SAFETY IN STRUCTURAL DESIGN

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INTRODUCTION

The main purpose of design and analysis of structures is to evaluate safety and economy. It would be of interest to relate the problem of safety in Civil Engineering Structures with those in other fields, such as in transportation vehicles, i.e., automobiles, trains, and aircrafts. The problem of reliability has been thoroughly studied in these fields especially in aeronautical engineering based on statistical approach by comparing the probabilities of failure with the actual rate of accidents.

This leads to the question of the conditions to be satisfied in design and construction to obtain structures that are safe and economic. The question cannot simply be attended to in a universal manner since there are several factors that must be considered such as social organization, technical knowledge, availability of materials and workmanship, inspection and quality control, and research facilities.

The accidents due to failure of structures cannot be avoided by increasing the factors of safety only. To reduce accidents, design rules have to be defined and have to be followed by competent designers. It is observed, for example, that the number of failures due to gross errors are found to be lower in technically developed societies, thus lowering the percentage of the probability of failure.

General Design Methods

Usually, structures are designed by choosing the sections, dimensions and materials of the structure in the first approximation. With combination of all possible loads, the behaviour of the structure is analyzed for its serviceability such as deformations, stresses, cracking and strength. If necessary revisions are made until the design regulations are satisfied. The behaviour of structures could be assessed by indirect methods such as tests on models.

An example of direct method is the Working Stress Design Method, which assumes linear relationship between stress and strain within the elastic limit. If the maximum stress under the assumed load is lower than the allowable stress, the structure is then considered to be safely designed.

The Limit State Design Method, on the other hand, assumes non-linear relationship between stress and strain. Attempts in improving design methods based on allowable stresses have shown that in practice the data and the results are dispresed. This has encouraged the use of statistical approach for studying the problem of structural safety.

The statistical concept of safety may be introduced by defining statistical distribution of loads and structural behaviour by using statistical theories instead of the usual deterministic approach.

REPRESENTATION OF REALITY-DECISION RULES

Types of Idealization

It is a known fact that the accuracy of forcast improves with increase in the size of the data. The probabilistic approach gives an accurate idealization of reality. Idealization can be deterministic, statistic or strategic. Mathematical analysis is an example of deterministic idealization. Vectors and numbers are, for example, deterministic quantities. In statistics the quantities used are defined by distribution functions or densities of probability. On the other hand, the theory of games deals with strategic decisions. The decision rules to be used for the solution of any problem should be related to the type of idealization adopted.

Decision Rules for Statistical Idealization

Decision rules help to identity the criteria for arriving at a suitable solution for a given problem. Economic criteria related with deterministic idealization would correspond to the following rules of decision.

Rule 1: Among the different solutions choose the one incurring minimum cost. In this rule no reference to safety is included.

Rule 2: Assume those events occuring with high value of probability as events that would actually take place. This rule is to be used when economic quantities are statistically defined.

Rule 3: With respect to an exhaustive set of mutually exclusive events, the event with the highest probability is expected to be certain.

Rule 4: Maximize the estimated gain.

An interpretation in terms of structural safety for the above rules is as follows:

(a) According to Rule 2, failure shall be considered not to occur if the probability of failure is very small. The disadvantage of Rule 2 is that it involves a subjective judgement of what is considered to be a very low probability of failure.

(b) Rule 3 would imply increased safety so that failure does not occur.

Other opinions about the problem of choosing decision rules are expressed as follows:

(a) Obtain a probability of failure equal to zero. Theoretically this result could be obtained if the distribution functions of load and resistance did not intersect.

(b) Minimize the generalized cost where generalized cost is defined by Eq. 1.

$$C_g = C_i + \Sigma P_f C_f \qquad (1)$$

where,

C_g = generalized cost

C_i = initial cost of construction plus maintenance

P_f = Probability of one type of failure

C_f = cost due to this type of failure obtained by adding construction and demolition cost

(c) The rule of minimizing the generalized cost is not acceptable, if the cost of reconstruction is a large part of the original construction (before failure).

When psychological factors are considered, the tendency is to increase the generalized cost to avoid accident or failure.

BASIC PROBLEM OF STRUCTURAL SAFETY

Computation of the Probability of Failure

Consider a set of structures loaded by a system of forces. The intensity of the forces, S, is measured by a variable s. Failure occurs when the variable s reaches the resistance R. The condition for safety, therefore, is when S is less than R. Failure will occur when the load is greater than the resistance. The quantities S and R are random, and the distribution functions F_S and F_R can be defined as

$$F_S(s) = P(S < s) \qquad (2)$$

and

$$F_R(s) = P(R < s) \qquad (3)$$

that is

$$P(S \geqslant s) = 1 - F_S(s) \qquad (4)$$

But, these distribution functions correspond to the probability density functions

$$f_S(s) = \dot{F}_S(s) \qquad (5)$$

and

$$f_R(s) = \dot{F}_R(s) \qquad (6)$$

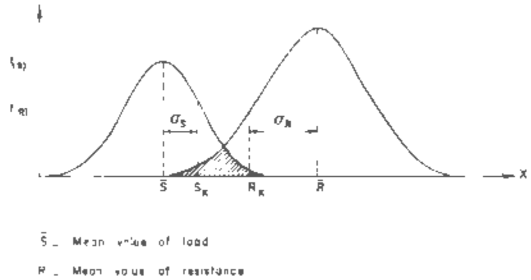

Fig. 1 Probability Curves

Since the distribution functions F_S and F_R are independent (Fig. 1), the probability of failure of the structure is given by

$$P_f = P(S > R) = \int_0^\infty F_R(s) f_S(s) ds \qquad (7)$$

and

$$P_f = P(S > R) = \int_0^\infty [1 - F_S(s)] f_R(s) ds \qquad (8)$$

There are various types of distributions — namely:

