ESTIMATION OF SETTLEMENT OF RIGID FOOTINGS

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INTRODUCTION

Even though there are various methods of calculating settlements, most civil engineers in this country are familiar with only one method of settlement calculation namely the method of compression index as formulated by Terzaghi [4].

The object of the present paper is to introduce other methods of calculating settlement which are widely used in Europe. The knowledge of the different methods is important in order to understand design works of engineers in the other part of the world. In addition some concepts which play an important role in settlement calculations are presented.

SETTLEMENT

The settlement of structures is mainly attributed to vertical compression of soils as a result of vertical stress. This vertical stress may in general produce immediate, consolidation (primary) and creep (secondary) settlements depending upon the type of soil and the duration of the applied load. One may formulate total settlement as follows:

$$s = s_o + s_1 + s_2 \qquad (1)$$

where

s = total settlement

s_o = immediate settlement

s_1 = consolidation (primary) settlement

s_2 = creep (secondary) settlement

These settlements would easily be identified in a time-settlement curve of a consolidation test as shown in Fig. 1. Since it is the consolidation settlement that is of primary importance for normally consolidated and pre-compressed clays, this paper will exclusively deal with the various methods of calculating this type of settlement.

METHOD OF CALCULATING CONSOLIDATION SETTLEMENT

All settlement calculations are based on parameters that are derived from consolidation test. In order to understand the principle involved in the calculation of settlement it is necessary to interprete correctly the compressibility curves of consolidation tests.

Compressibility Curves

The results of consolidation test may be presented either as relative settlement, s', against effective normal stress, $\bar{\sigma}$, or void ratio, e, against effective normal stress as indicated in Fig. 2. For ease of analysis, $\bar{\sigma}$ is normally made dimensionless by dividing it by a unit stress.

Relative Settlement Versus Effective Normal Stress

The compressibility curve obtained from the consolidation test may be expressed with sufficient accuracy by the equation of Ohde [2, 3] given by

