DETERMINATION OF THE BEHAVIOUR OF TRANSVERSE VIBRATION OF A TIMOSHENKO BEAM WITH AN OPEN CRACK OF UNIFORM DEPTH

Ermias Gebrekidan and Alem Bazezew
Department of Mechanical Engineering
Addis Ababa University

ABSTRACT

Beams are widely used as structural elements and machine elements in civil, mechanical, naval and aeronautical engineering with quite complex design features. These structural and machine elements are designed for different load conditions, with good range of safety factors, and are inspected regularly. Still, unexpected and sudden failures occur due to the presence of flaws or cracks. This paper discusses the behaviour of a cracked Timoshenko beam under vibration. Particularly, the effect of crack depth and location along the beam length is treated and results obtained by the finite element method are presented. The effect of crack is introduced in the system by considering stress intensity factors at the crack tip for different crack modes which give the energy release rate due to the crack. The energy release rate is then used to determine the compliance of the cracked element from which the stiffness of the beam element can be determined.

INRODUCTION

The need for quantitative damage investigation and detection methods that can be applied to complex structures has led to the continued development of methods which can be applied to examine static and dynamic characteristics of structures. In the past three decades, researchers have used open and closed crack models in their investigation of the behaviour of cracked beams subjected to vibratory motion. Dimarogonas and Chondros [4] used local flexibility matrix to simulate the stiffness of the shaft system with opening crack. Maiti [9], Tsai *et.al.* [20] and Ostachowicz *et.al.* [11], in their study of crack in a structural element assumed the crack to be open and that it remained open during vibration. Such an assumption avoids the complexities that may result from the non-linear characteristics present in a breathing crack.

Likewise, other researchers have implemented closed crack model in their work for investigation of crack behavior. Among them, Chondros and Dimarogonas [2], Dimarogonas and Paipetis [3], and Shen [16] dealt with closed crack model to study the dynamic response of structural members with variable elasticity.

To study the behavior of crack in structures, vibration parameters like compliance, mechanical impedance and damping factors have played great role. The presence of crack in a structure affects, directly or indirectly, these vibration characteristics. Specifically, the eigen frequency and mode shapes of structures are affected by the inclusion of cracks. For this reason researchers have focused on these parameters to investigate the behavior of crack. Pandey *et.al.* [13] investigated the behavior of crack as related to mode shape of structures. They have shown that the absolute changes in the curvature mode shapes are localized in the region of damage and hence can be used to analyze the damage in the structures. Sekhar *et.al* [15], Qain *et.al* [14], Sinha *et.al.* [17], Chinchalkar [1], Maiti *et.al* [8], Ostachowitcz *et.al* [12], Matijaz [18], Gouanaris *et.al* [5], Nikolakopouloz, *et.al* [10] have proposed different approaches to analyze crack problems of structural vibration. Skrinar [18] presented a generalization of a simple mathematical model based on the FEM for transverse motion of a beam with crack. Maiti *et.al.* [8] have used both the forward method, in the determination of vibration characteristics of a beam knowing the crack parameters, and the inverse method, determination of crack parameters from known vibration characteristics. Vibration characteristics of cracked Timoshenko beams have also been analyzed by using the finite element method [7].

In this paper investigation of crack behavior of a cantilevered Timoshenko beam is dealt with and solutions are obtained by using the finite element method. To avoid complexities that may arise from non-linearity of the system, the crack is modeled as an open crack. In addition, the beam supports a mass at the free end. The natural frequencies and mode shapes of vibration are determined which are helpful in the detection of the presence of crack without disassembling the system.

CRACK MODELING

In order to study the behavior of crack in the beam, some assumptions are introduced. The crack is considered as an open crack with a uniform transverse crack depth across the width of the beam. The flexural rigidity of the beam is constant except at the cracked element, where the moment of inertia will be affected due to the presence of the crack.

According to Saint-Venant's principle, the presence of the crack affects the stress field only in the region adjacent to the crack which in turn affects the element stiffness matrix in the vicinity of the crack. To find appropriate shape functions to express the kinetic energy and elastic potential energy of the cracked element is difficult because of the discontinuity of deformation due to the presence of the crack. However, the energy release rate due to the crack, obtained from studies made in fracture mechanics [19], can be utilized to determine the flexibility coefficients from the stress intensity factor by using Castigliano's theorem. These flexibility coefficients can then be used to determine the stiffness of the cracked element.

Consider a cantilever beam of length l with given stiffness properties and cross sectional dimensions $b \times h$, and a transverse crack of depth a (see Fig. 1). In general, the beam is subjected to the applied loads shown in the figure where P_1 is an axial load; P_2 and P_3 are shear forces; and P_4 and P_5 are bending moments. In the study of the effect of the presence of a crack in the transverse vibration of a cantilever beam presented in this paper, only bending moment about the z-axis P_5, and the shear force P_3 parallel to the y-axis due to the tip force are considered.

