THE USE OF SPRINGS IN STATIC ANALYSIS OF STRUCTURES TO ACCOUNT FOR SHORT-AND LONG TERM SUBGRADE DEFORMATIONS

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ABSTRACT

The theory of elasticity of continua is employed to show the background of spring formulas that are introduced at the bases of structures to account for immediate static deformation of soils. This is followed by providing sets of such valuable formulas for use in practical modeling of structures founded on deformable soils. Additional spring formulas that account for primary and secondary consolidation settlement are derived. It is shown that these vertical springs can be joined in series to account for all types of soil deformation - immediate, primary consolidation, and secondary consolidation or creep. The application of the springs is illustrated using a simple building frame subjected to gravity loads only. The internal forces in the structural members with and without flexible base elements showed notable differences. The significance of the introduction of flexible-base elements in taller and more rigid structures subjected to lateral loads can be expected to be even larger. The influence of the consolidation springs could particularly be more significant if different foundation elements of a structure rest on compressible layers of different properties and varying thickness. A companion paper deals with a parametric study on the influence of elastic base springs, in which the height and type of the structural system, the soil type, and the embedment depth of the foundation are varied.

INTRODUCTION

The analysis of structures is routinely conducted by treating their bases as firmly fixed so that no displacements and rotations are allowed. This conventional approach was justifiable in the distant past, when studies on soil-structure interaction (SSI) did not make significant advances. However, SSI problems have been topics of research for the past many decades since the pioneering works of the 1930's in the field [1]. Presently, a wealth of information has accumulated that can be used for purposes of routine design and further research [3,4,5,6].

The early works in this area focused on circular foundations resting on the surface of an elastic half space subjected to different modes of loading. Later on, subsequent studies considered effects of foundation shapes, foundation embedment, soil layering, dynamic loading, and even inelastic response. Presently, there is a wealth of information regarding the behavior of both statically and dynamically loaded foundations [3,4,5,6]. Results of such studies are commonly presented in form of relationships between the applied load and the resulting surface deformation. In other words, for static cases, the results are mathematical expressions for the coefficients of springs that could be introduced at the bases to account for the soil deformability. Therefore, the continued use of fixed-base models cannot be easily justified on the mere ground of unavailability of information.

This paper aims at:

- Providing a highlight of the background theory of spring formulas for immediate (short term) soil deformations to be used at bases of structures in static analyses
- Compiling important spring formulas found scattered in sources that are difficult to access by practicing engineers for the most important cases of soil and foundation conditions.
- Deriving relations for additional static spring coefficients that account for long-term static soil deformations including primary and secondary consolidation settlements.
- Illustrating the use of the springs in statically loaded structures so that their potential significance is understood.

Before directly embarking on the derivation and compilation of the spring formulas, the potential influence of flexible supports on internal stresses and strains of structures is first demonstrated in the following section by considering very simple cases of transversally and axially loaded beams.

FLEXIBLE-BASE PARAMETERS

The elements to be employed at the base of structural models to account for SSI effects are linear springs in the case of static loading and a combination of springs and viscous dashpots in the case of dynamic loading. The parameters of these elements are derived from comparison of the governing equations of the actual soil-foundation system with those of the corresponding mechanical models intended to replace the former. This work focuses on the use of linear springs at bases of structures.

The theoretical considerations underlying the derivation of spring coefficients for the case of immediate static soil deformation are provided in the following subsections. This will be accompanied by presentations of formulas for selected common cases with the intention of making them accessible for practical use. Additional spring coefficients are also derived that account for long-term soil deformations, namely one-dimensional primary consolidation and creep settlements.

1. Springs for Immediate Soil Deformability

(a) Circular Foundations on the Surface of an Elastic Half Space

The ideally flexible circular foundation of radius R shown in Fig. 1(a) resting on the surface of an elastic half space and subjected to a uniformly distributed vertical static load of q is considered. The spring coefficient $(K_v)_e$ is sought for use in the model rigid footing shown in Fig. 1(b) with the intention of replacing the half space.