$$E_s = \frac{d\sigma}{ds'} = v(\bar{\sigma})^w \qquad (2)$$

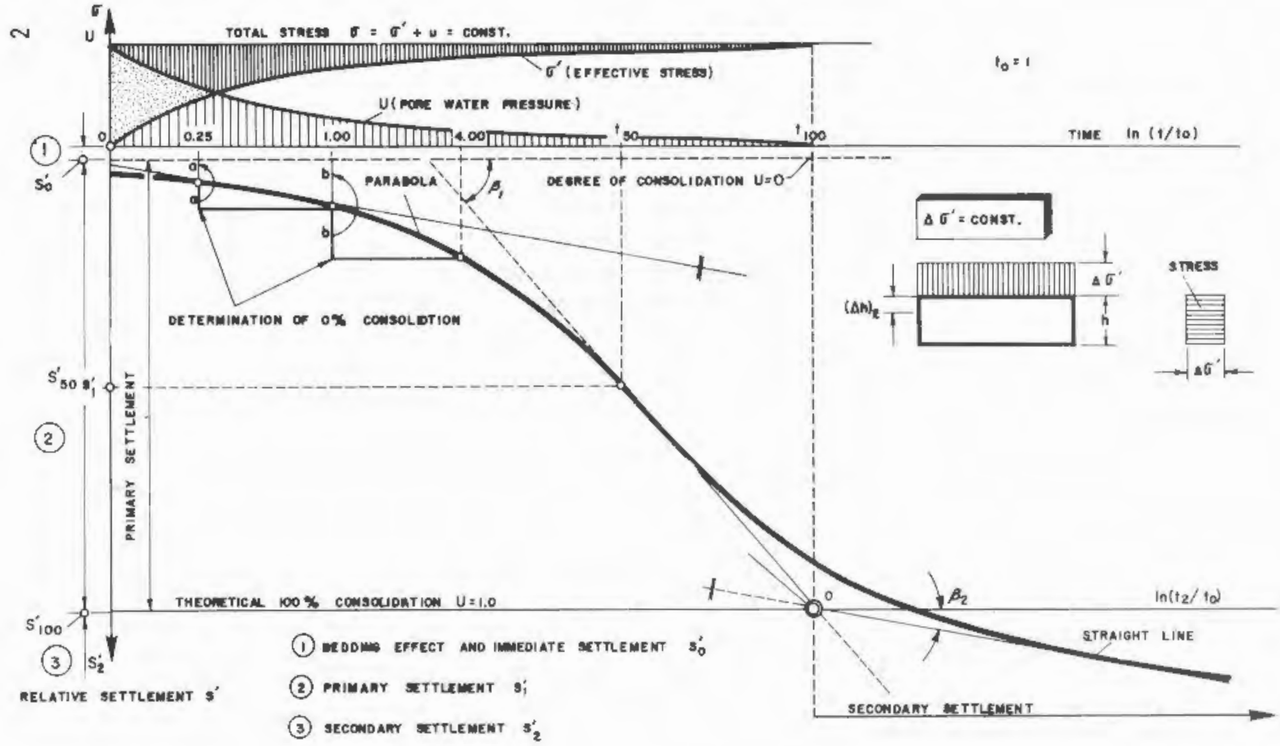


Fig. 1 Time-Settlement Cruve of a Consolidation Test

In the above equations s' is the relative settlement, v and w are coefficients where v has a unit of kPa. It depends on the void ratio, water (moisture) content and consistency of the sample. It could have values ranging from 50 to 30000 kPa.

The coefficient w is a dimensionless quantity which depends on the soil type. It could have values [3] ranging from 0 to 1.

The tangent of the compressibility curve, which is a function of $\bar{\sigma}$ gives the modulus of compressibility E_s (Fig. 2a).

From Eq. (2)

$$\frac{ds'}{d\bar{\sigma}} = \frac{1}{v(\bar{\sigma})^w} \quad (3)$$

$$ds' = \frac{1}{v}(\bar{\sigma})^{-w}d\bar{\sigma} \quad (4)$$

$$s' = \frac{1}{v}\int(\bar{\sigma})^{-w}d\bar{\sigma} \quad (5)$$

For the case $w \neq 1$

$$s' = \frac{1}{v(1-w)}(\bar{\sigma})^{1-w} + C \quad (6)$$

Defining $a = \frac{1}{v(1-w)}$ and $k = 1-w$, Eq. (6) becomes

$$s' = a(\bar{\sigma})^k + C \quad (7)$$

For the case when $w = 1$

$$\frac{ds'}{d\bar{\sigma}} = \frac{1}{v\bar{\sigma}} \quad (8)$$

$$s' = \frac{1}{v}\ln\bar{\sigma} + C \quad (9)$$

If a plot s' versus $\ln \bar{\sigma}$ is made, a straight line relationship is obtained for some cohesive soils. This would mean that the compressibility of the soil is described by Eq. (9). Other soils give straight line relationship when the results are plotted on a double logarithmic scale (Eq. 7).

Void Ratio Versus Effective Stress

Instead of the relative settlement s', one can use the void ratio e.

It can be shown that $s' = \dfrac{\Delta e}{1 + e_o}$ in which s' and Δe, ($\Delta e = e_o - e$) are related to the corresponding loading and e_o is the initial void ratio of the sample.

Similar to Eq. (2) one may write:

$$E_e = -\frac{d\bar{\sigma}}{de} = v' (\bar{\sigma}_e)^{w} \qquad (10)$$

$$\text{Since } s' = \frac{\Delta e}{1 + e_o} = \frac{e_o - e}{1 + e_o} = \frac{e_o}{1 + e_o} - \frac{e}{1 + e_o} \qquad (11)$$

then

$$\frac{ds'}{de} = -\frac{1}{1 + e_o} \qquad (12)$$

From Eqs. (10), (11), and (12) one may deduce

$$E_s = E_e(1 + e_o) \qquad (13)$$

Similarly from Eqs. (2), (10) and (13) one obtains

$$v' = \frac{v}{1 + e_o} \qquad (14)$$

Since by definition $a_v = -\dfrac{de}{d\bar{\sigma}}$ and $m_v = \dfrac{a_v}{1 + e_o}$

one would obtain [5] from Eq. (10) and (13)

$$E_e = \frac{1}{a_v} = \frac{E_s}{1 + e_o} \qquad (15)$$

$$E_s = \frac{1}{m_v} \qquad (16)$$

where

a_v = coefficient of compressibility

m_v = coefficient of volume compressibility

On an e versus $\log \sigma$ plot, ordinary clays show for the most part, straight line relationships (Fig. 2). This line may be represented by the following equation

$$e = e_o - C_c \log \frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o} \qquad (17)$$

where

$C_c = \tan \alpha$ = compression index (Fig. 2b)

