SYNTHESIS OF UNEQUALLY SPACED LINEAR ANTENNA ARRAYS

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ABSTRACT

In antenna synthesis an approximate analytical model is chosen to represent, either exactly or approximately, the desired radiation pattern and then the model is realized by an antenna model. This process is most conveniently achieved by antenna arrays. For antenna array consisting of identical elements, the major factors that determine its radiation characteristics are the inter-element spacing, the elements' excitation amplitudes and phases. Commonly, antehna array synthesis is carried by making the inter-element spacing constant and only varying either the excitation amplitude and/ or phase.

In this paper, antenna synthesis is achieved by varying all the array parameters- inter-element spacing, excitation amplitudes and excitation phases. The results show that the method is more efficient in synthesis accuracy and minimization of sidelobes. Also the method needs fewer number of array elements to synthesize an array exhibiting the same (or better) level of directivity and sidelobe level in comparison with uniform arrays. Reduction in number of array elements has a positive consequence in manufacturing cost effective arrays.

INTRODUCTION

The radiation pattern of a single element antenna (eg., a dipole) is relatively wide, and each element provides low value of directivity (gain). In many applications it is necessary to design antennas with very directive characteristics. This can only be accomplished by increasing the electrical size of the antenna. Enlarging the dimensions of single element often leads to more directive characteristics. Another way of enlarging the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multi-elements, is referred to as an *array*. In most cases, the elements of an array are identical. This is not necessary, but it is often convenient, simpler, and more practical. The individual elements of an array may be of any form (wire, apertures, etc.).

The total field of the array is determined by the vector addition of the fields radiated by the individual elements. To provide very directive patterns, it is necessarily that the fields from the elements of the array interfere constructively in the desired directions and interfere destructively in the remaining spaces.

In an array, there are five controls that can be used to shape the overall pattern of the antenna. These are: [1]

1. The geometrical configuration of the overall array (liner, circular, rectangular, spherical, etc.),
2. The relative displacement between the elements,
3. The excitation amplitude of the individual elements,
4. The excitation phase of the individual elements, and
5. The relative pattern of the individual elements.

Our study focuses on linear arrays of identical elements.

In the already established analysis and synthesis methods, the arrays are assumed to have:

1. Uniform inter-element spacing ,
2. Uniform progressive phase shift (constant phase difference between adjacent elements),
3. Either uniform or non-uniform excitation current.

The objective of the present work is synthesizing linear antenna arrays in which the above listed restrictions are lifted. Simultaneously varying *all* the array parameters (element position, excitation current and excitation phase) will make the synthesized array an efficient and cost-effective one.

REVIEW OF PREVIOUS WORKS ON UNEQUALLY SPACED ANTENNA ARRAYS

Over the past six decades, the theory of uniformly spaced antenna arrays have been studied in depth and is certainly well documented [1]-[6]. A considerable amount of work [7] has also been done on the synthesis of equally spaced linear arrays. For example, given a desired radiation pattern (e.g., pencil-beam, sectorial, $cosec^2$, etc.), inter-element spacing and the number of elements, it is possible to employ such traditional synthesis procedures as Binomial, Dolph-Chebyshev, Fourier inversion, Woodward-Lawson or numerical optimization to obtain the required array current distribution for a uniformly spaced array.

Spacing the elements unequally along a straight line in order to obtain better performance (e.g., sidelobe reduction) is relatively a younger field in the array synthesis theory. Probably the first work on unequally spaced arrays has been carried out by Unz [8], who developed a matrix formulation to obtain the current distribution necessary to generate a prescribed radiation pattern from unequally spaced linear arrays. Sahalos [9] further extended Unz's method to synthesize three-dimensional arrays. Ma [10] has used mini-max criterion and has applied the technique of approximation theory while Ishimiru [11] has described a theory of unequally spaced arrays based on the Poisson's summation formula. Ishimiru's classical analysis addressed the following aspect [20]: 1) sidelobe reduction in comparison with a linear array with uniform excitation; 2) secondary beam suppression of the linear array by use of the Anger function; and 3) azimuthal frequency scanning by means of an unequally spaced circular arrays. Mautz and Harrington [12] described some general numerical methods for pattern synthesis of *randomly spaced*[1] elements with and without constraints on the source norm. The problem of choosing the element positions than specifying them has been tackled by Perini and Idselis [13] by making use of the steepest descent technique. Redlich [14] has made use of a perturbation technique to arrive at a set of element positions and the corresponding currents, which minimize the mean-squared error between the desired and the synthesized pattern. Murth and Kumar [15] discussed array synthesis employing the L_2-norm as well as the L_∞-norm. They obtained approximation to the L_∞-norm making use of Lawson's algorithm. They also evolved a general iteration perturbation technique for pattern synthesis for the case when the antenna currents are alone varied and as well for the case when both the antenna current amplitudes and the element positions are simultaneously varied. The convergence of the iteration and the uniqueness of the solution were discussed. Arora, Daniel and Terrat [16] addressed the problem of synthesizing a small antenna array by using the locations and phases of the elements as variable while keeping the excitation amplitude constant. They used a combined Gauss-Newton and modified Newton iteration schemes in their optimization routine.

