AN AUTOMATED ANALYSIS-SYNTHESIS PACKAGE FOR DESIGN OPTIMIZATION OF FRAMED STRUCTURES

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ABSTRACT

An integrated analysis-synthesis software package that is based on the Fortran-90 standards is developed for the design optimization of framed structures — continuous beams, plane and space trusses and rigid frames, grids and composite truss-rigid frames. The package will enable the structural engineer to effectively and efficiently isolate designs that are better than alternative designs with minimal interaction and less computational effort. The capabilities of the package also include pure analysis and parametric studies.

A numerical example will be presented to show the potential capabilities of the package.

INTRODUCTION

Design in any engineering discipline is a process by means of which a product is generated to meet a certain goal while simultaneously satisfying some perceived requirements. Principles of design optimization are implemented in any design process to enable an effective use and allocation of scarce resources. Design optimization is a process by which the practically-best design is isolated from among several alternative designs. The formal representation of a design optimization process involves an objective to be met, conditions to be satisfied, and defining the domain of alternative designs. The portion of the structural design process that can be optimized automatically has been significantly increased in recent years due to rapid developments in structural analysis, digital computers, and optimization methods.

Several analysis packages (see, for example [1]) and some optimization packages (see, for example, [2]) are available. Each group of such packages is almost always called upon to perform a single task — either analysis or synthesis. Analysis packages are tailored towards providing response quantities such as axial, shear, bending and torsional stresses, various types of displacements, modal frequencies and other dynamic behaviours; nevertheless, they are not design-oriented and, consequently, are not suitable for integration into an iterative design environment. Synthesis packages, on the other hand, provide optimized structural topologies, geometries and member dimensions that will be capable of resisting effects of imposed design actions and other requirements. These latter group of packages demand additional effort for the determination of behaviour responses. Effective automation in the process of design optimization is achieved when the two categories of packages are integrated into one seamless unit. The purpose of this paper is to provide a package built around these principles and that possesses analysis-synthesis capabilities in a single optimization package for a complete set of framed structures — continuous beams, two and three dimensional trusses and rigid frames, grids as well as composite plane and space truss-rigid frame systems.

There are two major categories of analysis methods that have been widely practiced — flexibility (or force) and stiffness (or displacement) methods [3]. In this paper, the stiffness method of analysis will be adapted. Likewise, there are two categories of synthesis approaches for design optimization. These are the mathematical programming [4] and the optimality criteria [5] methods. The algorithm developed in this paper is based on the numerical approach of mathematical programming.

MATHEMATICAL FUNDAMENTALS

The process of structural design optimization and its problem formulation in the numerical optimization approach consists of three main components:- analysis, synthesis and convergence criteria. The three components employed in the present package will be discussed subsequently.

Structural Analysis: Structural analysis is a process by which structural response quantities are determined. These response quantities are necessary in the course of design optimization to assess the adequacy of a proposed design with respect to some established criteria and to guide the synthesis process. It has been pointed out that the stiffness method will be employed in the development of the analysis component of the optimization package presented in this paper. The general formulation for determining the unknown nodal displacements is based on equilibrium conditions and the general expression that forms the basis for the stiffness method for multiple loading condition is:

$$K\delta + P' = P_o \tag{1a}$$

where

- K: Stiffness matrix,
- δ : Matrix of unknown nodal displacements each column of which corresponds to one loading condition,
- P', P_o: Matrices of forces in the restrained structure and in the original structure, respectively, and corresponding to the unknown nodal displacements. Each column represents a loading condition.

Once the unknown nodal displacements δ are computed from Eq.(1a), other displacements and forces of interest at any point in the structure are evaluated based on the superposition principle. The final displacements Δ and forces N in a multiple loading system are determined from:

$$\Delta = \Delta' + \Delta''\delta \tag{1b}$$
$$N = N' + N''\delta \tag{1c}$$

where Δ' and N' are matrices of displacements and forces , respectively, due to loadings in the restrained structure while Δ'' and N'' are corresponding quantities due to unit values of the corresponding displacements δ.