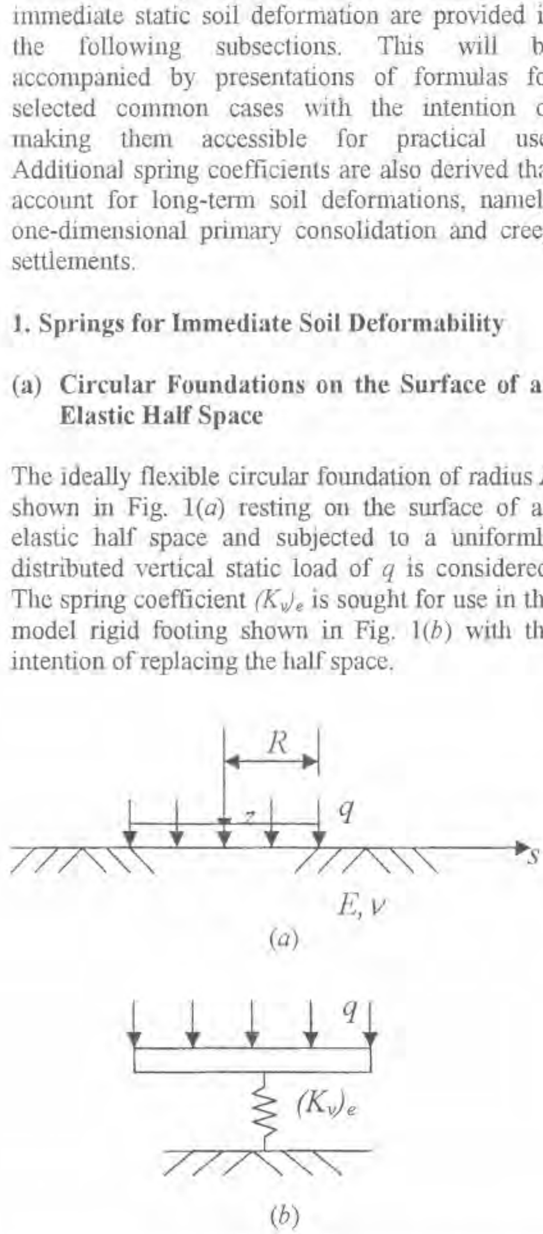


Figure 1 (a) An ideally flexible circular footing on the surface of an elastic half space;
(b) A model of the rigid circular footing supported by a linear spring

The stress and strain components at points below the center of the loaded circular region can be easily determined in closed forms by integrating Boussinesq's solution for a vertical point load on the surface of an elastic half space, which is based on the theory of elasticity. Ahlvin and Ullery, as cited in [2], solved this problem for all stress components at an arbitrary point in the half space using cylindrical coordinates. They provided formulas and tabular values of their accompanying influence factors [2].

By integrating the vertical strains over the depth, they also derived the following expression for the vertical elastic settlement of the surface of the half space[1]:

$$(S_e)_{z=0,flex} = qR\frac{1-\nu^2}{E}I_c \tag{1}$$

where E and ν are the elasticity modulus and Poisson ratio, respectively, of the half space. The values of the dimensionless influence factor I_c depend on the normalized coordinates s/R and z/R, where s and z are the radial and vertical coordinates of the point under consideration. Its value for the average settlement of the flexible foundation is 1.7. Schleicher as cited in [2], showed that the uniform vertical settlement of the ideally rigid foundation is about 7% less than the average settlement of the ideally flexible circular foundation [2]. Then, the uniform settlement of the rigid circular footing becomes

$$(S_e)_{rigid} = 1.58qR\frac{1-\nu^2}{E} \tag{2}$$

Equation (2) is now equated to the deflection of the spring supporting the model foundation shown in Fig. 1(b) under the action of the same load. After simplifying, rearranging, and introducing the shear modulus $G = E/2(1+\nu)$ of the elastic half space, the following expression for the spring coefficient is obtained:

$$(K_v)_e = \frac{4GR}{1-\nu} \tag{3}$$

This is the coefficient of vertical elastic spring one finds in the literature for a circular rigid footing resting on the surface of an elastic half space subjected to a central vertical loading for use in the model foundation of Fig. 1(b)[2].

[1] The presentation of the details of elasticity theory relationships underlying this equation is omitted.

[2] While Eq. (3) for circular foundations and Eqs. (9) and (10) in the following section for rectangular foundations are derived by the author himself from considerations of elasticity theory without inertia forces, the same relations found in the literature originate from dynamic considerations that involve inertia forces in the stress equations.

Proceeding in a similar manner of integrating appropriate strains, one can also determine the spring coefficients for the remaining degrees of freedom of the rigid foundation on the surface of an elastic half space. These are summarized in Table 1 for all degrees of freedom of a rigid circular foundation.

Table 1: Static spring coefficients for a rigid circular foundation resting on the surface of an elastic half space [3,4]

Vertical	Horizontal	Rocking	Torsion
$K_v = \frac{4GR}{1-\nu}$	$K_h = \frac{8GR}{2-\nu}$	$K_r = \frac{8GR^3}{3(1-\nu)}$	$K_t = \frac{16GR^3}{3}$

(b) Rigid Rectangular Foundations on the Surface of an Elastic Half Space

As in the case of the circular foundation, the problem of an ideally flexible rectangular foundation was solved by integrating Boussinesq's solution for a point load.