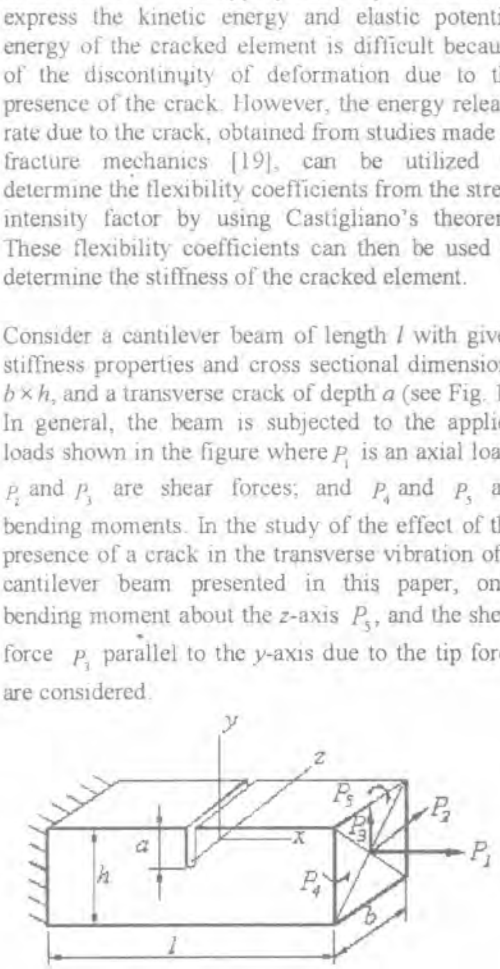

Figure 1 Loaded beam element with transverse crack

The displacement field u_i, due to the presence of the crack of depth a, has been derived by Paris [20] and is given by the equation

$$u_i = \frac{\partial}{\partial P_i}\left[\int_0^a J(a)da\right] \qquad (1)$$

where, $J(a)$ is the strain energy density function ($SEDF$) or the J-Integral,

P_i is the load corresponding to the displacement u_i, and

a is crack depth.

The local flexibility coefficients due to the crack can be obtained by using Castigliano's theorem, and are given by

$$c_{ij}^{cr} = \frac{\partial u_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j}\left[\int_0^a J(a)da\right] \qquad (2)$$

Integrating the strain energy density function along the width b, the local flexibility coefficients of the cracked element are obtained, in non-dimensional form, as

$$c_{ij}^{cr} = \frac{\partial u_i}{\partial P_j} = \frac{1}{b}\frac{\partial^2}{\partial P_i \partial P_j}\left[\int_0^b \int_0^a J(a)da\,dz\right] \qquad (3)$$

The J integral is evaluated from the stress intensity factors at the crack-tip. As concerns the crack-tip stresses, crack mode due to the bending moment P_5 is the opening mode, mode I, whereas the load P_3 induces both opening and sliding modes, modes I and II. The stress intensity factors that correspond to these crack modes are given by [20]

$$K_{I5} = \sigma_5\sqrt{\pi a}F_I\left(\frac{a}{h}\right), \text{ where } \sigma_5 = \frac{P_5}{\left(bh^3/12\right)}y = \frac{6P_5}{bh^2} \qquad (4)$$

$$K_{I3} = \frac{3P_3L}{bh^2}\sqrt{\pi a}F_I, \text{ the stress intensity due to shear force for mode } I \qquad (5)$$

$$K_{II3} = \sigma_3\sqrt{\pi a}F_{II}\left(\frac{a}{h}\right), \text{ where } \sigma_3 = \frac{P_3}{bh} \qquad (6)$$

The local flexibility coefficients c_{33}, c_{15} and c_{55} are obtained by combining Eq. (3) with Eqs. (4)-(6) which, in non-dimensional form, are given by

$$c_{ij}^{cr} = \frac{\partial u_i}{\partial P_j} = \frac{1}{b}\frac{\partial^2}{\partial P_i \partial P_j}\left[\int_0^b \int_0^a J(a)da\,dz\right] \qquad (7)$$

The J integral, which is a function of the crack length, is given by [20]

$$J(\alpha) = \frac{1}{E'}\left[\left(K_{I3} + K_{I5}\right)^2 + K_{II3}^2\right] \qquad (8)$$

Substituting for the stress intensity factors from Eqs. (4) – (6), the J integral is obtained as

$$J(a)=\frac{1}{E'}\left[\left(\frac{3P_3LF_I\sqrt{\pi a}}{bh^2}\right)^2+2\left(\frac{3P_3LF_I\sqrt{\pi a}}{bh^2}\right)\left(\frac{6P_5\sqrt{\pi a}F_I}{bh^2}\right)+\left(\frac{6P_5\sqrt{\pi a}F_I}{bh^2}\right)^2+\left(\frac{P_3\sqrt{\pi a}F_{II}}{bh}\right)^2\right] \quad (9)$$

Substituting for $J(a)$ in Eq. (7) and carrying out the integration, the flexibility coefficients, for an element of the beam of length l_e that carries the crack, are obtained to be

$$c_{33}^{cr}=\frac{\pi}{E'}\left[\frac{18F_I^2l_e^2}{b^2h^4}+\frac{2F_{II}^2}{b^2h^2}\right]\frac{a^2}{2} \quad (10)$$

$$c_{35}^{cr}=\frac{\pi}{E'}\left[\frac{18l_eF_I^2}{b^2h^4}\right]\frac{a^2}{2} \quad (11)$$

$$c_{55}^{cr}=\frac{\pi}{E'}\left[\frac{72F_I^2}{b^2h^4}\right]\frac{a^2}{2} \quad (12)$$

TIMOSHENKO BEAM FORMULATION

A cantilever beam of length l and cross sectional area $b\times h$ carrying a mass M_T at the tip is considered for analysis.