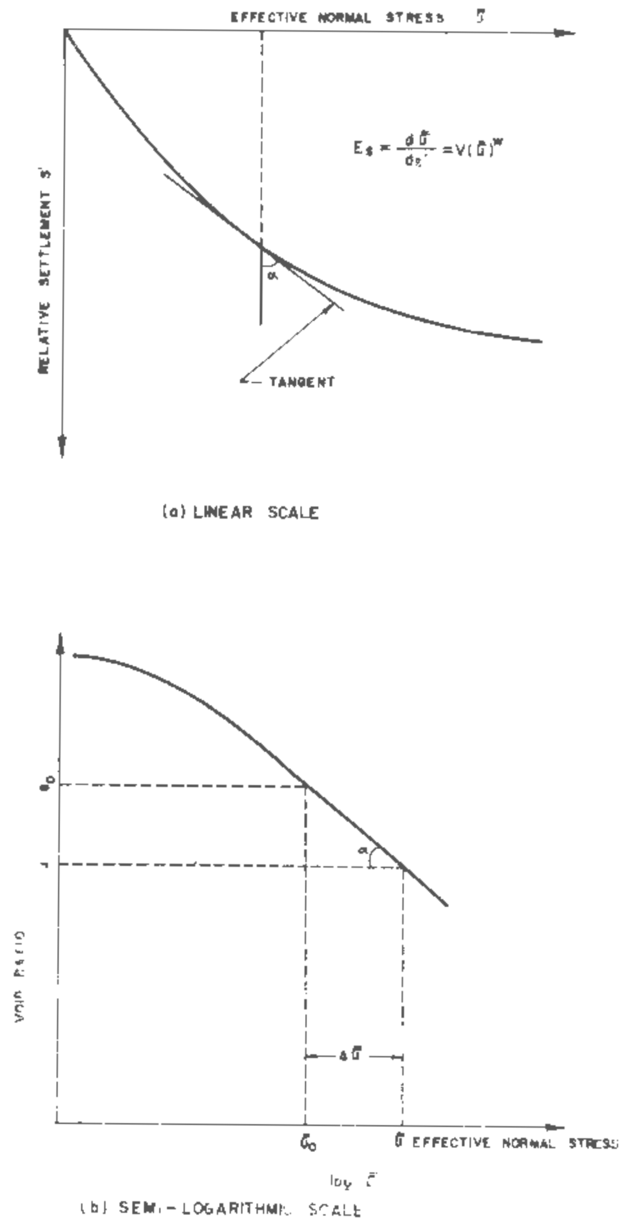


Fig. 2 Compressibility Curves

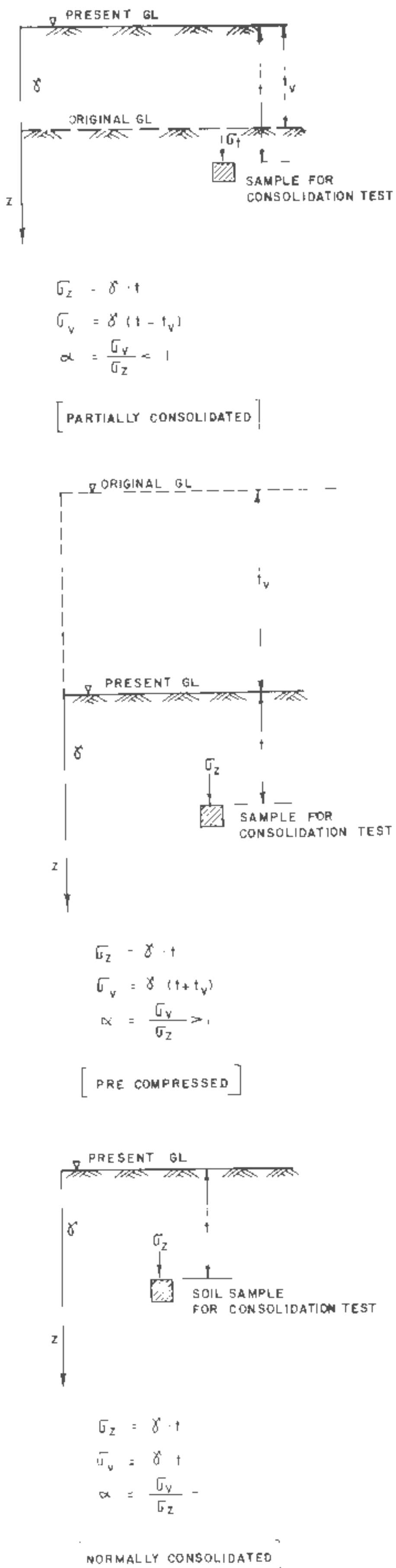


Fig. 3 Geological History of a Soil Sample [3]

Settlement Calculations

The above relationships are based on the assumption that the soils are normally consolidated. Basically soils in-situ may have experienced one of the three conditions in their geological history (Fig. 3). If the soil has been pre-compressed, the compressibility curves s' versus $\log \bar{\sigma}$ will not be straight lines. The curves manifest some kind of curvature. The equations derived from the straight line relationships cannot be used for the whole of the compressibility curve. It is therefore, necessary to determine the value of the pre-compression pressure $\bar{\sigma}_v$ from the consolidation test. There are various methods for determining σ_v. The two common methods are given in Fig. 4. Soil subjected to stress conditions having values less than the pre-compression pressure show little settlement. Stresses higher than the pre-compression pressure give settlement in accordance to the parameters discussed earlier.