For thinned array class of design, Skolnik [17] employed dynamic programming. This approach stresses the power of computer aided optimization tools to design unequally spaced arrays by treating the goal of the sidelobe reduction as an objective function and the limits in the placement of adjacent elements as optimization constraints. In this approach, the density of elements located within a given aperture is made proportional to amplitude distribution of the conventional equally spaced arrays. The latter method was extended by Mailloux and Cohen [18] who utilized statistical thinning of arrays with quantized element weights to improve sidelobe performance in large circular arrays.

Recent studies include that of Haupt [19] for thinned linear and planar array design using genetic algorithms. Kumar and Branner [20] also discussed the synthesis of unequally spaced arrays utilizing an inversion algorithm to obtain the element spacing from prescribed far-zone electric field and current distributions, or current distributions for prescribed far-zone electric field and element spacing.

THE NEED FOR A NEW WORK

Pattern synthesis using unequally spaced arrays still present a challenging problem having considerable practical advantages. These include minimization of sidelobes and reduction in the number of elements of the array. The design of non-uniformly spaced array is more challenging than uniformly spaced design based on several considerations.

- Since the element spacing occurs as exponential or trigonometric functions, element spacing synthesis is a *nonlinear* problem whereas the array current synthesis is a *linear* problem.

[1] Non-uniform arrays can be classified as *randomly spaced* and *thinned* arrays, the later derived by selectively zeroing some elements of an initial equally spaced arrays.

Constraints have to be placed on the solutions for the element spacing; viz., they *cannot be complex numbers* and the array element positions should be separated enough to reduce the array element coupling.

The works done so far address the problem of pattern synthesis using unequally spaced arrays *either by keeping the excitation amplitudes or the excitation phases or both as constants* (e.g., see [15], [16] and [20]). In our work these restrictions have been avoided, which means the element positions, excitation amplitudes and excitation phases are simultaneously employed as synthesis parameters. The results obtained highly confirm the claims made earlier, viz., minimization of sidelobes and reductions in the number of elements of the array.

In the present work a general method is developed wherein *all* the array variables (element positions, excitation amplitudes and excitation phases) are used as synthesis variables. The problem reduces to an unconstrained nonlinear least-squares minimization where the objective function to be minimized is the mean-squared synthesis error between the desired and the synthesized pattern. The Nelder and Mead nonlinear simplex algorithm [21] and exhaustive search algorithm are used in solving the optimization problem.

GENERAL FORMULATION OF THE ARRAY PATTERN SYNTHESIS AND ITS SOLUTION

The general relation between the source **f** of a radiating system and the field pattern **g** that it generates can be expressed in abstract vector space as:

$$T\mathbf{f} = \mathbf{g}, \qquad (1)$$

where T is a linear operator.

The synthesis problem may be stated thus: Given a desired field $\mathbf{g_o}$, to determine a source distribution **f** whose field **g** approximates $\mathbf{g_o}$ in some acceptable sense.

In the array context, (1) takes the form:

$$\mathbf{e}(\theta,\phi)\sum_{n=1}^{N} I_n e^{j\alpha_n} e^{jk(\mathbf{a}_r\cdot\mathbf{r}_n)} = \mathbf{g}_0(\theta,\phi), \qquad (2)$$

where:

$\mathbf{e}(\theta, \phi)$ the element pattern
I_n amplitude excitation of the n^{th} element
α_n excitation phase of the of the nth element
$\mathbf{r}_n$ the position vector of the nth element
$\mathbf{a}_r$ unit vector in the direction of observation
N the number of elements in the array, and
$k = 2\pi/\lambda$. wave number.