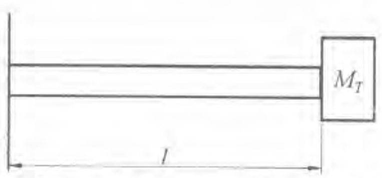

Figure 2 Cantilever beam with mass at the tip

In the formulation of transverse vibration of a Timoshenko beam, the effect of rotary inertia and shear deformation are included, which are ignored in Bernoulli beam analysis. Due to the shear deformation, a plane normal to the beam axis before deformation does not remain normal to the beam axis after the deformation. The kinematics of deformation of the beam is shown in Fig. 3. To derive the equation of motion of the beam, energy method is used.

Let u and v be the axial and transverse displacements of the beam, respectively. Because of the shear deformation, the slope of the beam θ is different from $\frac{dv}{dx}$. Instead, the slope is given by

$\theta=\frac{dv}{dx}-\gamma$ where γ is the transverse shear strain.

Consequently, the displacement field for the beam can be written as

$$u(x,y)=-y\theta(x) \quad (13)$$

$$v(x)=v \quad (14)$$

where the x-axis is located along the neutral axis of the beam.

From Eqs. (13) and (14), the axial and shear strains are

$$\varepsilon=-y\frac{d\theta}{dx} \quad (15)$$

$$\gamma=-\theta+\frac{dv}{dx} \quad (16)$$

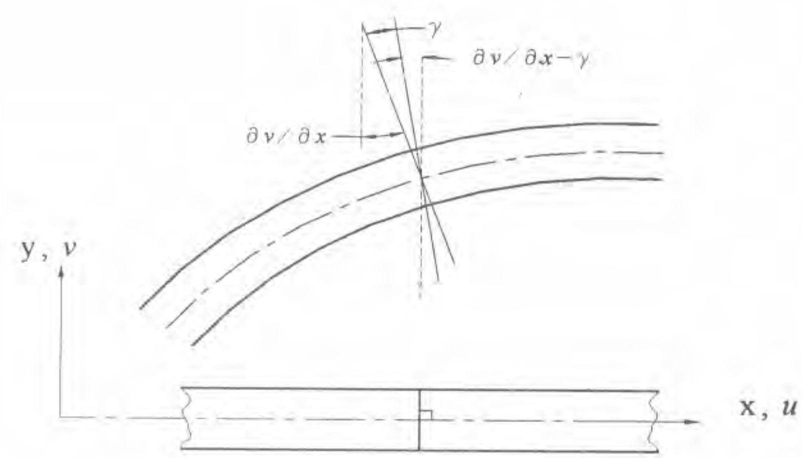

Figure 3 Kinematics of Timoshenko beam deformation

Element stiffness matrices of an un-cracked element

The stiffness matrix of an uncracked element is obtained from strain energy considerations. The strain energy for an element of length l_e is

$$U = \frac{b}{2}\int_0^{l_e}\int_{-h/2}^{h/2}\varepsilon^T E\,\varepsilon\,dy\,dx + \frac{b\kappa}{2}\int_0^{l_e}\int_{-h/2}^{h/2}\gamma^T G\,\gamma\,dy\,dx \tag{17}$$

The first term in Eq. (17) is the bending strain energy and the second term is the shear strain energy. κ is the shear correction coefficient that depends on the cross section. For rectangular cross sections, $\kappa = \frac{5}{6}$ [1, 3].

Substituting Eqs. (15) and (16) in Eq. (17) and integrating with respect to y gives

$$U = \frac{1}{2}\int_0^{l_e}\left(\frac{d\theta}{dx}\right)^T EI\left(\frac{d\theta}{dx}\right)dx + \frac{\kappa}{2}\int_0^{l_e}\left(-\theta+\frac{dv}{dx}\right)^T GA\left(-\theta+\frac{dv}{dx}\right)dx \tag{18}$$

where I and A are the moment of ineı ıa and cross sectional area of the beam, respectively.

To derive the element stiffness matrix for the beam, the variables v and θ are interpolated within each element. For a generic element, the nodal displacements, v_1, θ_1 for node 1, and v_2, θ_2 for node 2 are shown in Fig. 4.

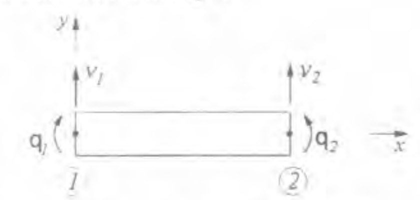

Figure 4 Element nodal displacements

Using these nodal displacements and linear shape functions, the displacements v and θ are

$$v = [H_1 \quad H_2]\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \tag{19}$$

$$\theta = [H_1 \quad H_2]\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} \tag{20}$$

where H_1 and H_2 are linear shape functions given by

$H_1 = \frac{x_2 - x}{l_e}$, $H_2 = \frac{x - x_1}{l_e}$ using physical coordinate, or

$H_1 = \frac{1}{2}(1-\xi)$, $H_2 = \frac{1}{2}(1+\xi)$ using an isoparametric element.