Structural Synthesis: This is the process by which a design that will meet any imposed restriction *(feasible design)* is obtained. When optimization is employed, the synthesis process further involves isolating a design that is at least as good as any other alternative design *(optimal design)* under the same set of conditions. The object of structural optimization is, therefore, to determine the values of the design variables in such a way that an objective function attains an extreme value while any imposed design requirements are satisfied. This can be posed mathematically as follows:

Given preassigned parameters and loading conditions, find the vector of design variables $X = \{X_1, X_2, \ldots, X_n\}$ so as to:

$$\text{Minimize } Z(X) \tag{2a}$$

Subject to

$$G_j(X) \le 0, \quad j=1,J \tag{2b}$$
$$H_k(X) = 0, \quad k=1,K \tag{2c}$$
$$X^l_i \le X_i \le X^u_i, \; i=1,N \tag{2d}$$

In Eq.(2), Z denotes the objective function, G_j and H_k represent inequality and equality constraints, respectively. X^l_i and X^u_i are, respectively, lower and upper bounds on the design variable X_i. J, K are the total number of inequality and equality constraints, respectively, and N is that of the design variables.

The objective function Z may represent cost, weight or even some behavior responses while the constraints G and H arise from functional and performance requirements and analysis conditions. The bounds on the design variables, X^l and X^u, commonly known as *side constraints*, usually arise from manufacturing conditions and practical considerations.

In a general structural optimization problem, the objective function and the constraints are nonlinear functions of the design variables and their continuity and differentiability are assumed in order to facilitate the solution process (Fig. 1). Several numerical methods based on mathematical programming approach are available to solve the problem of Eqs.(2) (see for example [6]). In this paper, the method of feasible directions [7] will be employed. According to this method, successive iterations are carried out starting from a feasible design point X^0 in a linear direction S that will provide better designs leading to feasible and improved designs at every successive iteration and finally to an optimal design X^*. The iteration process at any design stage can be expressed as:

$$X^{q+1} = X^q + \alpha S^q, \quad \alpha > 0 \tag{3}$$

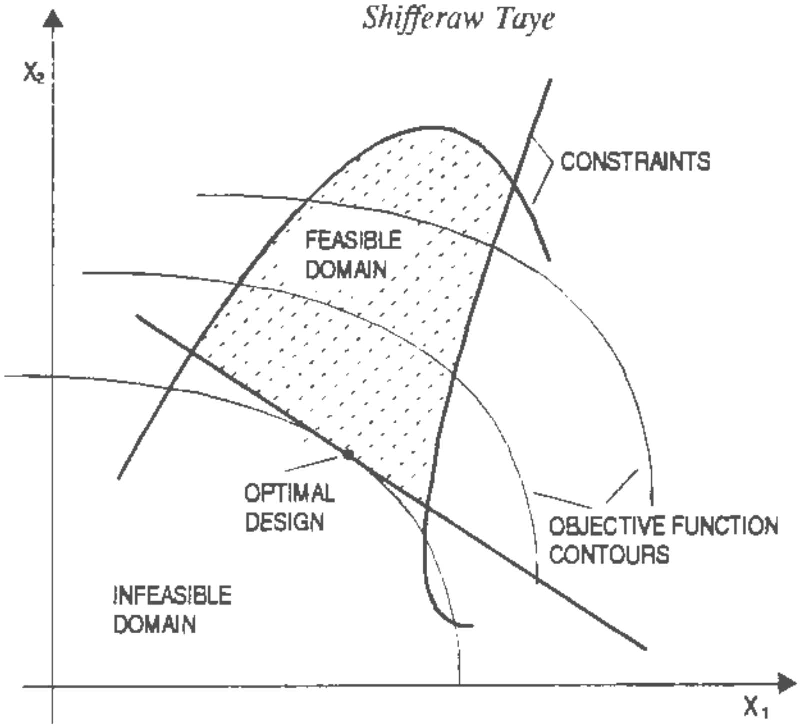

Figure 1 Definition and Elements of Structural Optimization

where X^q is the current design (iteration q) and α is defined as a *step-length* parameter.

During the solution process, attempt is made to stay away from constraint boundaries as much as possible. Such a direction S which does not immediately leave the feasible domain is defined as a *feasible direction.* Under the assumption of smooth constraints Eqs.(2), a feasible direction S^q at some design point X^q is one for which the following relations are satisfied [7]:

$$(S^q)^T \nabla g_j(X^q) < 0, \quad j=1,J \qquad (4a)$$
$$(S^q)^T \nabla h_k(X^q) \leq 0, \quad k=1,K \qquad (4b)$$

where J and K denote the total number of active inequality and equality constraints, respectively. Thus, any vector satisfying Eqs.(4) lies at least partly in the feasible domain.