Let $\mathbf{G}(\theta,\phi)=\mathbf{g}(\theta,\phi)/\mathbf{e}(\theta,\phi)$ be specified at M different angles (θ_1,ϕ_1), (θ_2,ϕ_2),…,(θ_M,ϕ_M). Then (2) may be written in matrix notation as

$$\begin{bmatrix} e^{jk(\mathbf{a}_{r_1}\cdot\mathbf{r}_1)} & e^{jk(\mathbf{a}_{r_2}\cdot\mathbf{r}_1)} & \cdots & e^{jk(\mathbf{a}_{r_N}\cdot\mathbf{r}_1)} \\ e^{jk(\mathbf{a}_{r_1}\cdot\mathbf{r}_2)} & e^{jk(\mathbf{a}_{r_2}\cdot\mathbf{r}_2)} & \cdots & e^{jk(\mathbf{a}_{r_N}\cdot\mathbf{r}_2)} \\ \vdots & \vdots & \vdots & \vdots \\ e^{jk(\mathbf{a}_{r_1}\cdot\mathbf{r}_M)} & e^{jk(\mathbf{a}_{r_2}\cdot\mathbf{r}_M)} & \cdots & e^{jk(\mathbf{a}_{r_N}\cdot\mathbf{r}_M)} \end{bmatrix} \begin{bmatrix} I_1 e^{j\alpha_1} \\ I_2 e^{j\alpha_2} \\ \vdots \\ I_N e^{j\alpha_N} \end{bmatrix} = \begin{bmatrix} G_0(\theta_1,\phi_1) \\ G_1(\theta_2,\phi_2) \\ \vdots \\ G_0(\theta_1,\phi_1) \end{bmatrix}, \qquad (3)$$

or concisely as

$$[\mathbf{B}][\mathbf{f}] = [\mathbf{G}_0], \qquad (4)$$

where $[\mathbf{B}]$ is an M x N matrix and $[\mathbf{f}]$ and $[\mathbf{G}_o]$ are N x 1 and M x 1 matrices, respectively, and M > N. Since an exact solution is not to be expected for such a problem, we should seek to minimize the mean-squared error $\|\mathbf{G}_0 - \mathbf{Bf}\|^2$. The equation is a type of unconstrained non-linear least-squares minimization problem of a function of several variables.

Since we are considering linear arrays, Eq. (2) reduces to

$$\mathbf{e}(\theta_m)\sum_{n=1}^{N} I_n e^{j\alpha_n} e^{jkd_n\cos\theta_m} = \mathbf{g}_0^*(\theta_m), \qquad (5)$$

where m= 1, 2, …, M. n = 1,2, …, N.

The corresponding matrix formulation becomes

$$\begin{bmatrix} e^{jkd_1\cos\theta_1} & e^{jkd_2\cos\theta_1} & \cdots & e^{jkd_N\cos\theta_1} \\ e^{jkd_1\cos\theta_2} & e^{jkd_2\cos\theta_2} & \cdots & e^{jkd_N\cos\theta_2} \\ \vdots & \vdots & \vdots & \vdots \\ e^{jkd_1\cos\theta_M} & e^{jkd_2\cos\theta_M} & \cdots & e^{jkd_N\cos\theta_M} \end{bmatrix} \begin{bmatrix} I_1 e^{j\alpha_1} \\ I_2 e^{j\alpha_2} \\ \vdots \\ I_N e^{j\alpha_N} \end{bmatrix} = \begin{bmatrix} G_0(\theta_1) \\ G_1(\theta_2) \\ \vdots \\ G_0(\theta_1) \end{bmatrix} \qquad (6)$$

Where d_n is the distance in wavelengths of the nth element from the center of the array, θ_m is the mth sample point elevation angle, n= 1,…,N and m=1,…,M. The linear array is symmetric in the azimus plane.

Solution to Eq. (6)

Since solving for the d_n , I_n *and* α_n's in closed form is difficult, we propose an iterative scheme.

Let:

$\theta = (\theta_1, \theta_2, ..., \theta_M)$, where θ_m is the mth sample angle

$\mathbf{d} = (d_1, d_2, ..., d_N)$, where d_n is the nth element position from z=0 on the z-axis.

