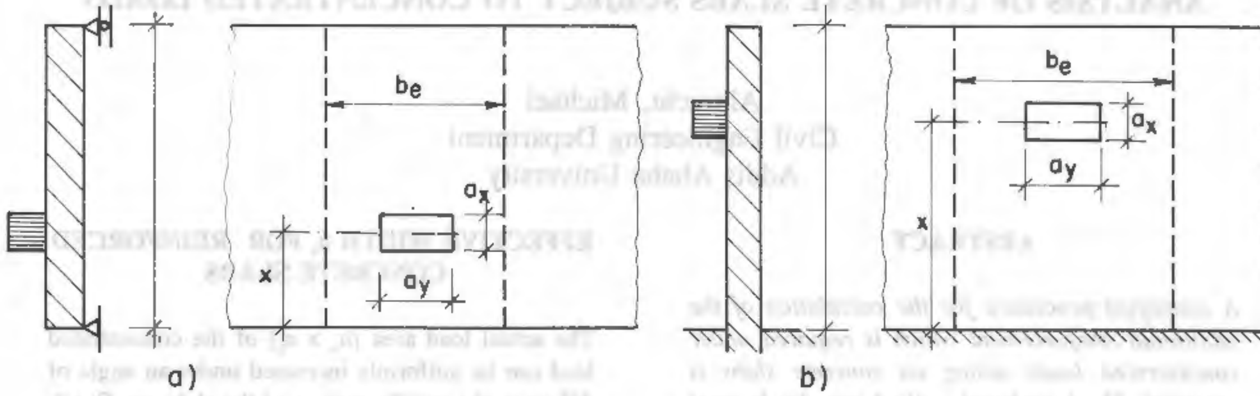


Figure 1 Effective Width b_e of Reinforced Concrete Slabs subject to a Concentrated Load
a) One-Way-Slab b) Cantilever Slab

$$m = \frac{M}{b_{e,M}} \tag{4}$$

$$v = \frac{V}{b_{e,V}} \tag{5}$$

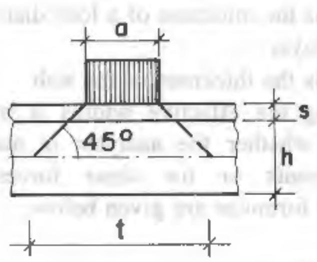


Figure 2 Increased Load Area

In each case the calculated effective width must not be larger than the actual one. If the loads are near to the edge of the slab or if they are near to each other, the effective width has to be taken as shown in Fig. 3. For more details refer to [9].

CONCENTRATED LOADS EXTENDING OVER THE WHOLE SPAN

If a slab is subject to a line load running over the whole span as it may be in case of a partition wall, the effective width b_e can be determined by values given in Table 3. The load contact area should approximately be described by the values a_x/l = 1.0 and a_y/l = 0.05. Using Tables 1, 2a, and 2b smaller values will be obtained in comparison to Table 3, but these values are valid for a large variation of loads and are on the safe side.

ARRANGEMENT OF REINFORCEMENT

The following recommendations, taken from [10], are given concerning the arrangement of that part of the reinforcement that is additionally required by the action of concentrated loads. For one-way slabs the additional reinforcement in the main bearing direction A_{sc} should be distributed over the whole effective width b_e. To care for transverse moments it is sufficient to arrange an additional transverse reinforcement A_{sct} = 0.6 A_{sc}. The ends of these bars should be staggered by $b_e/4$. The arrangement of the additionally required reinforcement is shown in Figure 4.

For concentrated loads located near to free edges the effective width reduces to the actual value. The additional reinforcement should be distributed from the free edge in the y-direction over a length of about 2 x *act* b_e, see Figure 5. If the length of the slab in the y-direction is less than 2 x *act* b_e the reinforcement can be distributed only over the actual length. The transverse moments in this case are negative. Therefore, a top reinforcement $A_{sct} \geq$ 0.1 A_{sc} should be arranged in the middle third of the span. The length of these bars should be about *act* b_e. In case of concentrated loads acting on cantilever slabs the reinforcement shall be distributed over b_e but should be concentrated in the middle third. A lower transverse reinforcement A_{sct} = 0.6 A_{sc} is required in the area where the concentrated load acts.