Besides the feasibility requirement on S, it usability is also an important factor. A move direction S^q is said to be a *usable direction* at the design coordinate X^q if a move along the direction for some step length $\alpha > 0$ improves the objective function Z. This requirement is met if there exists a direction S^q for which:

$$(S^q)^T \nabla Z(X^q) < 0 \qquad (5)$$

A direction satisfying both Eqs.(4) and (5) is said to be a *feasible-usable direction.*

The method of feasible direction at any design stage proceeds in two steps:
i. Finding a feasible-usable direction S^q,
ii. Computing the step length α in order to find a new feasible design X^{q+1} for which the objective Z attains its minimum values along the search direction S^q in (i).

The direction finding problem plays a central role in the method of feasible directions. There exist several directions which satisfy the conditions of equations (4) and (5). If there are no active constraints at the current design point X^q, the direction S^q may effectively be selected as the steepest descent direction, ie, $S^q = -\nabla Z(X^q)$. When, however, there exist active constraints at X^q, the requirements of Eqs.(4) must also be satisfied.

Among the infinitely many possible directions, one which is best in some sense must be identified for efficient computation. Two conditions govern the choice of such a direction: reducing the objective function as fast as possible while staying away from the constraint boundaries as far as possible. The key to the solution lies in finding a compromise between the two phenomena.

Table 1: Loadings on the Twenty-five Bar Transmission Tower

Loading Condition	Node	Load Components		
		x	y	z
1	1	1000	10000	-5000
	2	0	10000	-5000
	3	500	0	0
	6	500	0	0
2	1	0	20000	-500
	2	0	-20000	-500

Table 2. Design Summary of the Twenty-five Bar Transmission Tower

Design Description		Initial Design	Optimal Design	
Variable	Members		This work	Ref. [9]
1	1	10	0.1001	0.1000
2	2-5	10	0.3761	0.3759
3	6-9	10	0.4710	0.4716
4	10-13	10	0.1000	0.1000
5	14-17	10	0.1001	0.1000
6	18-21	10	0.2780	0.2778
7	22-25	10	0.3800	0.3799
VOLUME			911.803	911.800

The direction-finding problem at the current design coordinate X^q may, therefore, be formulated as a linear programming problem in which the feasible direction leading to the maximum decrease of the objective function is a solution of the following maximization problem; that is, one of choosing the vector S^q and the scalar β such that:

$$\text{Maximize } \beta \qquad (6a)$$

Subject to

$$(S^q)^T \nabla Z(X^q) + \beta \leq 0 \qquad (6b)$$

$$(S^q)^T \nabla g_j(X^q) + \theta_j \quad \beta \leq 0, j=1, \qquad (6c)$$

$$-1 \leq S^q_i \leq +1, \; i=1,N \qquad (6d)$$

Equation (6b) is the usability condition while (6c) is the feasibility condition. θ_j are arbitrary positive constants, called *push-off factors*, employed to control the extent to which the feasible direction S is deflected from the feasible constraint surface. If a constraint is strongly nonlinear and problematic in computational sense, it is recommended to choose a large θ_j for it. Conversely, a small θ_j is sufficient for linear or nearly linear constraints. Generally, unless the optimization problem exhibits special characteristics, it is recommended to assume $\theta_j = 1$ for each of the active constraints. The formulation of Eq.(6c) from those of Eqs.(4) is general in a sense that active equality constraints of Eq.(4b) may also be formulated as two opposite inequality constraints and incorporated into Eq.(6c).