$\mathbf{I} = (I_1, I_2, ..., I_N)$, where I_n is the nth element excitation amplitude.

$\alpha = (\alpha_1, \alpha_2, ..., \alpha_N)$, where α_n is the nth element excitation phase.

$E = \|\mathbf{G}_o - \mathbf{Bf}\|^2$, the mean-squared error between the desired and the synthesized pattern. It is the objective function to be minimized in the optimization routine.

$m = 1,2, ..., M$.

$n = 1,2, ..., N$.

The steps taken to solve Eq. (6) are:

1. Start with initial uniformly spaced configuration $\mathbf{d_o}$.
2. Compute **B** using θ and $\mathbf{d_o}$.
3. Solve for **f**.

   I = magnitude of **f**, and α = angle of **f**.

   Since M>N, i.e. over-determined simultaneous equation, we seek the least mean-squares solution[2]. Here is a useful simplification- for a given **d** there is one and only one best set of **I** and α in the least mean-squared error sense. In effect, it implies that **I** and α are taken out of the optimization routine. In a 15-element array, for example, only the 15 components of **d** are used in the optimization routine. Had it not been, the optimization algorithms employed - such as the simplex -would have to handle 45 variables. But now only 1/3 of the variables run into the optimization routine while the rest (**I**, α) are handled by Step 3.
4. Compute E using **B**, **f** and $\mathbf{G}_o$. This is the objective function to be minimized.
5. Use the optimization routine-simplex or golden section search and parabolic interpolation algorithms- to generate a new set of **d** which minimized E. The function which computes the corresponding optimum set of **I** and α and then E for the given **d** (steps 2-4) is called within the optimization routine..
6. Repeat steps 5 until E is less than the specified tolerance or the maximum number of function evaluations has been exceeded.
7. Take the values of **d**, **I** and α in the last iteration to be the optimal values.

Problem of Getting Global Minima and the Exhaustive Search Algorithm

The well-known problem of falling into local minima is to be expected here. As the number of variables keeps on increasing, the local minima get on exploding. Though the Nelder and Mead nonlinear simplex algorithm is strong at escaping local minima, there is no guaranty of getting global minima especially in areas where the number of variables is large as in array synthesis. Not only this- the number of iterations taken to converge runs into hundreds. To alleviate this problem, the exhaustive search algorithm is employed. It consists of evaluating the objective function at a predetermined number of equally spaced **d**'s in the interval $\mathbf{d_s}$ and $\mathbf{d_f}$, where the **d**'s are equally spaced. The **d** corresponding to the minimum objective function E is used as the initial $\mathbf{d_o}$ for the optimization iteration. This process guarantees global minimum and moreover the iteration step considerably drops. For example, a given problem which takes about 200 iterations and falling into local minima can be improved to 40 iterations with global minimum assurance.

RESULTS

Using all the array variables (**d**, **I** and α) in pattern synthesis of unequally spaced linear arrays highly confirm the claims made earlier-viz., minimization of sidelobes, reductions in number of elements of the array. Some examples will be considered.

1. The classical design methods such as Binomial, Dolph-Chebyshev, Fourier inversion and Woodward-Lawson are fit only for a broadside arrays. But using our synthesis method, a broadside beam can be synthesized with much

[2] For demonstration purpose M is taken to be 200. For 10 element array **B** will be 200x10 matrix. The unknown variables are 10 but with 200 simultaneous equations.

lower level of sidelobes. As an illustration, the synthesized pattern of:

$$G(\theta)=\begin{cases}1, & \frac{\pi}{2}\le\theta\le\frac{3\pi}{4}\\ 0, & \text{otherwise}\end{cases}$$

using Woodyard-Lawson with 10 elements results in a mean-squared error of 0.02 and sidelobe level of −13.5 dB (Fig. 1a), while using our method with the same number of elements results in a mean-squared error of 0.01 and sidelobe level of ~ − 24 dB (Fig. 1b.). As can be seen from the figures, there is a marked difference in sidelobe levels.