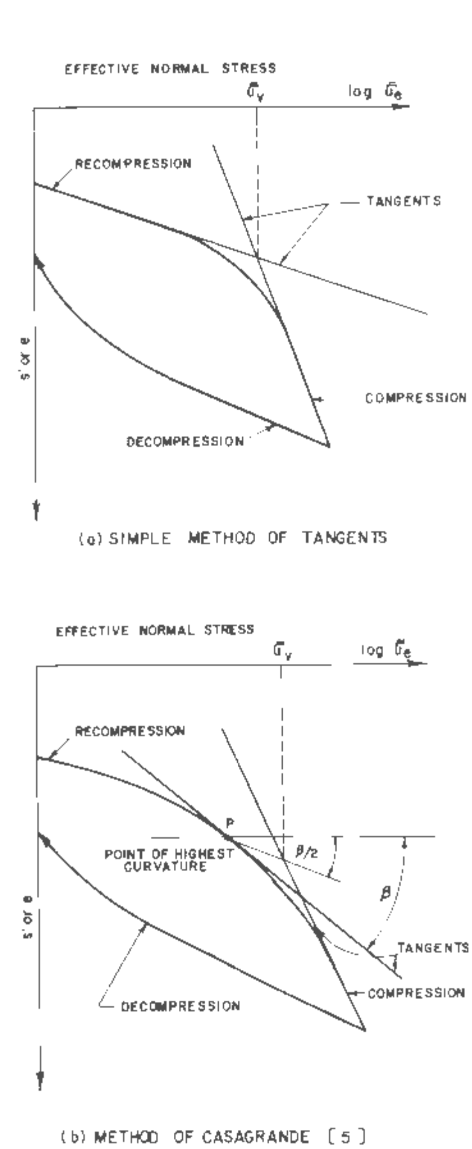


Fig. 4 Methods for Determining Precompression Pressure $\bar{\sigma}_v$

Methods of Calculation

Under this item, the three methods of calculation are discussed using the example shown in Fig. 5. Here it is required to determine the settlement of a structure erected on a normally consolidated soil layer.

(a) **Average Modulus of Compressibility Method.** The average modulus of compressibility E_s is obtained from the results of the consolidation curve. Two points, namely $\bar{\sigma}_{min}$ and $\bar{\sigma}_{max}$ are located on the curve and these two points are then joined by a straight line. The slope of the straight line gives E_s (Fig. 5)

$$E_s = \tan\alpha = \frac{\Delta\sigma}{\Delta s'} \tag{18}$$

The total settlement is then calculated by dividing the area of the pressure distribution curve, A, by E_s (Fig. 5).

