CREEP ANALYSIS OF BOILER TUBES BY FEM

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ABSTRACT

In this paper an analysis is developed for the determination of creep deformation of an axisymmetric boiler tubes subjected to axisymmetric loads. The stresses and the permanent strains at a particular time and at the steady state condition, resulting from loading of the tube under constant internal pressure and elevated temperature are evaluated taking into account the secondary creep characteristics of a given material. Finite element solution is presented and discussed for a class of problems in which no prior analytical solution may exist. The method of solution developed is an extension of the direct stiffness method. The body is replaced by a system of discrete triangular cross-section ring elements interconnected along circumferential nodal circles. The equations of equilibrium for the element are derived from the principle of minimum potential energy. The creep behavior of the body is formulated in terms of creep laws in current use. Starting with the elastic solution of the problem, creep strains are treated as initial strains to determine the new stress distributions at the end of the pre-set time interval. The procedure is repeated until the final time is reached and/or until the stress distribution reaches a steady stated condition. Numerical results obtained for a tube material 12CrMoV are presented at the end.

Nomenclatures

$[\mathbf{a}]$	*Generalized displacements*
A	*Surface area of an element*
E	*Modulus of elasticity*
G	*Shear modulus of material*
i, j, k	*Subscripts for unit vectors*
P	*Surface force*
p	*Pressure*
t	*Time*
T	*Temperature*
σ	*Stress*
ε	*Strain*
α	*Coefficient of thermal expansion*
ν	*Poison ratio*
U	*Strain energy*
V	*Volume of an element*
K, n, q	*Creep indexes of material*
r, θ, z, rz	*Subscripts for radial, hoop, axial and shear direction, respectively*
c, el, th, e	*Superscripts for creep, elastic, thermal and effective, respectively*
u, w	*Nodal radial and axial displacements, respectively*
$[D]$	*Material property in plane strain*
$[\Phi]$	*Nodal coordinate matrix*
$[u]$	*Element nodal point displacement*
$[\Gamma]$	*Element nodal coordinate matrix*
$[H]$	*Transformation matrix* $= [\Gamma]^{-1}$
$[k]$	*Element stiffness matrix*
$[f]$	*Force matrix*
$[K]$	*Global stiffness matrix*

INTRODUCTION

Successful boiler operation involves taking heat energy in its available form (for example, coal, oil, etc.) and converting it into a form that can be conveniently used. This may be done by heating water in a boiler and then using the hot water or steam for a desired purpose. Boiler tubes may fail in service condition due to many reasons. Some of them are tube surface pitting, corrosion cracking, creep rupture, carbide graphitization, oxidation, sulfidation, embrittlement, etc. These conditions that give rise to early failure of tubes are attributed to one or a combination of the following reasons [2, 3].

i. The environmental conditions within the boiler can be highly aggressive and alter the microstructure of tubing.

ii. Stresses caused by external loads, or induced by cold forming operations, uneven cooling or welding, may substantially lower the resistance of tubing to be attacked by certain corrosive media.

During the last hundred years, extensive researches were carried out and findings were published on creep mechanics mostly influenced by various approaches in establishing suitable constitutive equations. Bailey [1] is the pioneer in presenting a useful work in the study of the design aspect of creep in 1935. He proposed a general relationship for creep in terms of principal stresses based on simple tension test. The consideration of primary creep in the design of internal-pressure vessels was proposed by Coffin et al [4]. They evaluated the permanent strains and stresses developed in a thick-walled cylinder at a particular time resulting from loading under constant internal pressure at elevated temperature, considering the primary creep characteristics of the tube material. A study on an axisymmetric method of creep analysis for primary and secondary creep was also undertaken by Jahed and Bidabadi [5].

In this paper the analysis of the effect of long-term overheating known as creep is discussed. A computation technique is developed for the evaluation of axisymmetric creep deformation of boiler tubes by using the FEM [8]. Long-term overheating occurs over a period of months or even years. Boiler tubes commonly fail after many years of service as a result of creep. Creep is a time-dependent deformation that occurs when a material is stressed at high temperature over a period of time with a continued load, in which the material will eventually rupture. The temperature at which creep becomes important depends on the particular metal. For carbon steel, creep rupture becomes a design consideration at 425°C, for alloy steels at about 480°C and for austenitic stainless steels at about 560°C [1]. Tubes that fail by creep exhibit minimal swelling and a longitudinal split that is narrow when compared to short-term overheating [1].