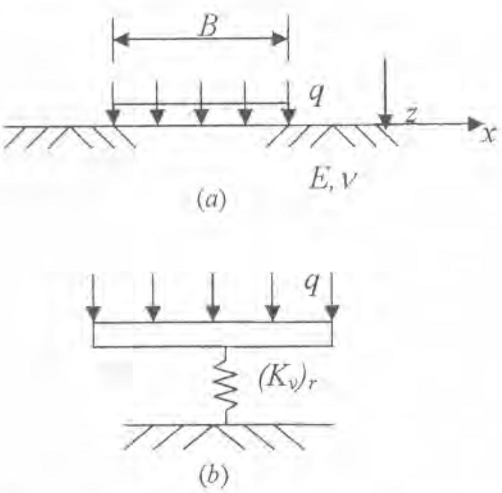


Figure 2 (a) An ideally flexible rectangular footing on the surface of an elastic half space;
(b) A model of the rectangular footing supported by a linear spring

As cited by Das [2], the problem of vertical settlement of points below the corners of a loaded rectangular region of plan proportion $B \times L$ as shown in Fig. 2 was solved by Harr (1966), who presented his solution in a closed form as

$$(S_e)_{flex,corner} = qB\frac{1-\nu^2}{2E}I_r \tag{4}$$

The dimensionless influence factor I_r is given by

$$I_r = \frac{1}{\pi}\left[\ln\left(\frac{\sqrt{1+m^2}+m}{\sqrt{1+m^2}-m}\right) + m\ln\left(\frac{\sqrt{1+m^2}+1}{\sqrt{1+m^2}-1}\right)\right], \tag{5}$$

in which, $m = L/B$, B is the shorter and L is the longer side of the loaded rectangular region [2].

Equations (4) and (5) can be systematically used to calculate settlements of any point on the surface. The settlement of the center of the rectangular region in particular can be determined by noting that this point can be considered as the corner of the four equal rectangles of side lengths *B/2* and *L/2* making up the bigger rectangle. This yields for the center settlement

$$(S_e)_{\text{flex,center}} = 4\left[q\frac{B}{2}\frac{1-\nu^2}{2E}I_r\right] = 2(S_e)_{\text{flex,corner}} \tag{6}$$

Noting that the average settlement of the loaded region is 85% of the settlement of the center, and that the uniform settlement of the rigid foundation is 7% less than the average settlement of the flexible foundation as in the case of the circular foundation, one obtains for the settlement of the rigid rectangular foundation

$$(S_e)_{rigid} = 0.79qB\frac{1-\nu^2}{E}I_r \tag{7}$$

This is now equated to the vertical displacement of the model foundation of Fig. 4(b) supported by a vertical spring under the action of the same loading. With the introduction of the shear modulus, this results in

$$(K_v)_{e,rect} = \frac{2.53GL}{(1-\nu)I_r} \tag{8}$$

For the special case of a square footing of side length B and $L/B=1$, Eq. (5) yields I_r=1.123 so that

$$(K_v)_{e,square} = \frac{2.25GB}{(1-\nu)} \tag{9}$$

It is worth recalling here that, in the times preceding the availability of rigorous solutions for rectangular footings, it was a common practice to make use of the solutions of circular footings for footings of other shapes, where an equivalent radius is used in Eq. (4). It is of interest to compare such an approximate expression with the rigorous solution.

The equivalent radius for the vertical displacement of the square footing is obtained by equating areas to get $R = B/\sqrt{\pi}$. Inserting this in Eq. (4), one obtains

$$\left(K_v\right)_{e,square} \approx \frac{2.26GB}{(1-\nu)} \qquad (10)$$

This is practically the same as the rigorous solution of Eq. (9). The same cannot, however, be concluded for rectangles of other side proportions in general, because the difference can become significant with increasing side ratios.

Presumably with the intention of avoiding the inconvenience in using the lengthy logarithmic expressions of Eq. (6) for calculating I_r, Pais and Kausel [3] recently proposed the following relatively simpler, but approximate, expression for the vertical spring coefficient of rigid rectangular foundations:

$$\left(K_v\right)_{e,rect} \approx \frac{GB}{2(1-\nu)}\left[3.1(L/B)^{3/4}+1.6\right] \qquad (11)$$

For a square footing, this reduces to

$$\left(K_v\right)_{e,rect} \approx \frac{2.35GB}{(1-\nu)} \qquad (12)$$

In contrast to Eq. (10), this is in discrepancy with the rigorous solution of Eq. (9) by about 4.4%. Larger discrepancies can be expected for rectangles of larger side ratios. However, errors of such an order may not be significant as far as the form of Eq. (11) is found more convenient in practical use.

Expressions for the spring coefficients of the remaining degrees of freedom of rectangular foundations that are obtained in a similar manner are provided in Table 2.

(c) Springs for More General Cases of Elastic Soil Deformation

A much more general problem of practical interest is the case of a foundation of arbitrary shape placed below the surface of a layered soil formation. Following the development of the relatively simple spring coefficients like those provided in the preceding sections, numerous studies have been conducted that considered various combinations of influencing factors like embedment depth, soil layering, and foundation shape. Valuable results are obtained, but are found scattered in various technical papers of journals and conference proceedings, which are difficult to access. Some efforts have been made to compile them [3,4,5,6].

Spring coefficients for circular foundations embedded a depth H in a flexible soil layer of thickness D that overlies a rigid half space are presented in Table 3. The elastic parameters with the subscript 1 refer to the upper elastic layer. Table 4 provides static spring coefficients for rectangular footings embedded a depth H in an elastic half space. It is believed that these formulas could be of importance in solving practical problems.

Additional formulas for spring coefficients of foundations of arbitrary shape are provided recently by Gazetas [5,6]. However, they are much more approximate than those provided here for circular and rectangular shapes.