From Eq. (18), the element stiffness matrix due to bending is obtained to be

$$[K_b^e] = \frac{1}{2}\int_0^{l_e}\left(\frac{d\theta}{dx}\right)^T EI\left(\frac{d\theta}{dx}\right)dx \tag{21}$$

Substituting for θ from Eq. (20) and carrying out the integration, after simplifications the element stiffness matrix is obtained to be

$$[K_b^e] = \frac{EI}{l_e}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{22}$$

Similarly, the element stiffness matrix due to shear strain energy is given by

$$[K_s^e] = \frac{\kappa}{2}\int_0^{l_e}\left(-\theta+\frac{dv}{dx}\right)^T GA\left(-\theta+\frac{dv}{dx}\right)dx \tag{23}$$

Substituting for v and θ, and upon integration, the element stiffness matrix due to shear strain is

$$[K_s^e] = \frac{\kappa GA}{4l_e}\begin{bmatrix} 4 & 2l_e & -4 & 2l_e \\ 2l_e & l_e^2 & -2l_e & l_e^2 \\ -4 & -2l_e & 4 & -2l_e \\ 2l_e & l_e^2 & -2l_e & l_e^2 \end{bmatrix} \tag{24}$$

Stiffness matrix of the element that carries the crack

The loading condition of the element of the beam that includes the crack is shown in Fig.5.

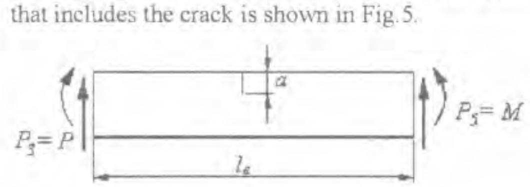

Figure 5 Schematic representation of an element of the beam with crack

The stiffness of this element is obtained by determining the compliance of the element from strain energy considerations. The strain energy of the element is obtained by considering first the uncracked situation to which the crack strain energy is then added. The strain energy of the element without the crack is obtained by considering bending strain energy and shear strain energy.

The bending strain energy is

$$U_b = \frac{bh}{2E}\int_0^{l_e}\left[\left(\frac{Mh}{2I}\right)^2 + 2\frac{Mh}{2I}\frac{P(l_e - x)h}{2I} + \left(\frac{P(l_e - x)h}{2I}\right)^2\right]dx$$

or,

$$U_b = \frac{3}{2EI}\left[M^2 l_e + MPl_e^{\,2} + \frac{P^2 l_e^{\,3}}{3}\right] \tag{26}$$

The shear strain energy can be expressed by [3]

$$U_s = \frac{1}{2}\int_0^{l_e} \kappa\,\tau\, A\, dx \tag{27}$$

where κ is the shear coefficient, A is area of the beam cross-section of beam, γ is the shear angle, and τ is the shear stress. The shear angle and the shear stress are given by, respectively, $\gamma = \varphi - \frac{\partial v}{\partial x}$, where φ is the rotation of cross-section; and $\tau = \frac{3P}{2A}$

The total strain energy is the sum of strain energy due to bending and shear.

$$U = U_b + U_s$$

$$U = \frac{3}{2EI}\left[M^2 l_e + MPl_e^2 + \frac{P^2 l_e^3}{3}\right] + \frac{9}{8}\frac{\kappa P^2 l_e}{GA} \tag{28}$$

The flexibility coefficient for the element without considering the crack is

$$c_{ij}^{(nc)} = \frac{\partial^2 U}{\partial P_i \partial P_j}, \text{ where } P_3 = P,\ P_5 = M;\quad i, j = 3, 5 \tag{29}$$

Hence, the flexibility coefficients are obtained to be

$$c_{33}^{(nc)} = \frac{\partial^2}{\partial P_3 \partial P_3}\left[\frac{3}{2EI}\left[M^2 l_e + MPl_e^2 + \frac{P^2 l_e^3}{3}\right] + \frac{9}{8}\frac{\kappa P^2 l_e}{GA}\right] = \frac{l_e^2}{EI} + \frac{15 l_e}{8GA},$$

$$c_{35}^{(nc)} = \frac{\partial^2}{\partial P_3 \partial P_5}\left[\frac{3}{2EI}\left[M^2 l_e + MPl_e^2 + \frac{P^2 l_e^3}{3}\right] + \frac{9}{8}\frac{\kappa P^2 l_e}{GA}\right] = 3\left(\frac{l_e^2}{EI}\right) = c_{53}^{(nc)},$$

$$c_{55}^{(nc)} = \frac{\partial^2}{\partial P_5 \partial P_5}\left[\frac{3}{2EI}\left[M^2 l_e + MPl_e^2 + \frac{P^2 l_e^3}{3}\right] + \frac{9}{8}\frac{\kappa P^2 l_e}{GA}\right] = \frac{3 l_e}{2EI}.$$

The flexibility coefficient matrix of an element without considering the effect of the crack is given by

$$c_{ij}^{nc} = \begin{bmatrix} c_{33}^{(nc)} & c_{35}^{(nc)} \\ c_{53}^{(nc)} & c_{55}^{(nc)} \end{bmatrix} \tag{30}$$