FINITE ELEMENT ANALYSIS

Modeling of the Tube

We can model a boiler tube as a thick-walled cylinder subjected to internal and/or external pressure at a temperature in the creep range. For our analysis, we will consider a normal boiler tube surface, i.e. surface with no soot or scale deposit and no crack and scar existing. Then the stresses and strains due to thermal loading, external load and creep phenomenon will be determined by FE formulation.

In order to make analysis of stress and strain under creep, we consider an axisymmetric hollow circular cylinder of inner and outer radii a and b, respectively, under the following basic assumptions [9]:

i. radial deformation is small, and the radius of the deformed cylinder is nearly the same as that of the un-deformed cylinder;

ii. tube material is homogeneous, isotropic and incompressible;

iii. all material properties are temperature dependant;

iv. Prandtl-Reuss work-hardening flow rule, Von Mises yield criterion and Ramberg-Osgood equations are applicable;

v. the Mises-Mises theory of creep and Norton's Law for creep are applicable; and

vi. inertial forces and the coupling term in the governing equations are neglected and plane-strain condition is valid.

Uniaxial Creep Curves at Constant Stress

It has been determined from experiments that if a metal which creeps is subjected to a constant uniaxial stress, and then the accumulation of creep strains with time has the form illustrated in as shown by Fig. 1 [1, 4].

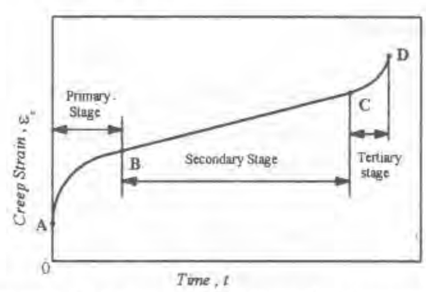

Figure 1 Creep curve obtained at constant temperature under constant load

OA - is an instantaneous deformation that occurs immediately upon application of the load and may contain both elastic and plastic deformation.

AB - *is the primary stage in which the creep is changing at a decreasing rate as a result of strain hardening. The deformation is mainly plastic.*

BC - *is the secondary steady state stage in which the deformation is plastic. In this stage the creep rate reaches a minimum and remains constant as the effect of strain-hardening is counter balanced by an annealing influence. Here the creep rate is a function of stress level and temperature.*

CD - *is the tertiary stage in which the creep continues to increase and is also accompanied by a reduction in cross-sectional area and the onset of necking, hence increase in necking; thereby, resulting in fracture.*

Derivation of Element Stiffness relation

The method of analysis used in this paper is the direct stiffness or displacement method. It can best be described as a variational procedure. In classical elasticity theory one of the most widely used variational principle is the theorem of minimum potential energy [10]. This theorem states that of all displacement functions that satisfy the boundary conditions, the one that satisfies the equilibrium equation makes the potential energy an absolute minimum. The direct stiffness method is a systematic procedure for the application of this theorem.

In continuum problems, the body is approximated by a set of simple sub-regions, called finite elements. Within each element, the displacements are assumed to be linear combinations of functions with undetermined coefficients, and are chosen so that continuity is preserved along the edges of adjacent elements. The assumed displacement functions for any element are related to the displacement at some particular points of the element, which are known as the nodal points of the element. These nodal points are usually taken as the corners of the element. The internal strain energy for each element of the body is then expressed in terms of the nodal point displacements. The potential energy for the complete body is determined by summing the strain energy for all the elements and subtracting the work done by the external loads.

Finally, minimization of the potential energy with respect to the nodal displacements yields the desired system of equation for determining the unknown nodal displacements. For programming and computational reasons, the method of constructing the governing equations is to consider each element separately. The potential energy for each element is minimized. This gives a system of equations in terms of the nodal displacements for that element and the applied forces acting on the element. The coefficient matrix for these equations is called the element stiffness matrix. The governing equations for the body are obtained by superposition of the element stiffness matrices subject to the condition that the displacement at any given node must be the same for all elements attached to that node. The resulting equations must satisfy the boundary conditions, and are solved to yield the unknown nodal point displacements. The element strains and stresses are then calculated from the known displacements.