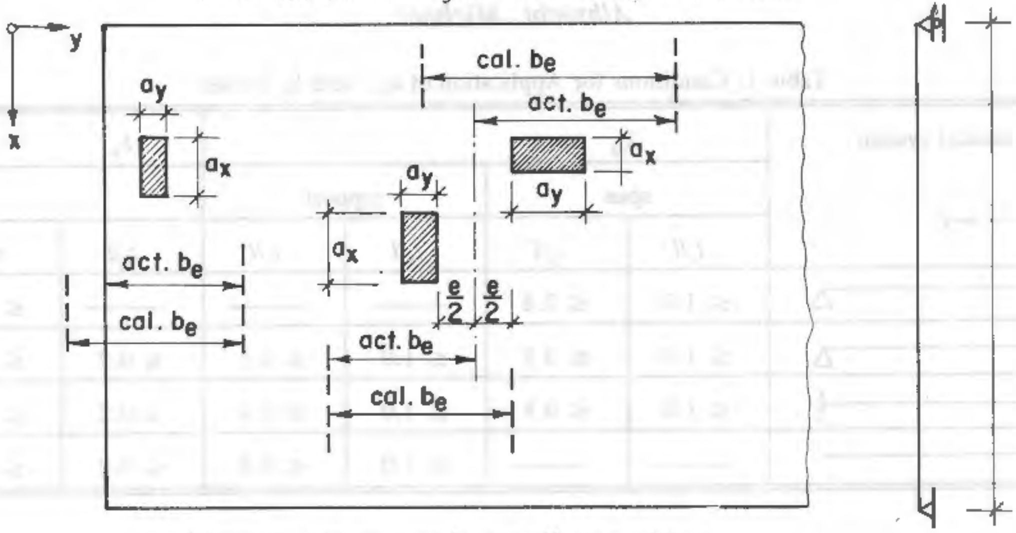


Figure 3 Assumptions for the effective width b_e

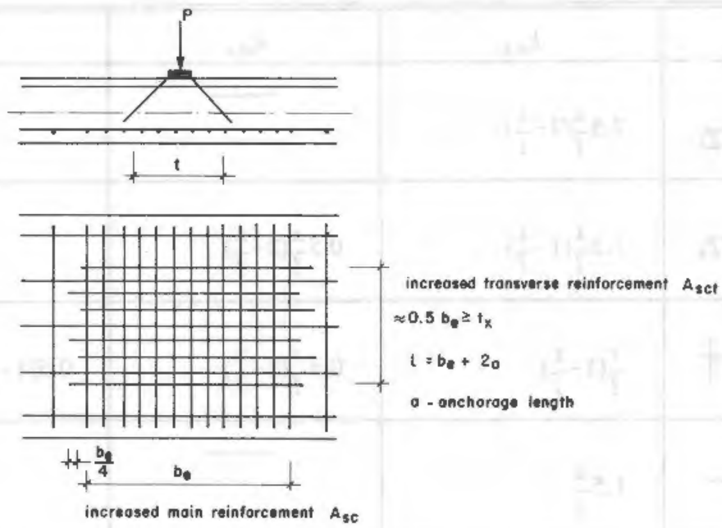


Figure 4 Arrangement of Additional Reinforcement Required for Concentrated Loads

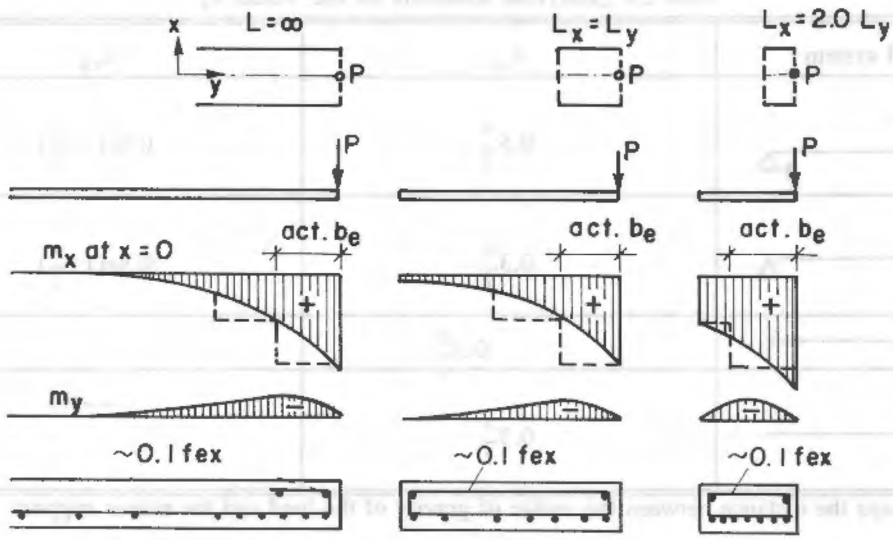


Figure 5 Distribution of Reinforcement Required for a Concentrated Load Acting Near the Free Edge

Table 1: Conditions for Application of k_M - and k_V Values

statical system 	k_M				k_V	
	span		support			
	t_x/l	t_y/l	t_x/l	t_y/l	t_x/l	t_y/l
	≤ 1.0	≤ 0.8	——	——	——	≤ 0.8
	≤ 1.0	≤ 0.8	≤ 1.0	≤ 0.8	≤ 0.2	≤ 0.4
	≤ 1.0	≤ 0.8	≤ 1.0	≤ 0.4	≤ 0.2	≤ 0.4
	——	——	≤ 1.0	≤ 0.8	≤ 0.2	≤ 0.4

Table 2a: Analytical Relations for the Values k_M

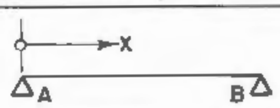
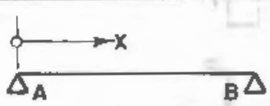
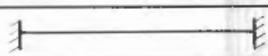
statical system	$k_{M,S}$	$k_{M,A}$	$k_{M,B}$
	$2.5\frac{x}{l}(1-\frac{x}{l})$	——	——