The total flexibility coefficient will then include the part due to strain energy without crack and the strain energy with crack and is given by

$$[c] = c_{ij} = c_{ij}^{(nc)} + c_{ij}^{(cr)} \tag{31}$$

where $c_{ij}^{(nc)}$ is the compliance of beam due to strain energy without crack,

$c_{ij}^{(cr)}$ is the compliance of beam due to strain energy of crack

From equilibrium condition of the element we obtain

$$\begin{Bmatrix} P_i \\ M_i \\ P_{i+1} \\ M_{i+1} \end{Bmatrix} = [N]\begin{Bmatrix} P_{i+1} \\ M_{i+1} \end{Bmatrix}^T \tag{32}$$

where $[N]$ is the nodal transfer matrix given by

$$[N] = \begin{bmatrix} -1 & 0 \\ -l_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The stiffness matrix of the cracked element is obtained from the flexibility matrix by applying the given transformation matrix [11].

$$\left[K_{cr}^e\right] = [N][c]^{-1}[N]^T \tag{33}$$

Having obtained the stiffness matrices for the cracked beam element $\left[K_{cr}^e\right]$, and for the uncracked elements $\left[K_s^e\right]$, the element stiffness matrices are assembled to give the global stiffness matrix $[K]$.

The consistent mass matrix of an element of the beam

The consistent mass matrix of the beam is computed from relations for the kinetic energy of the system. The kinetic energy of the beam is

$$T = \frac{1}{2}\int_0^{l_e}\int_{A(x)} \rho\left(\dot{u}^2 + \dot{v}^2\right) dA\, dx$$

Evaluating $\dot{u}$ and $\dot{v}$ from Eqs. (8) and (9), the kinetic energy becomes

$$T = \frac{1}{2}\int_0^{l_e}\int_{A(x)} \rho\left(y^2\dot{\theta}^2 + \dot{v}^2\right) dA\, dx \tag{35}$$

Introducing $I(x)=\int_A y^2 dA$, moment of inertia of the cross section, $m = A\rho$, mass per unit length of beam element, and $r^2 = \frac{I}{A}$, where r is the radius of gyration of the cross section,
the equation of the kinetic energy of a beam element becomes

$$T = \frac{1}{2}\int_0^{l_e} m\dot{v}^2 dx + \frac{1}{2}\int_0^{l_e} mr^2\dot{\theta}^2 dx \qquad (36)$$

In Eq (36) the first term is due to the translatory inertia and the second term is due to rotary inertia. Introducing the shape functions in Eq. (36), the mass matrix can be computed from the translatory mass M_{tr}^e and rotary mass M_{rot}^e of the element and are given by

$$M_{tr}^e = \int_0^{l_e} mB^T B dx,$$

$$M_{rot}^e = \int_0^{l_e} mr^2 B^T B dx,$$

where $[B]$ is the derivative of the shape function matrix.

Carrying out the integration, the mass matrices due to the translatory inertia and rotary inertia are obtained to be

$$M_{tr}^e = \frac{ml_e}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and}$$

$$M_{rot}^e = \frac{mr^2}{l_e}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The element mass matrix of the beam element that carries the applied mass M at the tip should include the applied mass and is given by

$$M_{tr}^e = \int_0^{l_e} mB^T B dx + M_T [B(l_e)]^T [B(l_e)]$$

which, upon simplification yields

$$M_{tr}^e = \frac{ml_e}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + M_T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

EQUATION OF MOTION

Once the mass and stiffness matrices are obtained, the equation of motion for free vibration of the beam is given by

$$[M]\{\ddot{\phi}\} + [K]\{\dot{\phi}\} = \{0\} \qquad (38)$$

The system characteristic equation for free vibration is

$$([K] - \omega^2[M])\{\phi\} = 0 \qquad (37)$$

where $[K]$ is the global stiffness matrix, $[M]$ is the global mass matrix, ω is the angular frequency, and $\{\phi\}$ is the modal shape.

NUMERICAL EXAMPLE AND DISCUSSION OF RESULTS

The method discussed in the paper has been applied to a cracked cantilevered Timoshenko beam for two cases: the first beam considered is a test beam for experimental investigation, and the second one is an actual beam of significant dimensions.

Effect of crack position and crack depth ratio on the natural frequency ratio

In order to make a detailed discussion of the effect of crack depth ratio on the natural frequency ratio of a cracked beam to that of an uncracked beam, a numerical example is presented. A test beam with geometric properties l=0.2m, h=0.0078m, b=0.025m, and material properties ρ=7850kg/m^2, E=216MPa and G=90MPa has been considered. Fig. 6 exhibits the results obtained, i.e. the ratio of the frequency of the cracked beam to that of the uncracked beam, for different crack depth ratios a/h. From the figure it can be observed that, for the same crack depth ratio, the frequency ratio decreases as the crack position gets closer to the built-in end, indicating that the dangerous crack position is near the fixed end. The results obtained closely tally with those obtained by Kisa et al [6] who derived the stiffness matrix by dividing the beam into two components connected by a spring.