The inclusion of creep behavior in the finite element approach developed in this paper is handled by using an incremental approach and treating the creep strains as initial strains. The solution of the problem begins by obtaining the elastic solution. Based on these stresses, the creep strains for a small time interval are computed. These are regarded as initial strains for the next time interval and are included as fictitious creep forces at the nodal points in the evaluation of the nodal displacements and element stresses and strains. The solution for the next time increment proceeds in the same manner. The basic assumption used in this approach is that the change in stress during any time increment is small compared to the stress at the beginning of that increment. The error introduced in this procedure can be made as small as one may wish by reducing the time increment.

In applying the direct stiffness method, the first step is to choose a finite element that will be used to represent the body. Because of the need of representing complex geometries for analysis using the finite element method, the triangular element has been used widely. For axisymmetric bodies, the easiest element to use is a triangular cross-sectional circular ring. In this paper we use the discretization shown in Fig. 2.

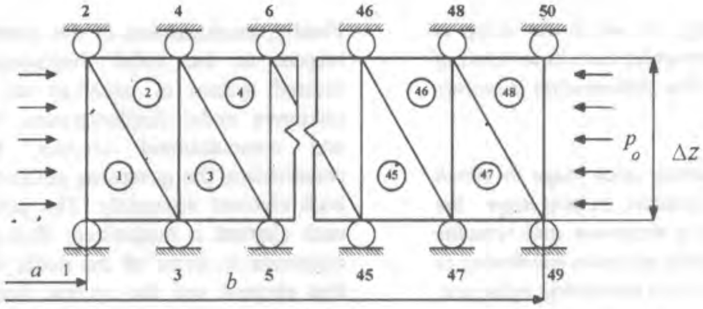

Figure 2 Infinite thick-wall tube discritized by triangular element

The choice of the displacement functions for the element is the prime consideration in the derivation of the element stiffness matrix. The stiffness characteristics are completely specified by the choice of displacements, which directly influence the accuracy of the solution. In general, the number of independent displacement functions to be used should equal the number of degrees of nodal point displacement of the element. The displacement functions are chosen so as to preserve continuity between elements, and there by assuring convergence of the finite element solution to the exact solution as the number of elements are increased.

Consider the triangular element as shown in Fig. 3. The coordinate system for this element is taken as a cylindrical system with the z-axis coincident with the axis of symmetry. The simplest way to satisfy the requirements stated in the preceding paragraph is to take the displacements as linear functions of the coordinates, i.e.

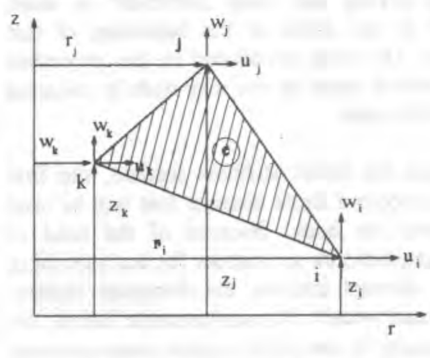

Figure 3 Triangular element

$$\begin{Bmatrix} u(r,z) \\ w(r,z) \end{Bmatrix} = \begin{bmatrix} 1 & r & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r & z \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{Bmatrix}, \text{ or}$$

$$\{\mathbf{u}\} = [\Phi]\{\mathbf{a}\} \tag{1}$$

By evaluating Eq. (1) at each of the three vertices of the triangle, the following relation is obtained

$$\begin{Bmatrix} u_i \\ w_i \\ u_j \\ w_j \\ u_k \\ w_k \end{Bmatrix} = \begin{bmatrix} 1 & r_i & z_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_i & z_i \\ 1 & r_j & z_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_j & z_j \\ 1 & r_k & z_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_k & z_k \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{Bmatrix} \text{ or}$$

$$\{\mathbf{u}\} = [\Gamma]\{\mathbf{a}\} \tag{2}$$

The generalized displacement vector $\{\mathbf{a}\}$ is now expressed in terms of the nodal point displacement by inverting Eq. (2), i.e.