The area may be approximately calculated with the help of Kepler's formula (Fig. 5).

$$A = \frac{Z_{(3)} - Z_{(1)}}{6}\,(\sigma_{(1)} + 4\cdot\sigma_{(2)} + \sigma_{(3)}) \tag{19}$$

(b) **Variable Modulus of Compressibility (Graphical) Method.** The average modulus of compressibility method approximates the curve between $\bar{\sigma}_{min}$ and $\bar{\sigma}_{max}$ with a striaght line. However, as could be seen in Fig. 5(c) the slope of the consolidation curve varies from point to point. The Variable Modulus of Compressibility method considers this variation in calculating the settlement. In this respect, this graphical method is more accurate than the constant E_s method. However, the accuracy depends, like all graphical methods, on the scale of the drawing.

From the consolidation cruve (Fig. 5) the total relative settlement S_T' corresponding to the total effective stress $\bar{\sigma}_T$, ($\bar{\sigma}_T = \bar{\sigma}_o + \Delta\bar{\sigma}$) and the relative settlement s_o' corresponding to the effective stress due to the overburden pressure would then be

$$\Delta s' = S_T' - s_o' \tag{20}$$

The settlement for a thin layer of soil with a thickness of Δd_s would be

$$\Delta s = \Delta s_m' \cdot \Delta d_s \tag{21}$$

where

$\Delta s_m'$ = the relative settlement $\Delta s'$ at the middle of the layer Δd_s