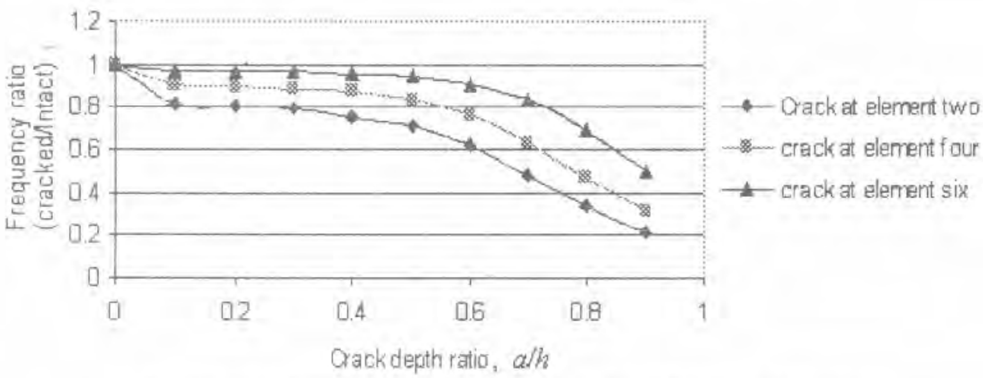

Figure 6 Fundamental (first) frequency ratios for different crack positions

Effect of crack position and crack depth ratio on the natural frequencies and mode shapes

To conduct a detailed discussion of the effect of crack depth ratio on the natural frequencies of a cantilever beam, a beam with geometric properties l=0.6m, h=0.05m, b=0.06m, and material properties ρ=7850kg/m^3, E=216MPa and G=90MPa is considered.

The comparison of the first and second natural frequencies obtained for the cracked and uncracked beams are given in Table 1 for different crack depth ratios. Moreover, the effect of the tip mass M_T is also included. From the figures given in the table, one can conclude that the presence of the crack reduces the natural frequency, and as the depth crack ratio increases, the natural frequency decreases significantly. The addition of the tip mass reduces the natural frequency significantly, which has been aggravated by the presence of the crack.

The effect of the presence of the mass M_T is depicted in Fig. 7. As can be clearly observed from the figure, the fundamental frequency of the cracked beam is reduced appreciably by the presence of the tip mass for all depth ratios.

Table 1: Natural frequencies of the cracked beam for varying crack depth ratios, for a crack located at element seven

	First Natural Frequency				Second Natural Frequency			
Crack Depth ratio	Beam with no crack and without mass	Beam with crack and without mass	Beam with no crack and with a mass at the tip	Beam with crack and a mass at the tip	Beam with no crack and without mass	Beam with crack and without mass	Beam with no crack and with a mass at the tip	Beam with crack and a mass at the tip
0	262.68	262.68	94.72	94.72	1680.38	1680.38	1213.64	1213.64
0.1	262.68	259.044	94.72	91.64	1680.38	1464.60	1213.64	1050.47
0.2	262.68	258.920	94.72	91.54	1680.38	1458.40	1213.64	1046.09
0.3	262.68	258.666	94.72	91.32	1680.38	1445.98	1213.64	1037.29
0.4	262.68	258.154	94.72	90.90	1680.38	1421.89	1213.64	1020.40
0.5	262.68	257.034	94.72	90.00	1680.38	1373.17	1213.64	986.86
0.6	262.68	254.311	94.72	87.87	1680.38	1273.19	1213.64	920.38
0.7	262.68	247.271	94.72	82.81	1680.38	1094.50	1213.64	808.28
0.8	262.68	229.839	94.72	72.35	1680.38	867.83	1213.64	676.28

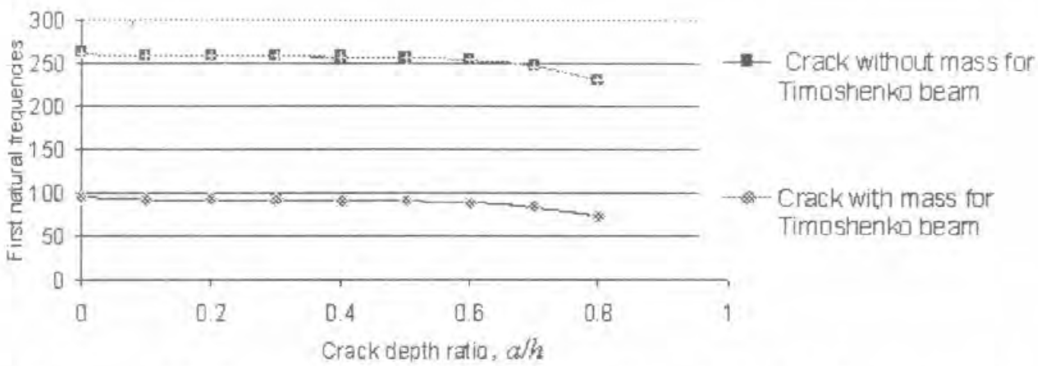

Figure 7 Changes of the first natural frequencies as a function of the crack depth at element seven with and without tip-mass

Mode shapes of the cracked and uncracked beam with and without a mass at the free end have been studied. The results are shown in Fig. 8 where the deviation of the mode shapes for the fundamental and second modes of the cracked and uncracked beam are presented. For all cases the position of the crack is located at mid-span of the beam. As can be noted from the mode shape diagrams, the displacement of the cracked beam is higher in comparison to that of the uncracked beam, clearly indicating that the effect of crack increases the displacement of the beam. It can also be observed that application of the tip mass also significantly increases the displacement of the cracked beam.