$$\{\mathbf{a}\} = [\Gamma]^{-1}\{\mathbf{u}\} = [\mathbf{H}]\{\mathbf{u}\}, \tag{3}$$

$$[\mathbf{H}] = [\Gamma]^{-1} = \frac{1}{2A} \begin{bmatrix} r_j z_k - r_k z_j & 0 & r_k z_i - r_i z_k & 0 & r_i z_j - r_j z_i & 0 \\ z_j - z_k & 0 & z_k - z_i & 0 & z_i - z_j & 0 \\ r_k - r_j & 0 & r_i - r_k & 0 & r_j - r_i & 0 \\ 0 & r_j z_k - r_k z_j & 0 & r_k z_i - r_i z_k & 0 & r_i z_j - r_j z_i \\ 0 & z_j - z_k & 0 & z_k - z_i & 0 & z_i - z_j \\ 0 & r_k - r_j & 0 & r_i - r_k & 0 & r_j - r_i \end{bmatrix} \tag{4}$$

and, $2A = r_i(z_j - z_k) + r_j(z_k - z_i) + r_k(z_i - z_j)$

The strain-displacement relations in cylindrical coordinates are given by

$$\varepsilon_r = \frac{\partial u}{\partial r},\quad \varepsilon_\theta = \frac{u}{r},\quad \varepsilon_z = \frac{\partial w}{\partial z},\quad \gamma_{rz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \qquad (5)$$

Substituting Eq. (1) into (5) yields

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_{rz} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{r} & 1 & \frac{z}{r} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{Bmatrix} \quad \text{or}$$

$$\{\varepsilon\} = [\mathbf{N}]\{\mathbf{a}\} \qquad (6)$$

For the solution of the creep problem, the strain $\{\varepsilon\}$ is assumed to be composed of the elastic strain, creep strain and thermal strain, i.e.

$$\{\varepsilon\} = \{\varepsilon^{el}\} + \{\varepsilon^{c}\} + \{\alpha\Delta T\} \qquad (7)$$

From equation (7) the elastic strain is given by

$$\{\varepsilon^{el}\} = \{\varepsilon\} - \{\varepsilon^{c}\} - \{\alpha\Delta T\} \qquad (8)$$

The stress-strain relations for an elastic isotropic material is [10]

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \sigma_{rz} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_r^{el} \\ \varepsilon_z^{el} \\ \varepsilon_\theta^{el} \\ \varepsilon_{rz}^{el} \end{Bmatrix} \qquad (9)$$

Substituting Eq. (8) into (9) and using the relation that the creep strains are incompressible yields

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \sigma_{rz} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_{rz} \end{Bmatrix} - \frac{E}{1+\nu} \begin{Bmatrix} \varepsilon_r^{c} \\ \varepsilon_z^{c} \\ \varepsilon_\theta^{c} \\ \varepsilon_{rz}^{c} \end{Bmatrix} - \frac{E\alpha T}{1-2\nu} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\{\sigma\} = [\mathbf{D}]\{\varepsilon\} - \frac{E}{1+\nu}\{\varepsilon^{c}\} - \frac{E}{1-2\nu}\{\varepsilon^{th}\} \qquad (10)$$

The stiffness matrix for the element may now be determined from the theorem of minimum potential energy. The principle states that *of all kinematically admissible configurations, the deformation producing the minimum total potential energy is the stable equilibrium condition.* Therefore:

$$\delta U = 0 \qquad (11)$$

The potential energy U for the element under consideration is:

$$U = \frac{1}{2}\int_V \{\varepsilon^T\}[\mathbf{D}]\{\varepsilon\}dV - \frac{E}{1+\nu}\int_V \{\varepsilon^T\}\{\varepsilon^c\}dV - \frac{E}{1-2\nu}\int_V \{\varepsilon^T\}\{\varepsilon^{th}\}dV - \int_{A_p} \{\mathbf{u}^T\}\{\mathbf{P}\}dA_p \qquad (12)$$

Substituting Eqs. (9), (6) and (12) into (11) gives

$$\delta U = \delta\{\frac{1}{2}\int_V \{\varepsilon^T\}[\mathbf{D}]\{\varepsilon\}dV - 2G\int_V \{\varepsilon^T\}\{\varepsilon^c\}dV - \frac{E}{1-2\nu}\int_V \{\varepsilon^T\}\{\varepsilon^{th}\}dV - \int_{A_p} \{\Phi^T\}\{\mathbf{P}\}dA_p\} = 0 \qquad (13)$$