Table 2: Static spring coefficients for a rigid rectangular foundation on the surface of an elastic half space [3]

Vertical spring		$(K_v)_e = \frac{GB}{2(1-\nu)}\left[3.1(L/B)^{0.75}+1.6\right]$
Torsional spring		$(K_t)_e = \frac{GB^3}{8}\left[4.25(L/B)^{2.45}+4.06\right]$
Horizontal spring	Short direction (x)	$(K_x)_e = \frac{GB}{2(2-\nu)}\left[6.8(L/B)^{0.65}+2.4\right]$
	Long direction (y)	$(K_y)_e = (K_x)_e + \frac{0.8GB}{2(2-\nu)}\left(\frac{L}{B}-1\right)$
Rocking spring	Around x-axis	$(K_{rx})_e = \frac{GB^3}{8(1-\nu)}\left[3.2(L/B)^{3/4}+0.8\right]$
	Around y-axis	$(K_{ry})_e = \frac{GB^3}{8(1-\nu)}\left[3.73(L/B)^{2.4}+0.27\right]$

Table 3: Static spring coefficients for a rigid circular foundation embedded in an elastic stratum over a rigid half space [4]

Direction	Spring coefficient for H/R<2 and H/D≤0.5
Vertical	$(K_v)_e = \frac{4G_1R}{(1-\nu_1)}[1+1.28(R/D)]\left(1+\frac{H}{2R}\right)\left[1.85-0.28\frac{H}{R}\frac{H}{D}\frac{1}{(1-H/D)}\right]$
Horizontal	$(K_h)_e = \frac{8G_1R}{2-\nu_1}\left(1+\frac{R}{2D}\right)\left(1+\frac{2H}{3R}\right)\left(1+\frac{5H}{4D}\right)$
Rocking	$(K_r)_e = \frac{8G_1R^3}{3(1-\nu_1)}\left(1+\frac{R}{6D}\right)\left(1+\frac{2H}{R}\right)\left(1+\frac{0.7H}{D}\right)$
Torsional	$(K_t)_e = \frac{16G_1R^3}{3}\left(1+\frac{2.67H}{R}\right)$
Coupled horizontal-rocking	$(K_{kh})_e = 0.4H(K_h)_{emb}$

Table 4: Static spring coefficients for a rigid rectangular foundation embedded in an elastic half space [3]

Vertical	$(K_v)_{emb} = (K_v)_{surf}\left[1+\left(0.25+\frac{0.25}{L/B}\right)\left(\frac{H}{B}\right)^{0.8}\right]$
Horizontal	$(K_h)_{emb} = (K_h)_{surf}\left[1+\left(0.33+\frac{1.34}{1+L/B}\right)\left(\frac{H}{B}\right)^{0.8}\right]$
Torsion	$(K_t)_{emb} = (K_t)_{surf}\left[1+\left(1.3+\frac{1.32}{L/B}\right)\left(\frac{H}{B}\right)^{0.9}\right]$
Rocking	$(K_{rx})_{emb} = (K_{rx})_{surf}\left[1+\frac{H}{B}+\left(\frac{1.6}{0.35+(L/B)^4}\right)\left(\frac{H}{B}\right)^2\right]$
	$(K_{ry})_{emb} = (K_{ry})_{surf}\left[1+\frac{H}{B}+\left(\frac{1.6}{0.35+L/B}\right)\left(\frac{H}{B}\right)^2\right]$
Coupled Horizontal-rocking	$(K_{rx,h})_{emb} = \frac{H}{3B}(K_{rx,h})_{surf}$; $(K_{ry,h})_{emb} = \frac{H}{3B}(K_{ry,h})_{surf}$

2. Vertical Springs for Long-term Soil Deformability

(a) A Vertical Spring to Account for Primary Consolidation Settlement

The consolidation settlement of a compressible stratum is commonly estimated on the basis of one-dimensional consolidation test results, which among others yield data on void ratio and effective vertical stresses. One form of presenting these data is in form of plots of void ratio versus effective vertical stress on a natural or semi-logarithmic scale.

On the basis of the plot on a natural scale, the primary consolidation settlement, s_c, of a compressible stratum of thickness H under an average superimposed vertical stress of Δp reaching the layer can be easily derived and is given by

$$s_c = m_v H\Delta p = \frac{H\Delta p}{E_c} \tag{13}$$

where, $m_v = a_v/(1+e_0)$ is the coefficient of volume compressibility; a_v is the coefficient of compressibility; E_c is the modulus of compressibility; and e_0 is the in-situ void ratio of the compressible stratum.

Equating this settlement with the deformation of the spring of the model foundation subjected to the same loading, one can readily determine the following expression for the spring coefficient:

$$K_p = \frac{A_f E_c}{\alpha H(\Delta p/q)} \tag{14}$$

where, A_f is the plan area of the foundation; $\Delta p/q$ is the fraction of the average superimposed vertical effective pressure reaching the compressible stratum; q is the contact pressure; and α is the degree of consolidation in decimals.