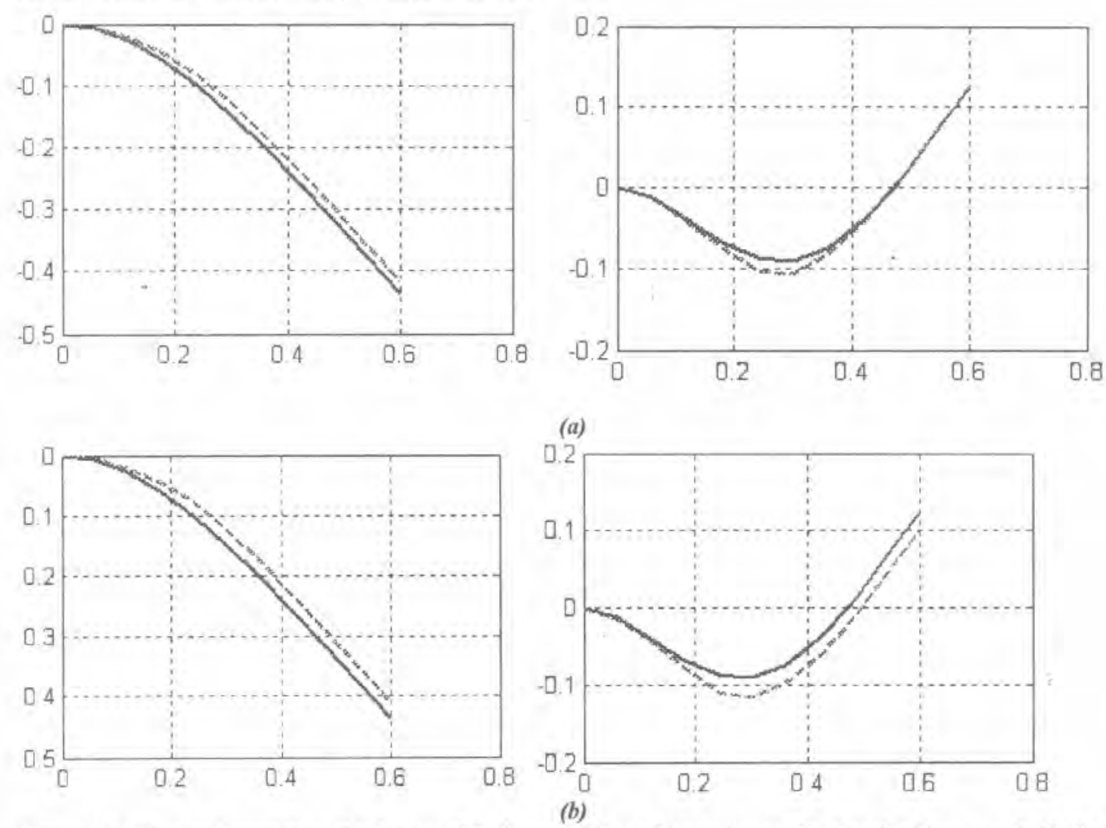

Figure 8 a) First and second mode shapes of the beam with crack (continuous line) and without crack (dashed line) with no tip mass; b) mode shapes with crack (continuous line) and without crack (dashed line) with a tip mass

Figure 9 shows the deviation of the displacement of the cracked beam from that of the uncracked beam for the fundamental mode, i.e. the mode shape deviation of the fundamental mode. It is evident from the mode shape deviation that there is a sharp change in the slope at the crack location as indicated by the discontinuity at mid-span.

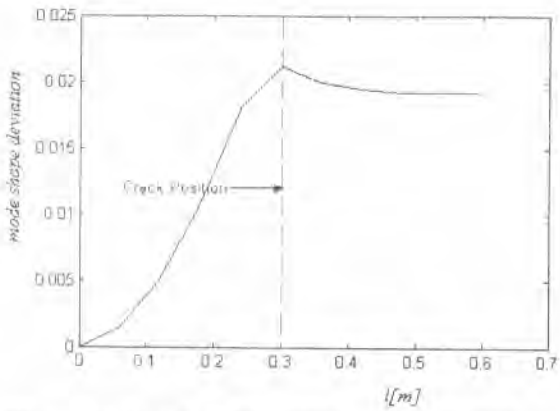

Figure 9 Deviation of first mode shape due to crack for beam without mass

CONCLUSION

In this paper the vibrational behaviour of a cracked Timoshenko cantilever beam has been analyzed. The method used has a noted advantage in determining the stiffness of the cracked beam by using the energy release rate. Numerical examples are included and results obtained are discussed by making comparison of cracked and uncracked beams, as well as comparison with results obtained by other researchers. From the results obtained, some conclusions can be made regarding transverse vibration of a cantilevered Timoshenko beam with crack.

i) The presence of crack in any beam subjected to transverse vibrations lowers the natural frequency of the beam which may be further aggravated by the location of the crack along the length of the beam and the depth of the crack. The closer the crack is to the built-in end, the higher the lowering of the natural frequency is. Likewise, the deeper the crack the lower the natural frequencies are. For some applications, lowering of the natural frequencies may be undesirable.