2. Arora and others [16] synthesized the sidelobe free pattern

$$G(\theta)=\begin{cases}\left|\frac{\sin(2\pi\cos\theta)}{2\pi\cos\theta}\right|, & \frac{\pi}{3}\le\theta\le\frac{2\pi}{3}\\ 0, & \end{cases}$$

otherwise

using 5 elements with mean-squared error of 0.00426 and sidelobe level of ~ −13.5 dB, but in our case with the same number of elements the mean-squared error drops to 0.000378 (91.1 % improvement) and sidelobe level of ~ − 38 dB (see Fig. 2.)

3. An interesting topic among array pattern synthesizers is synthesizing unequally spaced arrays with reduced sidelobe in comparison with a linear array of uniform excitation and spacing. Kumar and Branner [20] were able to reduce peak sidelobe levels by ~6.5 dB in comparison with uniformly excited pencil-beam arrays and up to ~7 dB for flat-top beam arrays while essentially maintaining the same beamwidth. Using genetic algorithms [19] yields ~-20 dB. Again our results are highly successful in this regard. For instance, a broadside array of 30 elements and uniform inter-element spacing 0.25λ has sidelobe level of ~ −13.5 dB and directivity of 15.1611. But in our case keeping the directivity the same and reducing the sidelobe level to ~ -23 dB, only 10 elements are needed with a mean-squared error of 0.000978 (see fig. 3). The number of elements has been reduced to a third to that of a uniform array implying that the method is cost effective.

4. Also an end-fire array having 40 elements and uniform inter-element spacing 0.25λ has sidelobe level ~ −13.5 dB and directivity 40.0672. But in our case with much higher directivity and sidelobe level of ~ -24 dB, only 8 elements are needed with a mean-squared error of 0.011479 (see fig. 4). Note also the reduction in the number of elements (40 viz 8) as in the above result.

Table 1: Summary of the above results

Result No.	Array Type (method)	Number of Elements	Mean Squared Error (MSE)	Directivity	Sidelobe Level (dB)
1	Woodward-Lawson	10	0.02	-	-13.5
	Our Method	10	0.01	-	-24.0
2	Arora	5	0.00426	-	-13.5
	Our Method	5	0.000378	-	-38.0
3	Broadside (uniform)	30	-	15.1611	-13.5
	Our Method	10	0.000978	15.1611	-23.0
4	End-fire (uniform)	40	-	40.0672	-13.5
	Our Method	8	0.011479	>40.0672	-24.0

Synthesis of Unequally Spaced Linear Antenna Arrays

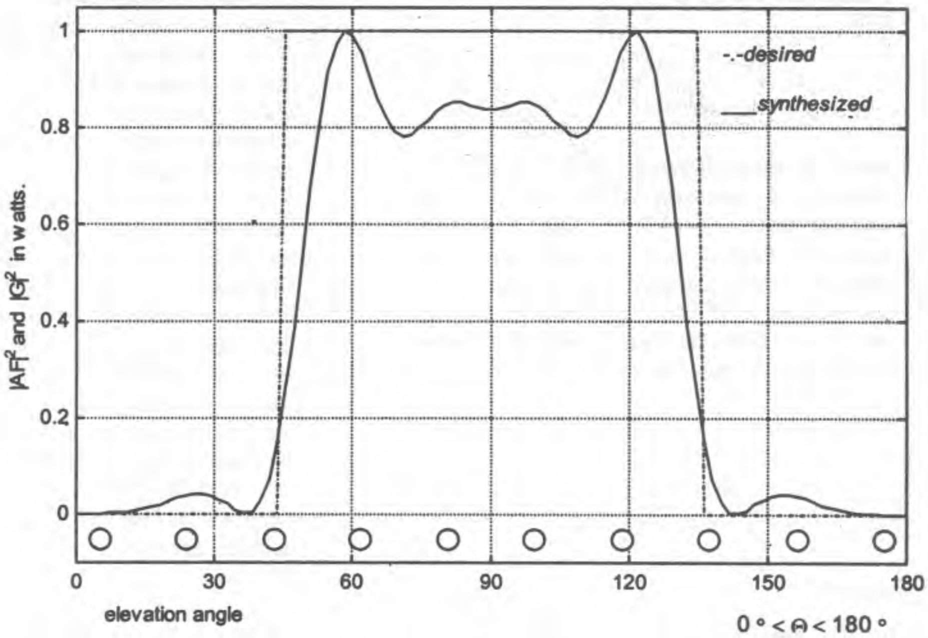

Figure 1a Woodward-Lawson synthesis, MSE[3]= 0.02. N=10, $d = 0.25\lambda$. Plot of $|AF|^2$ in watt vs. θ