Alternatively, the primary consolidation settlement can be estimated on the basis of the semi-logarthmic plot to obtain different expressions for normally consolidated and overconsolidated soils, the details of which are omitted. The spring coefficient, K_p, is then determined from

$$K_p = \frac{A_f q}{s_c} \qquad (15)$$

Equation (14) is easier to use in that it only demands prior estimation of the foundation size, similar to the immediate-deformation case of the previous section, and the fraction of the effective pressure reaching the compressible stratum.

The spring so established can be attached in series with the spring for the immediate soil settlement in the vertical direction. This will be discussed in a later section. The spring coefficients for the immediate/elastic deformation in the remaining degrees of freedom remain unaltered because of the inherent assumption of one-dimensional volume change in one-dimensional consolidation theory.

(b) A Vertical Spring to Account for Secondary Consolidation Settlement

Secondary consolidation settlement, also known as creep, is a result of further rearrangement of the soil grains and compression of the individual grains under sustained loading after the end of the primary consolidation. It has importance in cohesive soils with some organic content.

Creep (secondary) settlement, s_s, is estimated from a relation derived from the plot of dial reading versus log (t), which is routinely prepared as part of consolidation test reports. This relation is given by

$$s_s = c_\alpha H \log\left(t/t_p\right) \qquad (16)$$

In Eq. (16), c_α is the coefficient of secondary consolidation determined as the slope of the bottom straight portion of the plot; t is the time at which the secondary settlement or the corresponding structural response is considered; and t_p is the time at the end of primary consolidation.

It is reasonable to base the determination of the coefficient of secondary settlement on the plot for the load increment corresponding to the total of the overburden plus the anticipated superimposed pressure.

The spring coefficient obtained in an analogous manner to the previous cases becomes then

$$K_s = \frac{P}{s_s} = \frac{A_f q}{c_\alpha H \log\left(t/t_p\right)} \qquad (17)$$

This spring is attached in series with the springs for the primary consolidation settlement and for the immediate settlement in the vertical direction. The combination of the springs is presented in the following section.

3. Equivalent Spring Coefficient for the Vertical Direction

The equivalent spring coefficient in static analysis for the vertical degree of freedom is determined readily from linear superposition of the component settlements that yields

$$\frac{1}{K_{v,eq}} = \frac{1}{K_e} + \frac{1}{K_c} + \frac{1}{K_s} \qquad (18)$$

which results in

$$K_{v,eq} = \frac{K_e K_c K_s}{K_c K_s + K_e K_s + K_e K_c} \qquad (19)$$

In cases where the secondary settlement is found insignificant, the two springs for the immediate and primary consolidation settlements alone may be used.

It is worth reminding that with the use of such springs under each rigid foundation element, the settlement is directly output by most commercial software. If that is not the case, the component settlements of each foundation element are easily determined from the vertical reaction forces, P, and the spring constants without the need to resort to the settlement equations. Thus,

$$s_e = P/K_e; \quad s_c = P/K_c; \quad s_s = P/K_s; \qquad (20)$$

Similarly, the tilting, δ, and the horizontal rotation, β, of each foundation are determined from

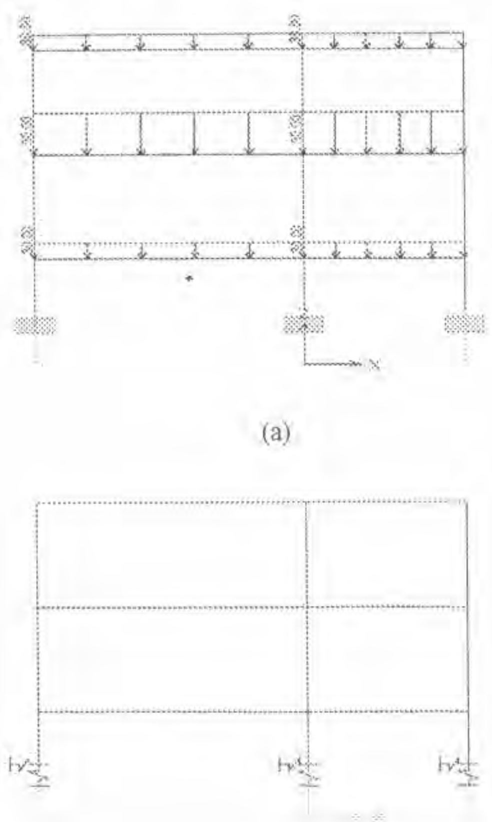
$$\delta = M_r/K_r; \quad \beta = M_t/K_t \qquad (21)$$

where M_r and M_t are the rocking and torsional moments, respectively.

It is worth pointing out at this junction that all spring coefficients both in the present and the previous sections are expressed in terms of the foundation size, which is not yet known at the analysis stage. Its prior estimation is necessary in order to quantify the spring coefficients. This, however, is not a difficult task if commercial software is used for the structural analysis. The foundation reaction forces of the fixed-base model of the structure analyzed in advance can be used for this purpose. If need be, the actual sizes can be adjusted later on based on the reaction forces of the flexible-base model of the structure.