Taking variations of Eq. (13) with respect to the generalized displacements $\{\mathbf{a}\}$ yields the system of equation

$$[\mathbf{k}]\{\mathbf{a}\} = \{\mathbf{f}\} \qquad (14)$$

where $\{\mathbf{k}\} = \int_V [\mathbf{N}^T][\mathbf{D}][\mathbf{N}]\,dV$, and

$$\{\mathbf{f}\} = 2G\int_V [\mathbf{N}^T]\{\varepsilon^c\}dV + \frac{E}{1-2\nu}\int_V [\mathbf{N}^T]\{\varepsilon^{th}\}dV + \int_{A_p} [\Phi^T]\{\mathbf{P}\}dA_p = \{\mathbf{f}^C\} + \{\mathbf{f}^{th}\} + \{\mathbf{f}^P\}$$

Thus far the stiffness matrix and load vector which have been presented are with respect to the generalized displacements $\{\mathbf{a}\}$. However, it is more convenient to work with the nodal displacements for combining stiffness matrices of adjacent elements. The nodal displacements are related to the generalized displacements through Eq. (3). Substituting Eq. (3) into Eq. (14) and pre multiplying by $[\mathbf{B}^T]$ yields the desired relationship:

$$[\mathbf{B}^T][\mathbf{k}][\mathbf{B}]\{\mathbf{u}\} = [\mathbf{B}^T]\{\mathbf{f}\} \;\; or \;\; [\mathbf{K}^{elm}]\{\mathbf{u}^{elm}\} = \{\mathbf{F}^{elm}\} \qquad (15)$$

where $[\mathbf{B}] = \frac{\partial}{\partial \mathbf{u}}[\mathbf{H}]$

Once the element stiffness matrix and load vector are evaluated, they are assembled into a set of $2 \times nel$ equations of equilibrium for the body, where nel = number of nodal points. Equilibrium requires that all element forces at node i be in equilibrium with the external forces at that node. This is accomplished by adding the equations of all elements associated with the forces at that point, i.e.

$$\{\mathbf{P}\} = \sum_{j=1}^{nel} \{\mathbf{F}^{elm}\}_j \tag{16}$$

Substituting Eq. (15) into (16), and specifying that the displacement at any given node be the same for all elements attached to that node, i.e. from conditions of compatibility,

$$\{\mathbf{P}\} = \sum_{j=1}^{nel} \left[\mathbf{K}^{elm}\right]_j \{\mathbf{u}^{elm}\} \text{ or } [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{P}\} \tag{17}$$

Imposing the boundary conditions, Eq. (17) is solved for the unknown displacement components $[\mathbf{u}]$.

Multi-axial Stress-Strain-Time Relationships

In order to determine the multi-axial creep strain at any point in the body, a stress-strain relation is needed. The multi-axial creep strains can be determined based on the assumptions introduced previously. These assumptions are satisfied by the stress-strain relationships for a cylindrical coordinate system with rotational symmetry [1] given by

$$\begin{aligned}
\Delta\varepsilon_r^c &= \frac{\Delta\varepsilon_e^c}{2\sigma_e}(2\sigma_r - \sigma_\theta - \sigma_z) \\
\Delta\varepsilon_\theta^c &= \frac{\Delta\varepsilon_e^c}{2\sigma_e}(2\sigma_\theta - \sigma_r - \sigma_z) \\
\Delta\varepsilon_z^c &= \frac{\Delta\varepsilon_e^c}{2\sigma_e}(2\sigma_z - \sigma_r - \sigma_\theta) \\
\Delta\varepsilon_{rz}^c &= \frac{3\Delta\varepsilon_e^c}{2\sigma_e}\sigma_{rz}
\end{aligned} \tag{18}$$

where the effective stress, σ_e, is defined as

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_r - \sigma_z)^2 + 6\sigma_{rz}^2} \tag{19}$$

The effective incremental creep strain $\Delta\varepsilon_e^c$ is equivalent to the uniaxial creep strain increment and is obtained from experiments and given by the empirical relationship [1]

$$\varepsilon_e^c = K\sigma_e^n t^q \tag{20a}$$

In general, the effective incremental creep strain is a function of the effective stress σ_e, the total effective strain ε_e^c, the temperature T, the time t, and the strain history of the material, i.e.

$$\Delta\varepsilon_e^c = f\left(\sigma_e, \varepsilon_e^c, T, t\right) \tag{20b}$$

and Eq. (20b) is the element creep law of the material.