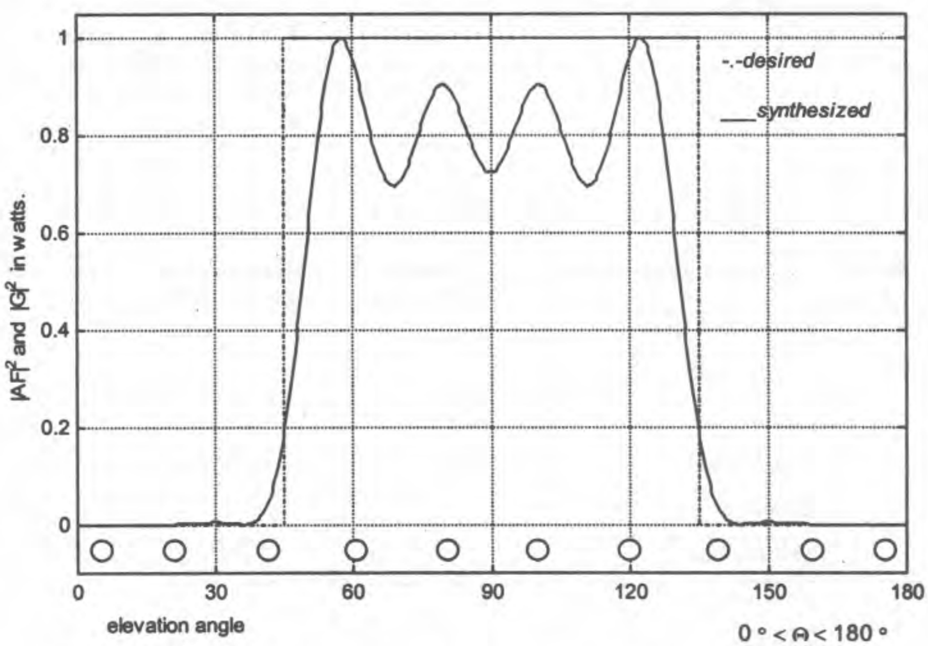

Figure 1b Synthesis using our program, MSE= 0:01.,N=10. Plot of $|AF|^2$ in watt vs. θ

[3] MSE- *M*ean *S*quared *E*rror

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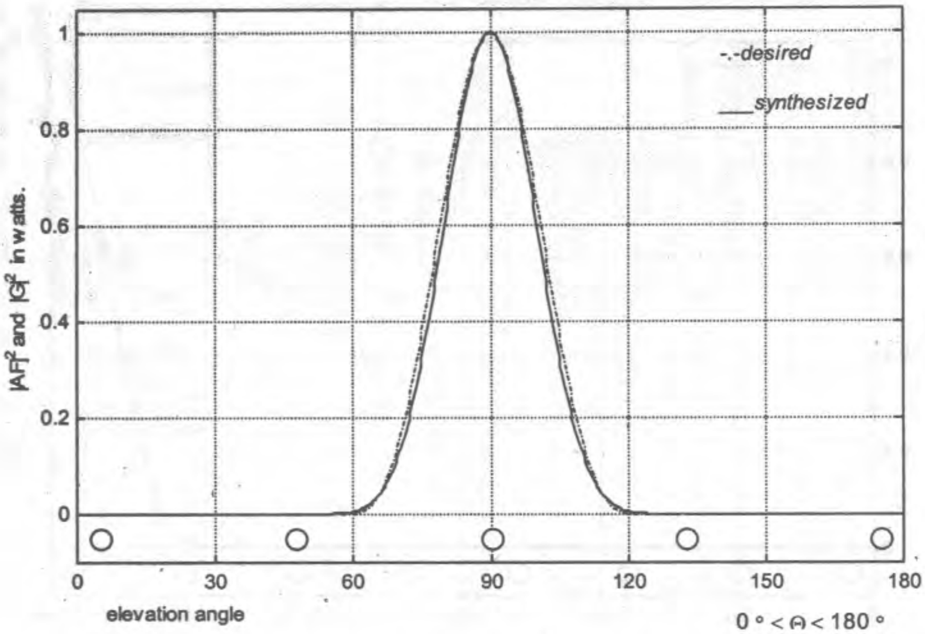