ILLUSTRATION

In order to illustrate the use of the spring formulas presented in the foregoing sections, an example of a simple two-dimensional, two-story and two-bay building frame is considered as shown in Fig. 5(*a*). Both stories are 4 meters high. The left bay is 7.5 meters and the right bay 4.5 meters wide. The footings of the building are all to be embedded 2 meters into a normally consolidated clay layer, which overlies a non-horizontal rigid formation sloping at 1 vertical to 6 horizontal. The ground water table is at 2 meters below the ground surface. The characteristics of the compressible clay layer and an idealization of the stratification are as shown in Fig. 6.



(a)

(b)

Figure 5 (a) The fixed-base frame with the loading; (b) The flexible-base frame

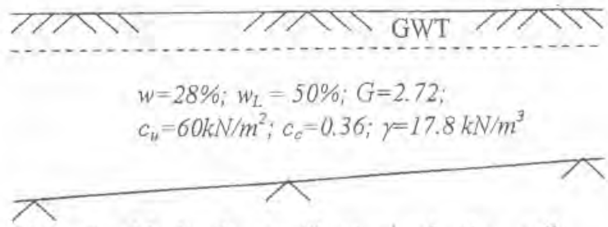


Figure 6 Idealized stratification of the soil formation

The conventional model of the frame with fixed bases was analyzed using SAP 2000. The resulting axial force, shear force, and bending-moment distributions are given in Fig. 7.

The bearing capacity of the footings is determined analytically as 160 kPa for the undrained condition providing for a safety factor of 3. The reaction forces and moments at the bases of the fixed-base model are employed to estimate the footing proportions, which are needed in the determination of the foundation spring coefficients for the flexible-base model. Since the moments are small compared to the vertical reaction forces, the foundations are proportioned as square footings. Accordingly, the left footing (F1) becomes 2m by 2m, the middle footing (F2) 2.45m by 2.45m, and the right footing (F3) 1.35m by 1.35m.

In order to make use of the formulas provided in Table 3 for circular foundations on an elastic layer underlain by a rigid formation, equivalent radii are determined. Empirical relations are employed to estimate the elastic parameters of the upper flexible soil layer [2]. This resulted in E=120 MPa and ν=0.313, and the shear modulus is calculated from these parameters as G=45.7 MPa.

A summary of the calculated spring constants for the three degrees of freedom of each footing is provided in Table E1.

Table E1: Calculated spring constants for the three footings for elastic deformation

Footing	Vertical Spring (MN/m)	Horizontal Spring (MN/m)	Rocking Spring (MNm)
F1	943	542	900
F2	1296	687	1719
F3	728	448	799