SOLUTION TECHNIQUE

Solution Steps

The solution of the creep problem begins with the assumption that if there is no temperature gradient, the total change in strain during a time interval is the sum of the changes in elastic and creep strain, i.e., $\{\Delta\varepsilon\} = \{\Delta\varepsilon^{el}\} + \{\Delta\varepsilon^c\}$, thus the change in the elastic strain is

$$\{\Delta\varepsilon^{el}\} = \{\Delta\varepsilon\} - \{\Delta\varepsilon^c\} \tag{21}$$

The incremental stresses are then related to the elastic strains by the general Hooke's law

$$\{\Delta\sigma\} = [\mathbf{D}]\{\Delta\varepsilon^{el}\} \tag{22}$$

Substituting Eq. (21) into Eq. (22) yields the relation between the incremental stress, total strain and creep strain

$$\{\Delta\sigma\} = [\mathbf{D}]\{\{\Delta\varepsilon\} - \{\Delta\varepsilon^c\}\} \tag{23}$$

The other field equations are obtained from the relationship between strain and deformation, the creep law of the material, and from the minimization of the potential energy of the body. These equations may be expressed as

$$\{\Delta\varepsilon\} = [\mathbf{B}]\{\Delta\mathbf{u}\} \tag{24}$$

$$\{\Delta\varepsilon^c\} = \{f(\sigma_e, t, T)\} \tag{25}$$

$$\{\Delta\mathbf{u}^c\} = [\mathbf{K}]^{-1}[\mathbf{F}^c] \tag{26}$$

To solve the creep problem, first the elastic stresses are determined at time $t = 0$. These stresses are assumed to remain constant during a small time increment, Δt, and the incremental creep strains are calculated from Eq. (25). The creep strains are substituted into Eq. (26) to find the total change of the nodal point displacements. These displacements are substituted into Eq. (24) to determine the total change in strain. Finally, the incremental stresses are obtained from Eq. (23) and are added to the previous stresses to yield the new stress distribution $\{\sigma_{t+\Delta t}\} = \{\sigma_t\} + \{\Delta\sigma_t\}$. As long as the incremental stresses, $\{\Delta\sigma_t\}$, are small compared to the previous stresses, $\{\sigma_t\}$, no basic assumptions are violated and the solution proceeds to the next time increment using the same procedure. If, however, the incremental stress is not small compared to the previous stresses, the time step can be repeated with a smaller time increment.

NUMERICAL EXAMPLE AND DISCUSSION OF RESULTS

To illustrate the method of solution developed in the paper, a numerical example is presented. The boiler tube material is 12CrMoV, [DIN X22CrV12I] having the following properties:

$$\varepsilon_e^c = K\sigma_e^n t^q$$

where $K = 19.64\times10^{-44}$, $n = 4.4$ and $q = 1.0$;

$$2a = 100mm,\ 2b = 150mm;$$

$$\nu = 0.33,\ E = 207GPa \text{ and } \alpha = 6.2\times10^{-6}$$

The loading condition considered is:

$T = 500°C$; $P = 100MPa$;

The stresses and creep strains for the load conditions considered are displayed in the output. Fig. 4 shows the elastic stress distributions, and we can see that the FE solution is in agreement with the analytical solution. Fig. 5 shows the dependence of the effective strain on time. From the diagram we can observe clearly when the effective strain reaches the limiting value. For most boiler tube materials the recommend strain after 10,000 hours of operation is about 1% [4]. Fig. 6 shows the total effective strain distribution for some selected time values. In Fig. 7 the contribution of the strain components at the selected time of 10,000 hours is shown. Fig. 8 shows the total strain for a selected radial position of $r/a = 1.25$, and as expected the strains increases with increasing time. Finally the comparison of the elastic stress and the steady state stress distribution, which includes the effect of creep and temperature, is shown in Figs. 9.

