TECHNICAL NOTES

SEISMIC SOIL-STRUCTURE INTERACTION AS A POTENTIAL TOOL FOR ECONOMICAL SEISMIC DESIGN OF BUILDING STRUCTURES

Asrat Worku[1]
School of Civil and Environmental Engineering
Addis Ababa Institute of Technology, Addis Ababa University
(Currently Operations Manager at Gibb International, Nairobi, Kenya)

ABSTRACT

Noting that contemporary seismic design codes have become more and more demanding in terms of requirements related to design forces and deformations for buildings, this paper attempts to demonstrate that it could be prudent to consider the introduction of soil-structure interaction (SSI) provisions into local codes in order to potentially offset the costs incurred by the high demand for base shear attributed to site amplifications by soft soil sites as per current code requirements. This mostly beneficial effect of site soils is as a result of lengthening of the fundamental period and of the mostly increased effective damping of the overall system due to SSI. After introducing the basic concepts of dynamic SSI, the paper demonstrates that if SSI provisions in some international codes are properly adapted, a substantial reduction in the base shear force could potentially be achieved so that the sizes of structural elements would also be proportionally less. With this, the paper attempts to address the legitimate concerns of many design engineers regarding the likely escalation of construction costs associated with the stringent requirements of contemporary seismic design spectra for soil sites that are expected to be introduced in the Ethiopian seismic code currently under revision.

Keywords*: soil-structure interaction, fixed-base structure, flexible-base structure, period lengthening, effective damping, base shear, site amplification*

INTRODUCTION

Site geotechnical condition is one of the three most important factors that impact the intensity of ground shaking due to earthquakes, the other two being source conditions and wave-path geology. Site effect is studied following one of two approaches. The first one is empirical and is based on comparison of recorded ground motions at nearby rock and soil sites of known geotechnical characteristics, if these are available. The results of such studies are presented in form of design spectra for different site-soil classes. These spectra are factored forms of the basic design spectrum for rock at the same site. The amplification factors are generally dependent on the nature of the site and the seismicity of the region. In the absence of recorded ground motions for a given seismic region, design spectra from regions of similar geological and tectonic setup are adapted.

The second approach is appropriate for site-specific studies and involves the modeling of the site soil like any other dynamic system subjected to the ground motion at the interface with the rock. The soil can be modeled as a continuous system, as a single-degree-of freedom (SDOF) system or even as a multi-degree-of-freedom (MDOF) system. The end result could be ground motion time histories, peak ground motions or response spectra.

Since this effect of site soils mostly amplifies the rock-level ground motion, it is regarded as detrimental to the structure founded on the soil.

Another important influence of site soils on structures is related to soil-structure interaction (SSI). When the ground motion, amplified by the site soil in the manner described above, strikes the foundation, two forms of SSI take place. The first is attributed to the difference in rigidity between the foundation unit and the soil, which causes reflection and refraction of the seismic waves back into the soil mass. As a consequence, the motions of the foundation and that of the free ground (without foundation) become different, the foundation motion being usually smaller. This aspect of SSI is known as *kinematic SSI*. Ideally, the foundation motion should be used as input motion in the analysis of the structure. However, studies have shown that the difference between the two motions can be regarded as negligible, thus the reason for using the free-ground motion in practice. The second, and more important, form of SSI is manifested when the superstructure starts to vibrate as a result of inertial forces, mostly horizontal, triggered by the excitation at the foundation level. The inertial forces distributed over the height of the structure cause a resultant base shear and an

[1]asratie@gmail.com:
aworku@gibbinternational.com

overturning moment at the foundation, which in turn cause deformation of the soil. This deformation initiates new waves propagating into the soil mass. The waves carry away some energy with them and act as a source of energy dissipation in addition to the material/hysteretic damping inherent in the system as in any other material like the superstructure itself. This form of SSI is known as *inertial SSI*. Its effect in most structures is to increase total displacement and to decrease the base shear demand due to the associated energy dissipation into the soil and to the increased period of vibration of the system.

The effect of SSI is thus beneficial for most building structures in terms of base shear, if not strictly always. Unfortunately, this beneficial aspect of site effect is mostly ignored by engineers with the conviction that the design would already be on the safe side without the additional computational effort needed to include SSI effects. This is despite the fact that code provisions related to this phenomenon have been availed since the late 1970s, though not as obligatory requirements. The original versions of these provisions have meanwhile been updated through results of calibration works conducted using actual records from strong earthquakes like the 1989 Loma Prieta and 1994 Northridge earthquakes[1-3]. Results of such works and experimental verifications are encouraging the use of the most recent code-based SSI provisions [2,4,5].

In relation to the work currently underway to revise the 1995 Ethiopian Building Code Standard (EBCS) [6], the site-dependent design spectra of either the current European code or of the American codes are expected to be directly adopted. These spectra are more demanding than those of the current EBCS 8 [6] in many aspects including the associated changes in return period of the design earthquake and the degrees of amplification due to the various classes of site soils [3,6]. Understandably, engineers have expressed their concern during the recent annual gathering of the Ethiopian Association of Civil Engineers, about the associated escalation of material and construction cost.