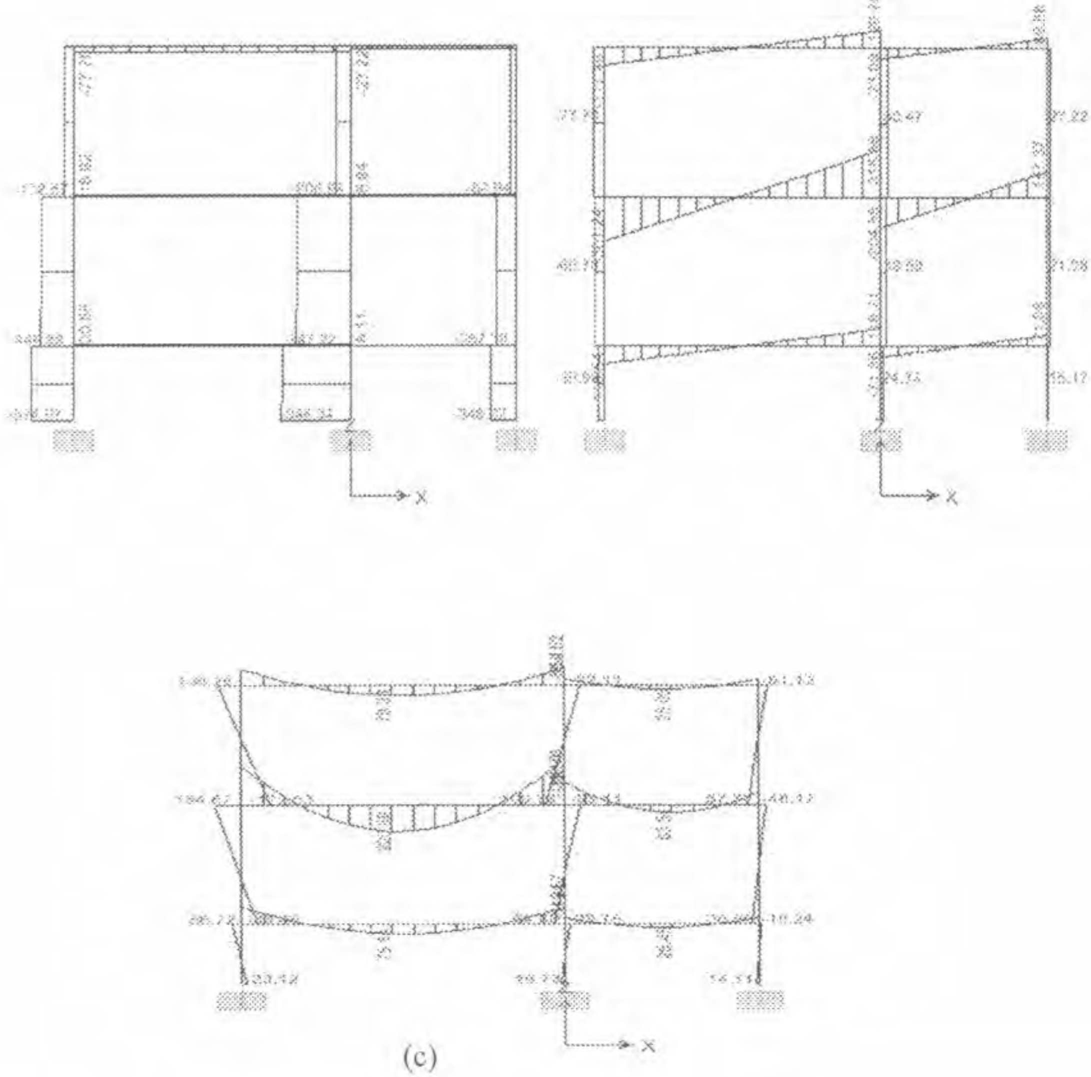


Figure 7 Internal forces and moments in the fixed-base model: (a) axial force; (b) shear force; (c) bending moment

With these springs introduced at the bases, the new flexible-base model is analyzed. The differences between the internal forces and moments of the frame so calculated and those of the fixed-base model presented in Fig. 7 are of little practical significance so that they are not presented here. The displacements and rotations of the footings are summarized in Table E2. These quantities are also quite small. This is to be expected because the influence of elastic soil-structure interaction on such flexible, low-rise, and light structures is normally insignificant.

Table E2: Displacements and rotations of the footings - elastic deformation case

Footing	Vertical displacement (mm)	Horizontal Displacement (mm)	Rotation (radians)
F1	-0.610	-0.068	-2.1×10^{-5}
F2	-0.729	0.034	9.95×10^{-6}
F3	-0.477	0.031	1.44×10^{-5}

Next, the case of end of primary consolidation is considered. The coefficient of the additional vertical spring is computed using Eq. (15) with α taken as unity for 100% consolidation. The average superimposed vertical stress is calculated by taking into account the difference in the thickness of the compressible layer under each footing. The modulus of compressibility is easily calculated from the given information. Finally, the spring coefficients for end of primary consolidation are readily calculated and presented in Table E3. Note that these springs are significantly softer than those for the immediate deformation case.

Table E3: Calculated spring constants for the three footings for end of primary consolidation

Footing	Spring for primary consolidation (kN/m)	Effective spring constant (kN/m)
F1	2823	2815
F2	5539	5515
F3	3166	3152

The effective spring constants in the vertical direction for the three footings are also calculated in accordance with Eq. (22) and given in the same table. It is important to note that the effective springs became very soft due to the consideration of the consolidation settlement of the footings (compare with values in Table E1).

The flexible-base frame is analyzed once again with the introduction of the modified vertical springs keeping the rest of the springs in the other degrees of freedom unaltered. The results of the analysis are given in Fig. 8 together with the deformed shape of the frame. The displacements and rotations of the footings are provided in Table E4.