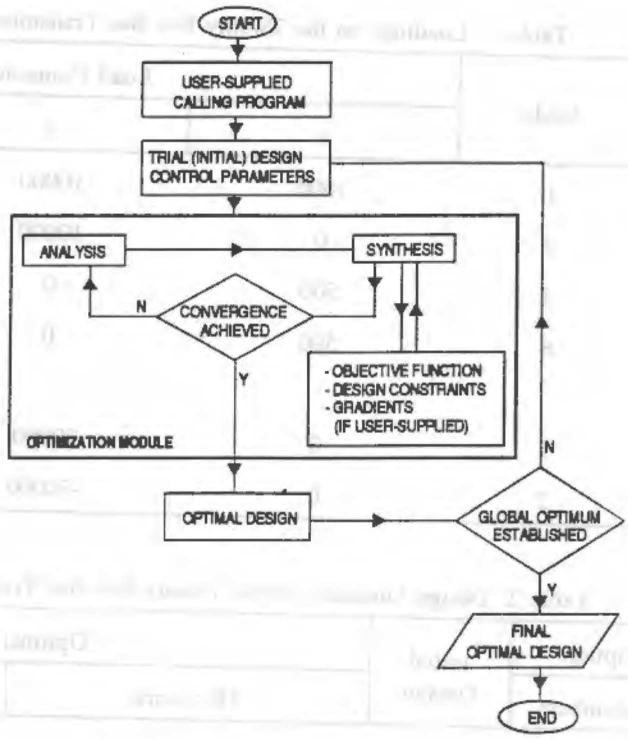

Figure 2 Program Organization of the Analysis-Synthesis Optimization Package

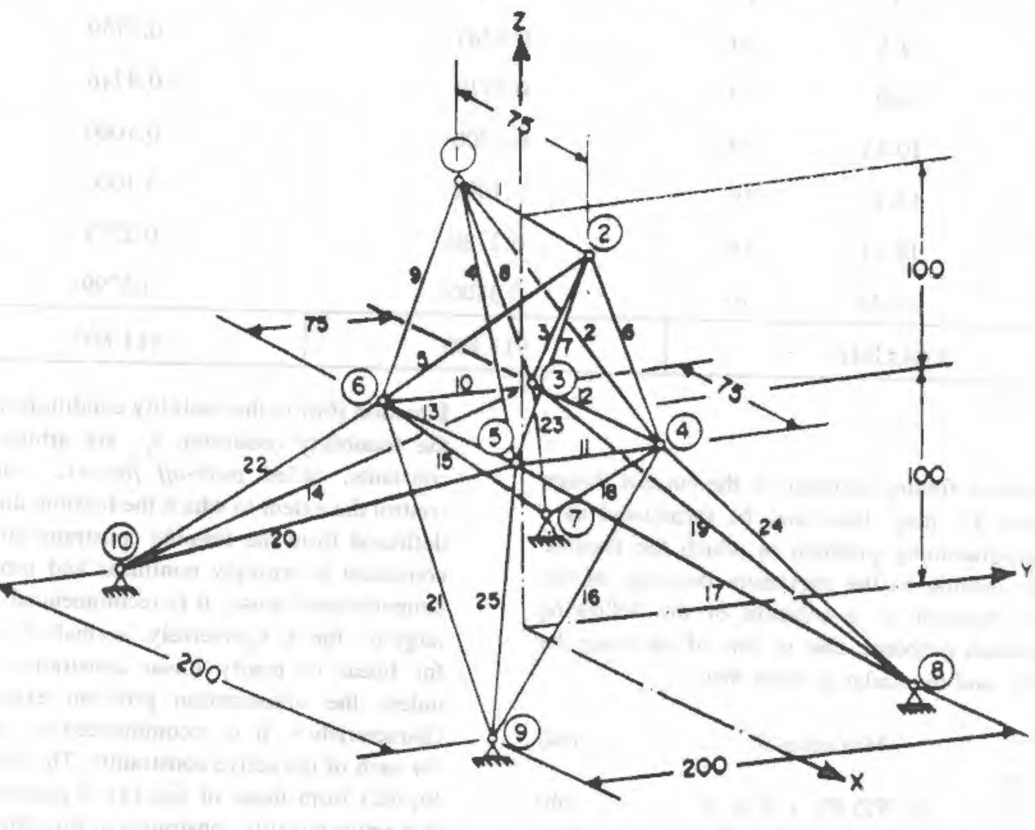

Figure 3 Example: Twenty-five Bar Transmission Tower

Convergence Criteria: Most numerical optimization techniques and procedures are iterative in nature and their solutions are obtained only to certain degrees of accuracy. An essential part of such an optimization process is, therefore, to determine when to stop the search for the optimum. The chosen termination criterion or criteria can have a major influence on the efficiency and reliability of the whole optimization process.