	$1.5\frac{x}{l}(1-\frac{x}{l})$	$0.5\frac{x}{l}(2-\frac{x}{l})$	——
	$\frac{x}{l}(1-\frac{x}{l})$	$0.5\frac{x}{l}(2-\frac{x}{l})$	$0.5[1-(\frac{x}{l})^2]$
	$1.5\frac{x}{l}$	——	——

Table 2b: Analytical Relations for the Values k_V

statical system	$k_{V,A}$	$k_{V,B}$
	$0.5\frac{x}{l}$	$0.5x(1-\frac{x}{l})$
	$0.3\frac{x}{l}$	$0.4x(1-\frac{x}{l})$
	$0.3\frac{x}{l}$	
	$0.3\frac{x}{l}$	

Note: x is always the distance between the centre of gravity of the load and the nearer support

Table 3: Effective Width for Line Loads

statical system	$b_{e,M}$ for $t_x/l = 1.0$ and $t_y/l = 0.05$	
	span	support
	$1.35\ l$	——
	$1.04\ l$	$0.65\ l$
	$0.86\ l$	$0.53\ l$
	——	$1.35\ l$

Table 2 can be used to get the expressions for k_M.

NUMERICAL EXAMPLE

A monolithic one-way slab is subject to three concentrated loads as shown in Figure 6. All measurements are in cm.

For each load the effective width of the slab in bending has to be determined.