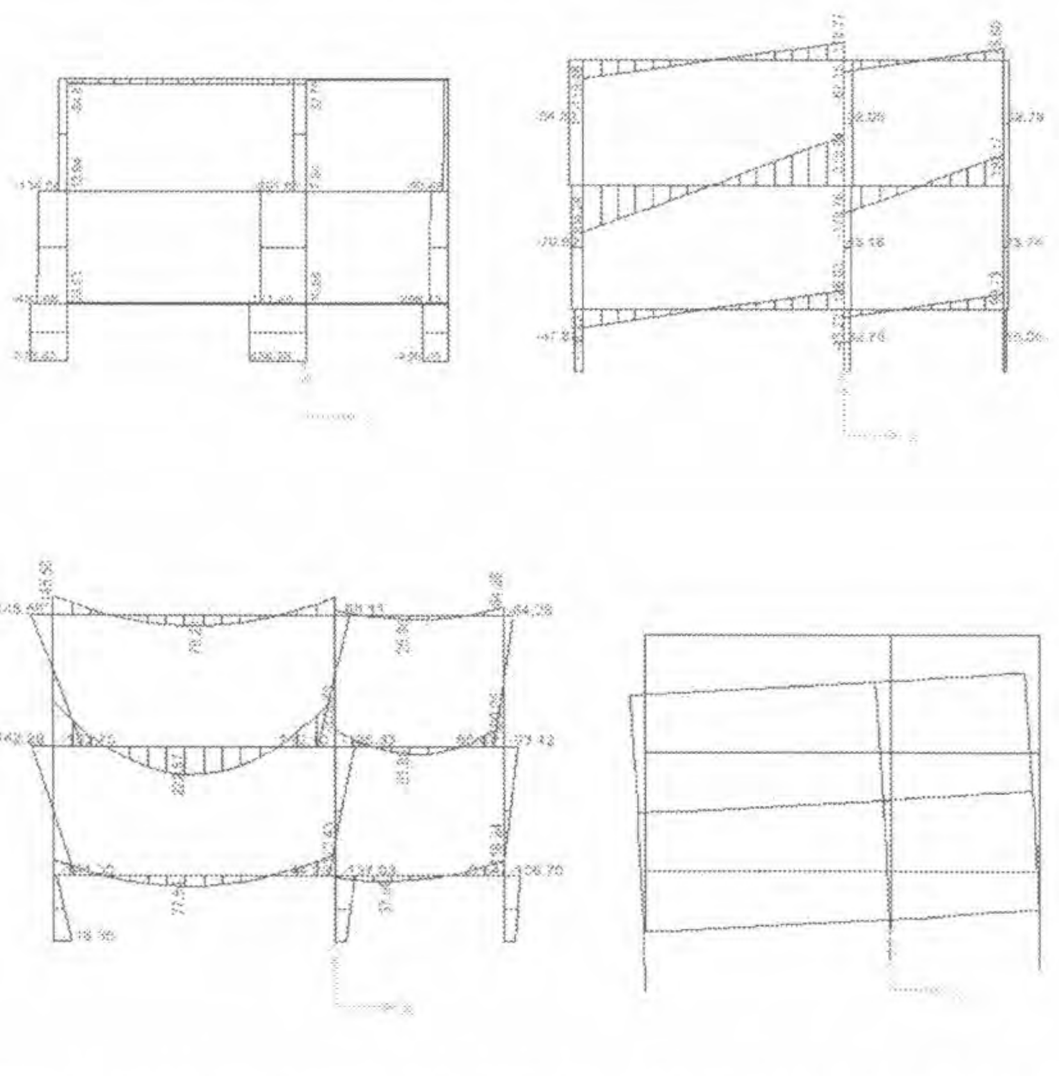


Figure 8 Internal forces and moments in the flexible-base model with springs for consolidation settlement: (*a*) axial force; (*b*) shear force; (*c*) bending moment; (d) deformed shape

Table E4: Displacements and rotations of the footings - consolidation settlement case

Footing	Vertical displacement (mm)	Horizontal Displacement (mm)	Rotation (radians)
F1	-203.78	-0.088	-1.293×10^{-4}
F2	-161.07	0.048	-4.16×10^{-5}
F3	-128.62	0.034	-9.83×10^{-5}

The following observations can be made from these results in comparison with those of the fixed-base model given in Fig. 5.

i. Generally, there is a modest difference in the magnitudes of the axial forces of the columns. The difference in axial forces is largest at the base columns.

ii. Whereas the vertical reaction forces at the left and middle footings decreased notably, the reaction force increased at the right footing, where the consolidation settlement is least.

iii. The shear forces in the columns increase consistently with the introduction of the consolidation springs.

iv. As a general trend, an increase in bending moments is observed with the introduction of the consolidation spring. This is most noted in the base columns and particularly at their junctions with the footings, where the increase is a minimum of about 275%. This has a practical significance in the proportioning of both the base columns and the footings.

v. Notable increases in the bending moments at the outer supports of the ground beams are also observed. Furthermore, the locations of the maximum span moments shifted significantly in these beams.

vi. The settlements of the footings are very significant. The differential settlements are in the order of 1/176 and 1/136 between F1 and F2 and between F2 and F3, respectively, demanding revision of the proportions of the footings.

Generally, the influence of the consolidation springs appears to be much more significant than that of the elastic-deformation springs, at least in this example.

It is to be noted that the structure considered in this example is a simple two-story, relatively flexible frame subjected to gravity loading only. It can be expected that the influence of the flexible elements at the bases would be much more significant in taller and rigid structures involving more rigid structural elements like shear walls and subjected to lateral loads in addition to gravity loads. A parametric study looking into this matter is presented in a companion paper, which considers buildings of different height and structural systems subjected to pseudo-static lateral earthquake loads.

CONCLUSIONS

The theoretical background of spring formulas that account for the immediate deformation of soils is presented and sets of these formulas are provided for use in practical modeling of structures founded on deformable soils. Additional spring formulas that account for primary and secondary consolidation are derived. It is shown that these latter springs can be joined in series with those for immediate soil deformation. The application of the springs is illustrated using a simple building frame subjected to gravity loads only. The internal forces in the structural members with and without flexible base elements show notable differences. Particularly, the absolute and differential settlements are found to be more significant suggesting that such flexible base elements could even be more important in taller and more rigid structures subjected to lateral loads. Especially, the influence of the consolidation springs is worth noting. The use of these springs in models of structures is a straightforward operation and is commonly supported by features of available commercial software. Their use in practical modeling of structures is recommended.

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