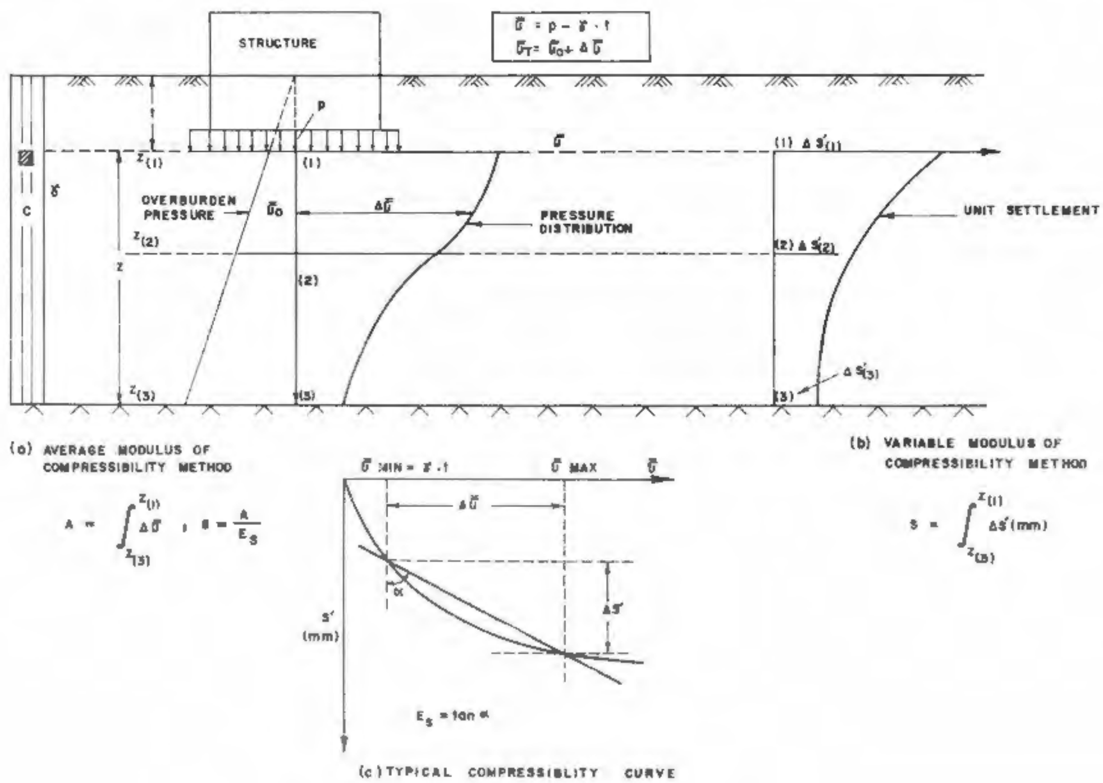


Fig. 5 Methods of Settlements Calculation