- Normal Distribution
- Log-normal Distribution
- Distribution of Extreme Type I (maxima)
- Distribution of Extreme Type I (minimia)
- Distribution of Extreme Type II (maxima)
- Distribution of Extreme Type III (minima)

Say, for normal type of distribution, if the distributions $F_R(s)$ and $F_S(s)$ are known, the probability of failure can be calculated by Eqs. 7 and 8.

It is observed that for high level of risk (e.g. $> 10^{-3}$), the calculated probability of failure does not vary much with the type of distribution. However, for a very small risk (e.g., $\leq 10^{-5}$), the failure probability is quite sensitive to these distributions. Since it is difficult to ascertain the correct distribution the true probability of failure will remain unknown and the calculated failure probability can serve only as a practical and consistent measure of risk, that is a relative measure of structural safety.

Factors of Safety

The central factor of safety, γ_o, is defined as the ratio between the mean values of $F_R(s)$ and $F_S(s)$. Safety is achieved if

$$\gamma_o \bar{S} < \bar{R} \qquad (9)$$

where,

$\bar{S}$ = mean value of load

$\bar{R}$ = mean value of resistance

The characteristic factor of safety, γ_k, is defined as the ratio between the lower 5% fractile of $F_R(s)$, and the upper 5% fractile of $F_S(s)$. This factor of safety is called characteristic because the 5% of resistances and loads are called characteristic values according to Comité Euro-International due Beton (CEB). The design factor of safety, γ_d, is defined as the ratio between the lower 5 per mil fractile of $F_R(s)$ and the upper 5 per mil fractile of $F_S(s)$. This factor of safety may be compared with the load factor γ_s as defined in CEB recommendations. The 5 per mil fractile of $F_R(s)$ may be considered to correspond to the design resistance R_d. Thus,

$$S_d = \gamma_s S_k \qquad (10)$$

where S_d and S_k are the design load and the characteristic loads, respectively. The design loads have to be compared with the design resistances in order to verify the safety conditions

$$R_d > S_d \quad \text{or} \quad R_d > \gamma_s S_k$$

Thus, the allowable value of γ_s is given by the ratio of the design resistance and the characteristic load. The definition of γ_s and γ_d coincide, and γ_d could be called design factor of safety instead of load factor.

Therefore, the safety could be expressed in terms of loads, load effects, and displacements and stresses, and, safety is achieved if $\gamma_o \bar{S} < \bar{R}$. Thus

$$S_k = \bar{S} + 1.64\sigma_s = S_{0.95} \qquad (11)$$

$$R_k = R_{0.05} \qquad (12)$$

$$\gamma_k S_k < R_k \qquad (13)$$

The design resistance is given by Eq. 14.

$$R_d = R_{0.005} = \frac{R_k}{m} \quad (14)$$

where,

m = reduction factor.

The design factor of safety is given by Eq. 15.

$$\gamma_d S_k < R_d \quad (15)$$

Using different types of safety factors, distributions and coefficients of variations, the following relation may be indicated:

(1) The relation between the probability of failure and the central factor of safety :

- As the coefficients of variation of the resistance or the load increase, the factor of safety for a given probability of failure also increases for normal distribution of resistance.
- For extreme type of distribution of resistance — as the coefficient of variation exceeds 10%, the probability of failure cannot be reduced below a certain limit, even if the central factor of safety is increased.

(2) The relation between the probability of failure and the characteristic factor of safety:

- For different types of distribution of loads and resistance — as the coefficient of variation of resistance greater than 10%, the influence of the variability of the loads is much smaller than the influence of the variability of resistances. An increase of the coefficient of variation of loads reduces the probability of failure. Although this seems a contradiction, the probability of failure must decrease when the difference between the mean values of the distribution of the resistance and load increases.

(3) The relation between the probability of failure and the design factor of safety:

- For normal distribution of loads and resistance - and for extreme type of distribution of loads and resistance a significant amount of increase is observed on the probability of failure since the normal distribution is replaced by extreme type distribution.

CONCLUSION

For evaluation of structural safety and performance, the concept of probabilistic method is the right approach. That is, for the formulation and evaluation of safety factors or load factors, at various levels of risk such that these factors are consistent with the various sources of uncertainities, the probabilistic method seems to be the most realistic solution.

In the absence of statistical data, probabilistic method and engineering judgment may be used. In short available data, experience and careful evaluation of the uncertaiuities of the design variables help one to arrive at an economical and practical risk based design.

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