Figure 2 Synthesis using our program, MSE= 0.000378, N=5. Plot of $|AF|^2$ in watt vs. θ

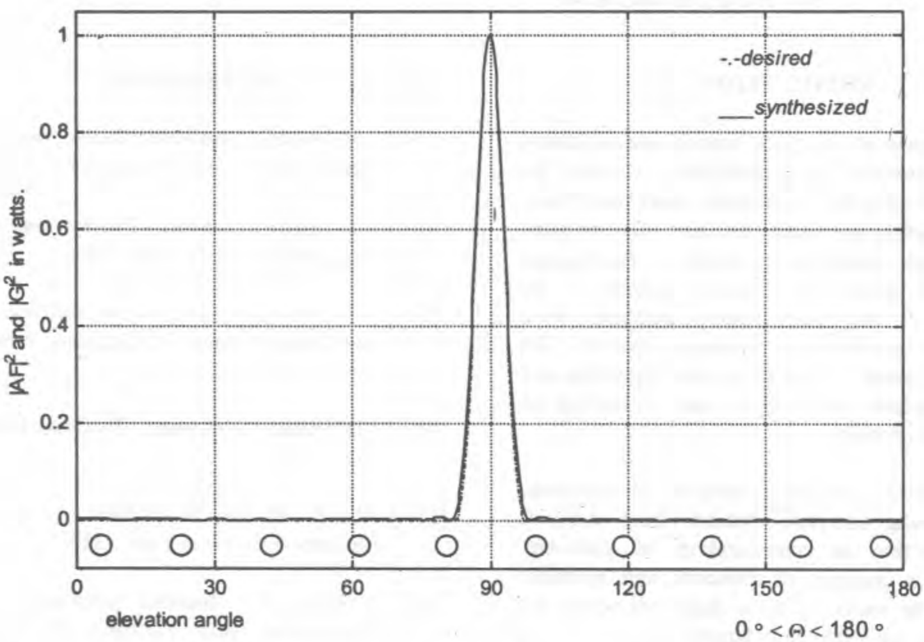

Figure 3 Synthesis using our program, MSE= 0.000978,N=10. The desired broadside array pattern has N=30. Plot of $|AF|^2$ in watt vs. θ

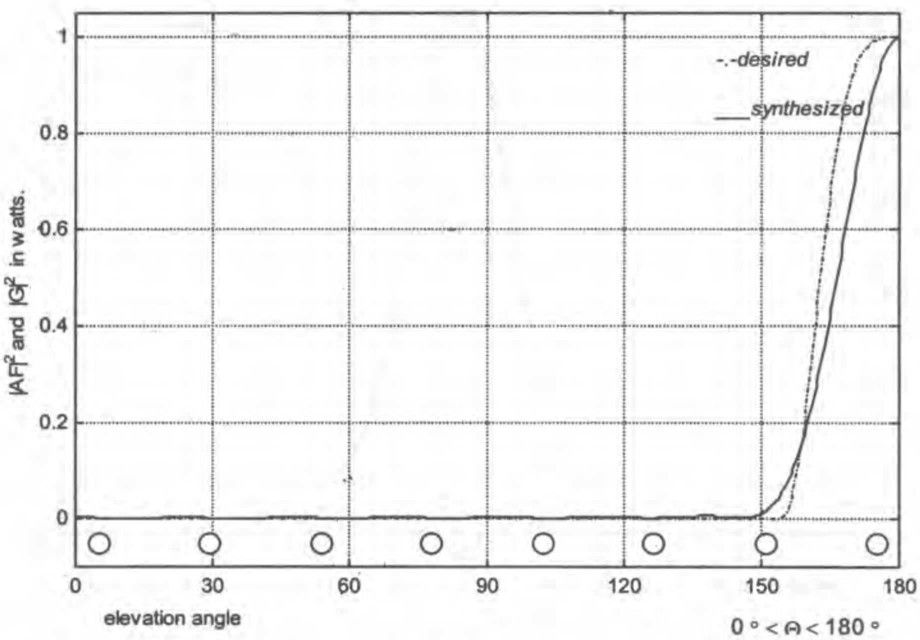

Figure 4 Synthesis using our program, MSE= 0.011479,N=8. The desired end-fire array has N=40. Plot of $|AF|^2$ in watt vs. θ

CONCLUSION

In the synthesis of unequally spaced antenna arrays a general method was developed wherein the excitation amplitudes, excitation phase and inter-element spacing are made to vary. The problem reduces to an unconstrained nonlinear least-square minimization where the objective function to be minimized is the mean-squared synthesis error between the desired and synthesized pattern. The Nelder and Mead nonlinear simplex algorithm and exhaustive search algorithm are used in solving the optimization problem.