ii) A mass applied at the tip of the beam enhances the effect of the crack in lowering the natural frequencies and increasing the mode shape displacements. Clearly, crack propagation due to the presence of the tip mass is an area of interest and further research can be conducted to study the behaviour of cracked beams for different loading and support conditions for which the method of analysis presented in this paper can be adopted.

iii) In cases where the location of a crack may not be known a-priori and determination of the crack is of interest, carrying out the analysis presented in this paper is helpful to get plots of the mode shape deviations. These plots can be used to locate the crack from the sharp changes in the slope of the mode shape deviations. Furthermore, the crack depth ratio may be used to determine the length of the crack.

REFERENCES

[1] Chinchalkar, S., "Determination of Crack Location in Beams Using Natural Frequencies", *Journal of Sound and Vibration*, Vol. 247, No. 3, PP 417-429, 2001.

[2] Dimarogonas A. D. and Chondros, T. G., "Vibration of Beam with A Breathing Crack", *Journal of Sound and Vibration*, Vol. 239, No. 1, PP 57-67, 2001.

[3] Dimarogonas A. D. and Paipetis, Stephen A., *Analytical Methods in Rotor Dynamics*, Applied Science Publishers Ltd, England, 1983.

[4] Dimarogonas, A. D. and Chondros, T. G., "Dynamic Sensitivity of Structures to Cracks", *Journal of vibration Acoustics*, Vol. 111, PP 251-256, July 1989.

[5] Gounaries, D., Papadopoulos, C. A. and Dimarogonas, A. D., "Crack Identification in Beam by Coupled Response Measurements", *Computer & Structure*, Vol. 58, No. 2. pp. 299-305, 1996.

[6] Kisa, M., Brandon, J. and Tipcu, M., "Free Vibration Analysis of Cracked Beam By A Combination of Finite Elements and Component Mode Synthesis Methods", *Computers and Structures*, Vol. 67, PP 215-223, 1998.

[7] Kwon, Young W. and Bang, Hyochoong, *The Finite Element Using MATLAB*, Second edition, CRC Press LLC, Boca Raton, Florida, 2000.

[8] Maiti, S. K and Nandwana, B. P., "Modeling of Vibration of Beam in Presence of Inclined Edge of Internal Crack for Its Possible Detection based on Frequency Measurements", *Engineering Fracture Mechanics*, Vol. 58, No, PP 193-205, 1997.

[9] Maiti, S. K. and Chaudhari, T. D., "Modeling of Transverse Vibration of Beam of Linearly Variable Depth with Edge Crack", *Engineering Fracture mechanics*, Vol. 63, PP 425-445, 1999.

[10] Nikolakopoulos, P. G., Katsareas, D.E. and Papadoulose, C.A., *Crack Identification in Structure*, CIVIL-COMP Ltd, Edinburgh, Scotland. 1994.

[11] Ostachowicz, W. M. and Krawczuk, M., "Analysis of The Effect of Cracks on The Natural Frequencies of A Cantilever Beam", *Journal of Sound and Vibration*, Vol. 150, No. 2, PP 191-201, 1991.

[12] Ostachowicz, W. M. and Krawczuk, M., "Transverse Natural Vibrations of A Cracked beam Loaded with A Constant Axial Force", *Journal of Vibration and Acoustics*, Vol. 115, PP 525-529, October 1993.

[13] Pandey, A. K., Biswas, M. and Samman, M. M., "Damage Deflection from changes in Curvature Mode Shapes", *Journal of sound and Vibration*, Vol. 145, No.2, PP 321-332, 1991.

[14] Qian, G. -L., Jiang, S. –N and J. –S., "The Dynamic Behavior and Crack Detection of A Beam With A Crack", *Journal of Sound and Vibration*, Vol. 138, No. 2, PP 233-243, 1990.

[15] Sekhar, A. S. and Prasad, P. Balaji, "Vibration Characteristics of A Cracked Rotor with Two Open Cracks", *Journal of Sound and Vibration*, Vol. 223, No.4, PP 497-512, 1999.

[16] Shen, M. H. H. and Chu, Y. C., "Vibration of A Cracked Beam", *Computer And Structures*, Vol. 45, No. 1, PP 79-93, 1992.

[17] Sinha, J. K., Friswell M. I. and Edwards, S., "Simplified Model for The Location of Cracks In Beam Structures Using Measurement Vibration Data", *Journal of Sound and Vibration*, Vol. 251, No. 1, PP 13-38, 2002.

[18] Skrinar, Matjaz, "A Simple FEM Beam Element with an Arbitrary Number of Cracks", Faculty of Civil Engineering, University of Maribor, Slovenia.

[19] Tada, Hroshi, Paris, Paul C. and Irwin, George R., *Analysis of Crack Handbook*, Third Edition, American Society of Mechanical Engineers, 2000.

[20] Tsai, T. C. and Wang, Y. Z., "Vibration Analysis and Diagnosis of Cracked Shaft", *Journal of Sound and Vibration*, Vol. 192, No. 3 PP 607-620, 1996.