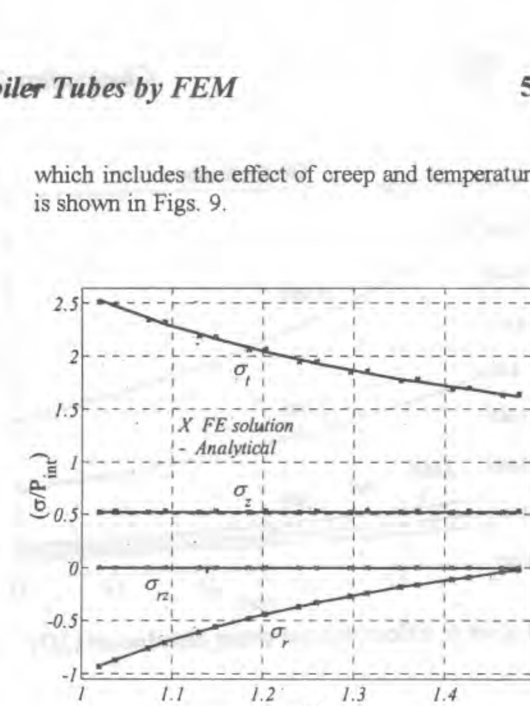

Figure 4 Elastic stress distribution

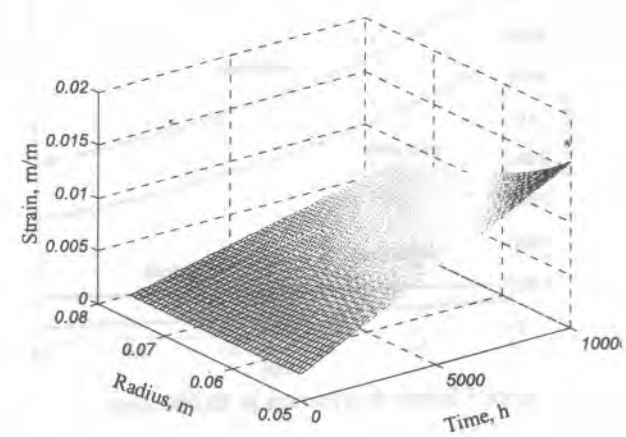

Figure 5 Elastic stress distribution

and the thermal strains remain more or less constant as long as there are no temperature and pressure variations. As can easily be noted from Fig. 7, at an elevated temperature, the strain component due to creep is more dominant. While the tube stretches due to the increase in total strain the stresses also change. The stress distributions are changed due to relaxation as shown in Fig. 9.

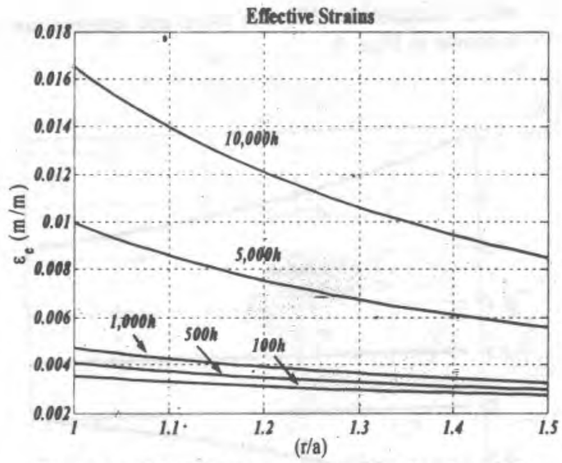

Figure 6 Effective total strain distribution (2D)