As the above sample examples demonstrate, synthesis using unequally spaced linear arrays is highly effective in minimization of sidelobes, reductions in number of elements and synthesis accuracy. The method can be employed where the above factors are a stressing demand.

REFERENCES

[1] C.A. Balanis, *Antenna Theory: Analysis and Design*, Wiley, New York, 1997.

[2] R.S. Elliot, *Antenna Theory and Design*, Prentice-Hall, New York, 1984.

[3] S. Silver (Ed.), *Microwave Antenna Theory and Design*, Dover Publications, New York, 1965

[4] J.D. Kraus, *Antennas*, McGraw-Hill, New York, 1950.

[5] W.L. Weeks, *Antenna Engineering*, McGraw-Hill, New York, 1968.

[6] Y.T. Lo (ed.), *Antenna Handbook – Theory, Application and Design*, Van Nostrand Reinhold Company, New York, 1968.

[7] R.E. Collin and Zucker, *Antenna Theory – Part I*, McGraw-Hill, New York, 1969.

[8] H. Unz, *Linear arrays with arbitrary distributed elements*, IEEE Traps. Antennas Propagat., vol. AP-8, pp. 222-223, Mar. 1960. Quoted by [15] and [20].

[9] J. Sahalos, *A solution of the general non-uniformly spaced antenna array*, Proc. IEEE, vol. 54, pp.53-54, Jan. 1966.

[10] M.T. Ma and L.C. Walters, *Synthesis of non-uniformly spaced antenna arrays yielding approximately equal sidelobes*, Radio Sci., vol. 1, pp.601-608, May 1966. Quoted by [15].

[11] A. Ishimiru, *Theory of unequally-spaced arrays*, IEEE Trans. Antennas Propagat., vol. AP-10, pp. 691-702, Nov. 1962. Quoted by [15].

[12] J.R. Mautz and R.F. Harington, *Computational methods for antenna pattern sysnthesis*, IEEE Trans. Antennas Propagat., vol. AP-23, no. 4, p. 507, July 1975.

[13] J. Perini and M. Idseli, *Note on antenna pattern sysnthesis using numerical iterative methods*, IEEE Trans. Antennas Propagat., vol. 19, pp. 284-286, March 1971.

[14] R.W. Redlich, *Iterative least square synthesis of non-uniformly spaced linear arrays*, IEEE Trans. Antennas Propagat., vol. AP-21, no. 1, pp. 106-108, Jan. 1973.

[15] P.K. Murthy and A. Kumar, *Synthesis of linear arrays*, IEEE Trans. Antennas Propagat., pp. 865-870, Nov. 1976.

[16] Rajendra K. Arora, J.P. Daniel, C.Terret, *Synthesis of linear arrays with unequally spaced elements and identical amplitude excitation*, IEEE Trans. Antennas Propagat., vol. 1, pp. 101-105, 1985.

[17] M.I. Skolnik, G. Nemhauser, and J.W. Sherman, III, *Dynamic programming applied to unequally spaced arrays*, IEEE Trans. Antenna Propagat., vol. AP-11, pp. 35-43, Jan. 1964.

[18] R. J. Mailloux and E. Cohen, *Statically thinned arras with quantized element weights*, IEEE Trans. Antennas and Propagation, vol. 39, pp. 436-477, Apr. 1991.

[19] R.L. Haupt, *Thinned arras using genetic algorithms*, IEEE Trans. Antennas and Propagation, vol. 42, pp. 993-999, July 1994.

[20] B. Preetham Kumar and G.R. Branner, *Design of unequally spaced arrays for performance improvement*, IEEE Trans. Antennas and Propagation, vol. 47, no. 3, pp. 511-523, March 1999.

[21] Nelder, J, A. and R. Mead, *A simplex method for function minimization*, Computer Jornal, Vol. 7, pp. 308-313, 1965.