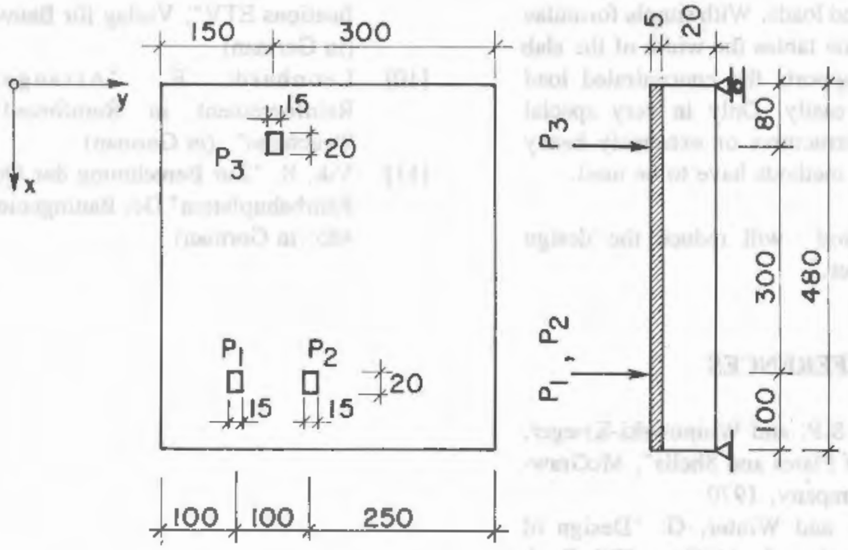


Figure 6 Statical System and Applied Loads for the Slab

Solution:

- measurements of the increased load area:

$t_x = 20 + 2 \times 5 + 2 \times 20/2 = 50$ cm

$t_y = 15 + 2 \times 5 + 2 \times 20/2 = 45$ cm

$t_x / L = 50/480 = 0.104 \leq 1.0$

$t_y / L = 45/480 = 0.094 \leq 0.8$

load	x/L	k_M	cal. $b_{e,M}$ act. $b_{e,M}$	load per m width [kN/m]
P_1	100/480=0.21	2.5x0.21(1-0.21)=0.415	45+0.415x480=244.1 100+100/2=<u>150</u>	32.5/1.50 = <u>21.7</u>
P_2	0.21	0.415	244.1 0.5(100+244)=<u>172.1</u>	32.5/1.721 = <u>18.9</u>
P_3	80/480=0.167	0.347	45+0.347x480=<u>211.7</u> 150+211.7/2=255.8	32.5/2.117 = <u>15.4</u>

The values for the effective width b_e, calculated with the presented procedure, can also be used for wheel loads. This is demonstrated in [11] by comparison with values obtained using more accurate procedures.

CONCLUSION

Very often the use of simplified design procedures is sufficient and even permitted by specifications. One example of an application of such a method is the treatment of reinforced concrete slabs which are subject to concentrated loads. With simple formulae and by means of some tables the width of the slab which effectively supports the concentrated load can be determined easily. Only in very special cases of important structures or extremely heavy loads more accurate methods have to be used.

The presented method will reduce the design efforts of the engineer.

REFERENCES

[1] Timoshenko, S.P. and Woinowski-Krieger, S. "Theory of Plates and Shells", McGraw-Hill Book Company, 1970

[2] Nilson, A.H. and Winter, G. "Design of Concrete Structures", McGraw-Hill Book Company, 1986

[3] Ferguson, Ph.D. "Reinforced Concrete Fundamentals", John Wiley & Sons, 1981

[4] Bares, R. "Tables for the Analysis of Plates, Slabs and Diaphragms based on the Elastic Theory", Bauverlag, Wiesbaden und Berlin, 1979

[5] AASHTO "Standard Specification for Highway Bridges", Washington, 1957

[6] Aswani, M.G.; Vazirani, V.N. and Ratwani, M.M. "Design of Concrete Bridges", Khana Publishers, 1978

[7] DIN 1045 "Concrete and reinforced Concrete; Design and Construction" (in German)

[8] TGL 33404/02 "Concrete Construction; Calculation of Internal Forces and Deformations; Tables and Data for Calculation. (in German)

[9] Drigert, K.; Benkert, K.; and Schröder, K. "Explanation to Reinforced Concrete Specifications ETV", Verlag für Bauwesen, 1982 (in German)

[10] Leonhard, F. "Arrangement of Reinforcement in Reinforced Concrete Structures" (in German)

[11] Vik, B. "Zur Berechnung der Querkräfte in Fahrbahnplatten" Der Bauingenieur 1964, S. 485 (in German)