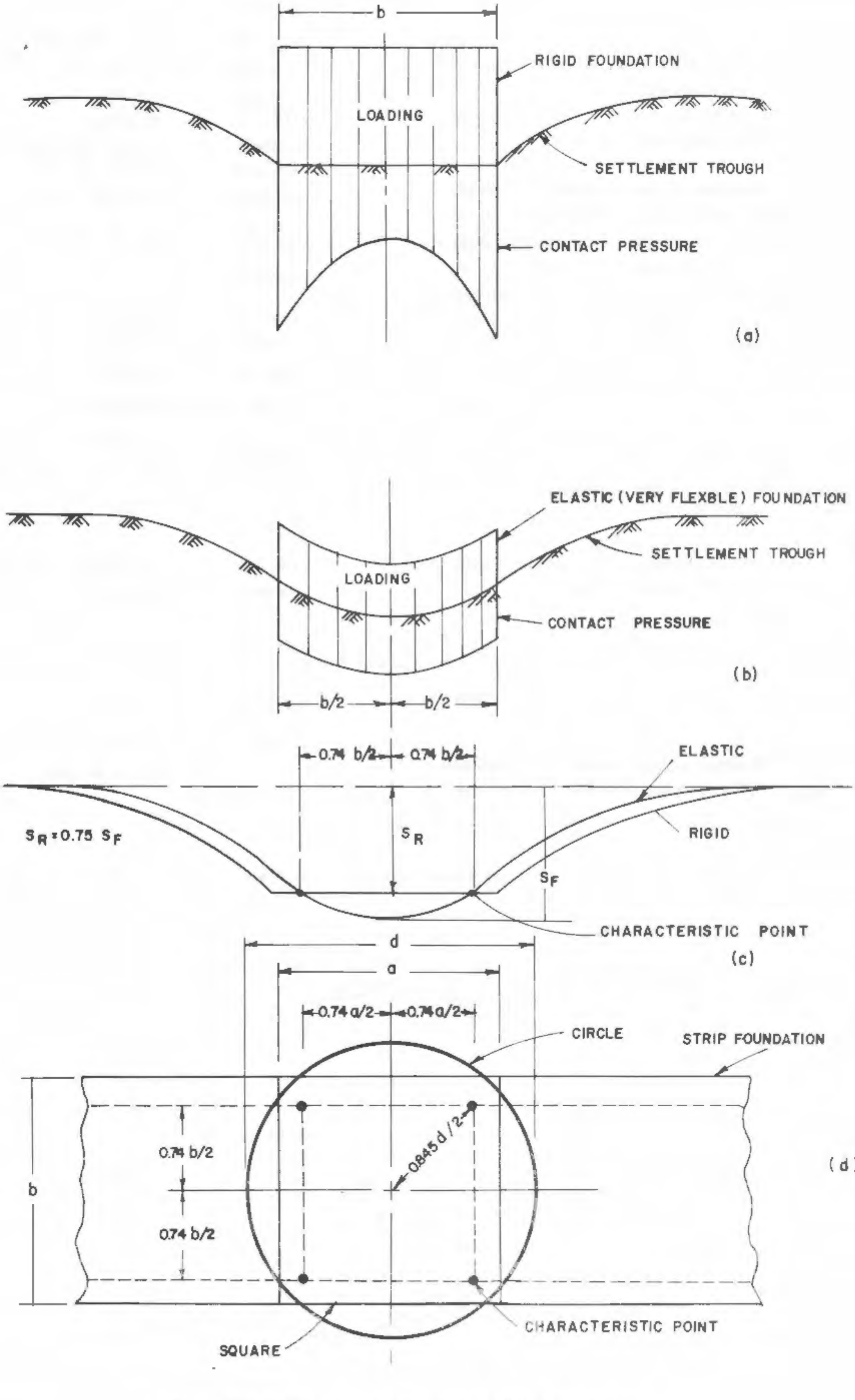
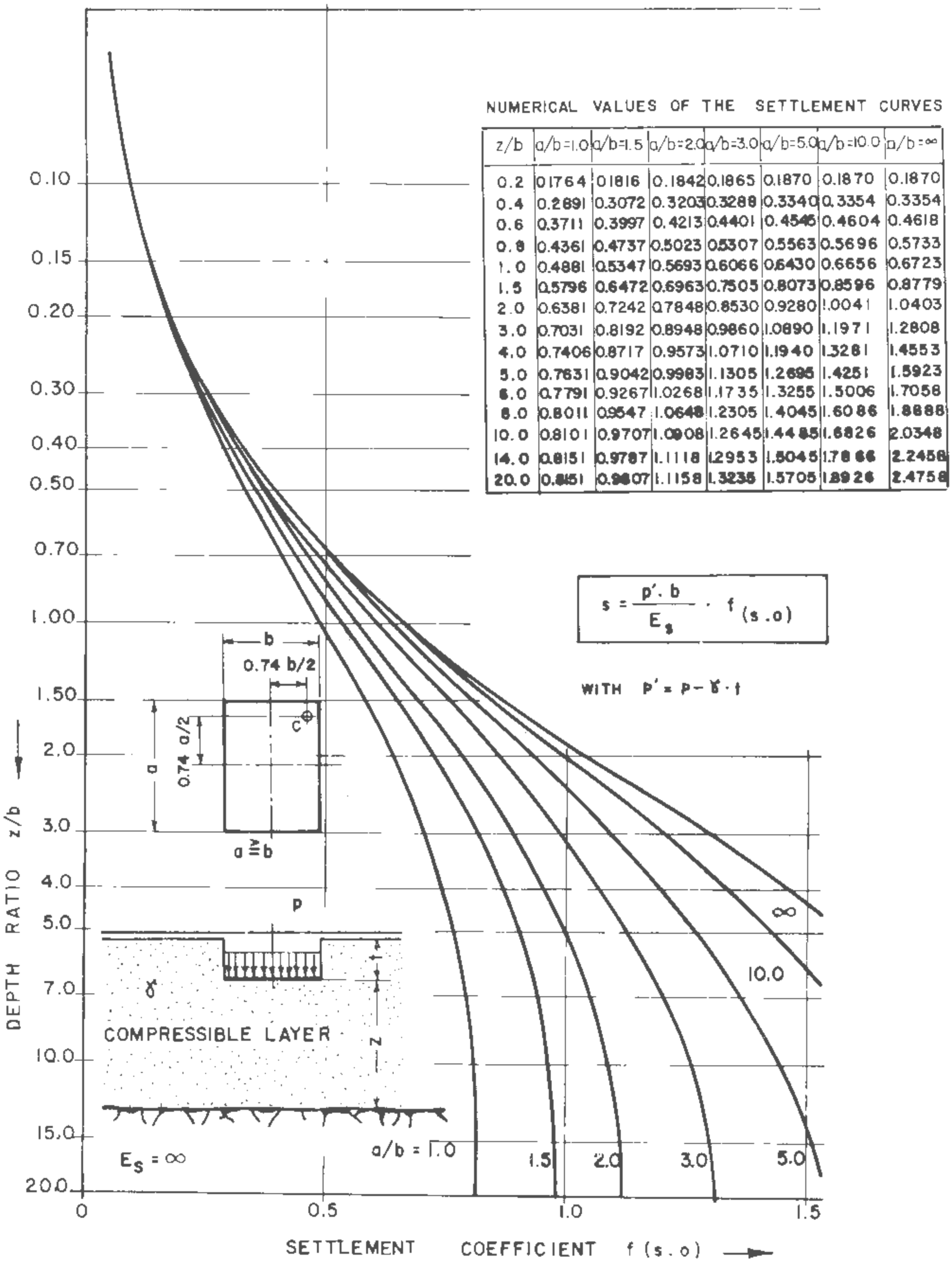