Among the several convergence/termination criteria employed in iterative optimization algorithms, the maximum permitted number of iterations, absolute and relative changes in the objective function, and the Kuhn-Tucker conditions have been incorporated in the development of the optimization package proposed in this paper. Each of these criteria has its own merits in an iterative design process and in assessing the optimality of the final design. Inclusion of the criterion of the maximum permitted number of iterations ensures that the iteration process will not continue indefinitely in cases of slow progress of convergence which may result from numerical difficulties or algorithmic errors.

The absolute change ϵ_a in the values of an objective function in successive iterations indicates convergence if for a specified small tolerance $\epsilon_{a,max}$ the following relation is satisfied:

$$\epsilon_a = \left| Z(X^q) - Z(X^{q-1}) \right| \leq \epsilon_{a,max} \qquad (7a)$$

In the criterion of relative changes ϵ_r in the objective function between successive iterations, convergence is indicated if for a specified tolerance $\epsilon_{r,max}$, the following fractional relation is satisfied:

$$\epsilon_r = \frac{\left| Z(X^q) - Z(X^{q-1}) \right|}{\max\left(\left| Z(X^q) \right| , \epsilon\right)} \leq \epsilon_{r,max} \qquad (7b)$$

ϵ in the denominator of Eq.(7b) is a very small positive number, say 10^{-10}, which has been employed to ensure that division by zero is avoided in the event $Z(X^q)$ approaches zero.

The criterion of Eq.(7a) is suited for problems with large objective function values while that of Eq.(7b) is for cases with known small objective-function values. Their simultaneous implementation ensures that both forms of objective functions will be taken care of.

OPTIMIZATION ALGORITHM AND METHODOLOGY

The procedure of applying the proposed optimization package for the design optimization of framed structures involves the following steps:

1. An initial trial design is provided via a user-supplied calling program.

2. The synthesis algorithm calls other user-snpplied routines to evaluate:
   i) Objective function,
   ii) Constraints, based on the results of integrated analysis routines,
   iii) All gradients required in the direction-finding problem and in assessing termination criteria unless they are evaluated by the built-in finite difference method.

3. The optimization process is terminated when any of the termination criteria is met for three successive iterations for a design.

4. The user provides several new starting designs and proceeds from Step (2) for each such attempt to establish the practical optimum.

5. The optimization process is terminated when the optimum design is established based on observations of the convergence behaviour.

The computational logic of the developed package is shown in Fig. 2.

NUMERICAL EXAMPLE

The proposed package has been tested on several types of framed structures. Its capabilities and potentials will be demonstrated on the twenty-five bar transmission tower shown in Fig. 3. The test structure has been used by several researchers (see, for example, [8]) and its optimal values have been well established. The following data have been assumed:

Loading: The test structure is subjected to two loading conditions as shown in Table 1.

Design variables: Cross-sectional area of the members linked into seven groups (see Table 2).

Objective function *Z:* Volume of construction material.

Constraints: Member axial stresses (a total of 100 constraints for the two loading conditions).

Side constraints $X^l = \{0.1\}$ for all members; no upper bound.

Allowable axial stress: $\sigma^u = -\sigma^l = 40000$ for all members

Termination Criteria:

Maximum number of iterations = 40

$\epsilon_{z,max} = 0.001 \ |(Z(X^0)|$, $\epsilon_{r,max} = 0.001$

Results are given in Table 2. Other starting designs have also converged to similar values. Consequently, it can be observed that effective automation has been achieved.

CONCLUSION

An optimization package that incorporates both the synthesis module based on methods of feasible directions and analysis capabilities based on the stiffness method has been developed for the design optimization of framed structures. The package can be used effectively and efficiently with minimal effort by the engineer to propose optimal structural systems for any desired quantifiable measure of optimality. The package is capable of processing large-scale structural systems and its capacity is limited only by the user's computer memory and by that of the compiler employed. It is believed that the package minimizes the large computational effort which otherwise might have been required to set up analysis equations manually for use in the behaviour constraints. This carefully coded package also eliminates any possible numerical and computational mistakes that creep into manual formulation of problems of this size.

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