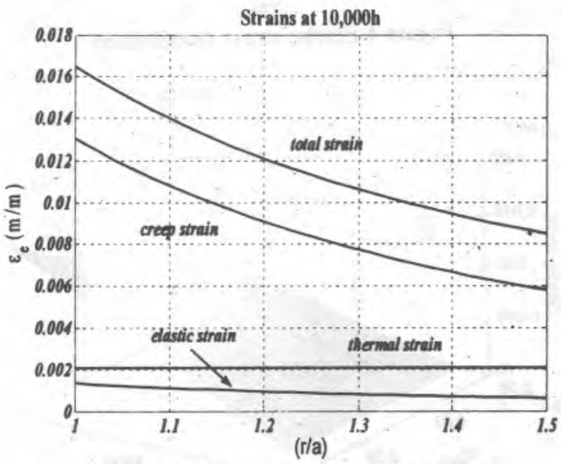

Figure 7 Strain distribution at 10,000 hour

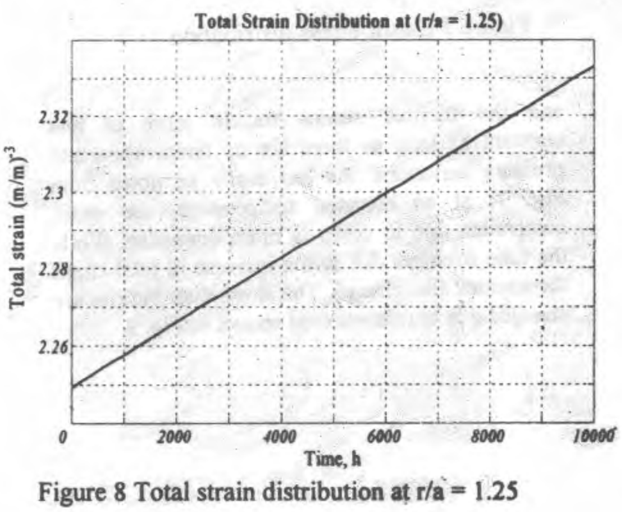

Figure 8 Total strain distribution at r/a = 1.25

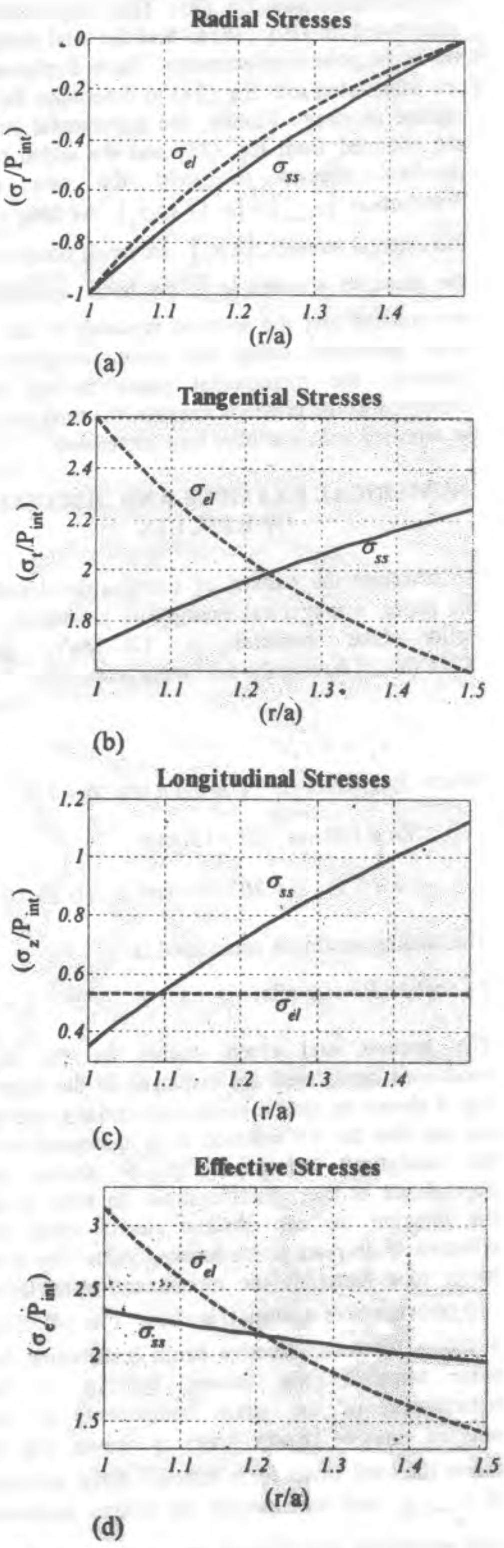

Figure 9 Elastic and relaxed stress distribution

CONCLUSION

In this paper we have treated creep problem of boiler tubes and have developed a finite element approach to solve the problem. In our analysis we have considered the surface of the tube to be clean with no crack initiated nor pitting formed on the surface. The results obtained compare favorably with the recommend strain values which gives strain values below 5% for 10000 hours of operation.

The method we presented here is comprehensive in the sense that material nonlinearity, loading nonlinearity and geometric complexities can easily be handled as long as the finite element mesh can be generated successfully. Since the method makes it possible to study creep behavior of complex geometries, the creep analysis of cracked tubes, corroded or eroded and/or pitted boiler tubes can be analyzed by developing similar finite element formulation of the problem. If actual boiler tubes considered, soot deposit, scale formation and cracks are commonly encountered on the surface of tubes. Transverse and longitudinal cracks and swelling of tubes due to localized heating are practically encountered. Thus, we think the method we proposed in this paper paves the way for further investigation and research for the cases that may be encountered in practice.

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