Fig. 6 Settlement Through Elastic and Rigid Foundation



NUMERICAL VALUES OF THE SETTLEMENT CURVES

z/b	a/b=1.0	a/b=1.5	a/b=2.0	a/b=3.0	a/b=5.0	a/b=10.0	a/b=∞
0.2	01764	01816	0.1842	0.1865	0.1870	0.1870	0.1870
0.4	0.2891	0.3072	0.3203	0.3288	0.3340	0.3354	0.3354
0.6	0.3711	0.3997	0.4213	0.4401	0.4546	0.4604	0.4618
0.8	0.4361	0.4737	0.5023	0.5307	0.5563	0.5696	0.5733
1.0	0.4881	0.5347	0.5693	0.6066	0.6430	0.6656	0.6723
1.5	0.5796	0.6472	0.6963	0.7505	0.8073	0.8596	0.8779
2.0	0.6381	0.7242	0.7848	0.8530	0.9280	1.0041	1.0403
3.0	0.7031	0.8192	0.8948	0.9860	1.0890	1.1971	1.2808
4.0	0.7406	0.8717	0.9573	1.0710	1.1940	1.3281	1.4553
5.0	0.7631	0.9042	0.9983	1.1305	1.2696	1.4251	1.5923
6.0	0.7791	0.9267	1.0268	1.1735	1.3255	1.5006	1.7058
8.0	0.8011	0.9547	1.0648	1.2305	1.4045	1.6086	1.8888
10.0	0.8101	0.9707	1.0908	1.2645	1.4485	1.6826	2.0348
14.0	0.8151	0.9787	1.1118	1.2953	1.5045	1.7866	2.2456
20.0	0.8151	0.9807	1.1158	1.3235	1.5705	1.8926	2.4758

Fig. 7 Settlement Cruves at the Characteristic Point According to Kany

If the whole soil layer is divided into n strips with widths Δds, then the total settlement would be:

$$s = \sum_{0}^{n} \Delta s' \qquad (22)$$

If the unit settlement distribution is continuous with depth, (Fig. 5) one can apply Kepler's formula:

$$S = \frac{Z_{(3)} - Z_{(1)}}{6} (\Delta s'_{(1)} + 4 \cdot s'_{(2)} + \Delta s'_{(3)}) \qquad (23)$$

(c) **Compression Index Method.** The settlement of a layer of normally consolidated clay having a depth of h may be calculated from the following relationship.

$$S = \sum_{o}^{n} h \cdot \frac{C_c}{1+e} \log_{10} \frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o} \qquad (24)$$

where

C_c = compression index

e_o = initial void ratio

$\bar{\sigma}_o$ = initial overburden pressure

$\Delta\bar{\sigma}$ = additional pressure due to loading

h = height of the consolidating layer under question

n = number of strips having a depth of h

SETTLEMENT CORRECTIONS

For calculating settlements under footings, one usually considers a uniform distribution of contact pressure under the footing irrespective of the degree of the rigidity of the structure. If one compares the contact pressure distribution under rigid and elastic footings, one would find that the contact pressure under a rigid footing is parabolically distributed and that under an elastic (very flexible) footing is uniformly distributed (Fig. 6). As a result the settlements vary. From the settlement troughs one observes that:

(a) the settlement of a rigid footing is 0.75 that of an elastic footing at the middle of the footing,

(b) the settlements of a rigid and elastic footings at a distance of $0.74b/2$ – called the characteristic point are the same (Fig. 6c).

Since the tables and formulas used for calculating stress distribution assume uniform contact pressure distribution (Table of Steinbrenner [3], Newmark's Chart [3]) the calculations of settlements for actual footings should be made either at the middle or at the characteristic point.

For uniform soil layer with a constant modulus of compressibility E_s, but underlain by a relatively rigid stratum (gravel, rockbed, etc.) one can directly determine the settlement under the characteristic point for a rectangular footing, using the Chart of Kany [1] as indicated in Fig. 7.

Apart from the above obvious corrections, there are other minior corrections to be undertaken [3] which will not be discussed here.

CONCLUSION

With this presentation, it is hoped to familiarize the reader with the different methods of settlement calculations. Which method to adapt should be left to the prevailing code and or to the discretion of the engineer.

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