With due account to this legitimate concern, the paper attempts to demonstrate that a good potential exists for some of the costs associated with the increased demand for base shear due to the new site amplification factors to be at least partially offset by the beneficial effects of inertial SSI, if the appropriate provisions are introduced in the new version of the code currently under revision and if the provisions are properly employed. The study shows that the savings could be of significant proportion in some cases, even though this must be ascertained by design engineers on a case-by-case basis.

INERTIAL SSI AND IMPEDANCE FUNCTIONS

In order to understand the influence of inertial SSI on the response of building structures subjected to seismic ground motions, it is helpful to briefly introduce the basic principles and concepts of dynamics of foundations supported by flexible media like soils. For this purpose, we consider the vibration of the rigid circular foundation of radius R_0 resting on the surface of the ground idealized as a homogenous elastic half space as shown in Fig. 1 and excited by the vertical harmonic load. Let the half space have an elastic modulus of E and a mass density of ρ. For purposes of simplicity and better insight, let us further represent the half space by the rudimentary model of the truncated solid cone of cross-sectional area of A_0 at the ground level which is the same as the contact area of the foundation. The cone defines the angle α with the horizontal and the height h_0 up to its apex above the ground [7].

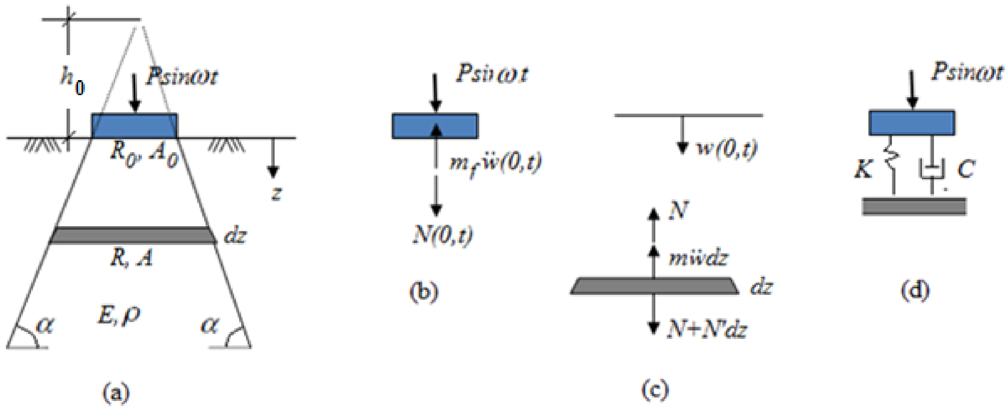


Figure 1 (a) A circular rigid foundation on the surface of a half space subjected to a harmonic load and its simplified conical representation; (b) free-body diagram of the foundation; (c) free-body diagram of a soil element from the soil column; (d) a SDOF model of the rigid foundation

After formulating the equation of motion of the conical soil beam based on the equilibrium of the differential soil element isolated in Fig. 1(c), one can readily derive the differential equation for the capping rigid circular foundation at *z=0* as

$$m_f \ddot{w}_0(t) + \frac{EA_0}{c_L}\dot{w}_0(t) + \frac{EA_0}{h_0}w_0(t) = P_o \sin \omega t \quad (1)$$

Where m_f is the mass of the foundation; $c_L = \sqrt{E/\rho}$ is the velocity of the longitudinal elastic wave travelling away from the foundation through the conical soil column; ω is the frequency of the harmonic load; and w_0 is the vertical displacement of the foundation [7].

This equation is similar to the conventional equation of motion of the single-degree-of-freedom (SDOF) model shown in Fig. 1(d) given by

$$m_f \ddot{w}_0(t) + C\dot{w}_0(t) + Kw_0(t) = P_o \sin \omega t \quad (2)$$

Comparison of Eqs. (1) and (2) results in the following expressions for the parameters of the SDOF model in terms of the geometry of the foundation and the elastic properties of the soil:

$$K = EA_0/h_0; \qquad C = EA_0/c_L \quad (3)$$

This interesting result obtained on the basis of a rudimentary idealization of the soil-foundation system as a truncated conical soil column capped by the rigid foundation (Fig. 1(a)) demonstrates the following fundamental facts:

- The semi-infinite continuum providing support to the foundation and subjected to dynamic loading can be replaced by a simple SDOF mechanical model of zero mass consisting of a spring and a dashpot of coefficients, *K* and *C*, respectively, arranged in parallel;

- The parameters of the SDOF model for the foundation soil can be expressed in terms of the foundation geometry, the elastic parameters of the continuum and a pertinent wave velocity; and

- Unlike in conventional dynamic models of structures, the damping term in Eq. (1) is not an assumed addition; it is a mathematical outcome showing that the damping is an intrinsic behavior of the system. This term, the second term in Eq. (1), represents an additional equilibrant force due to energy dissipation through waves propagating away from the foundation as represented by the wave velocity in the coefficient. It is in addition to the material damping of the continuum that is not considered in this discussion.

These important outcomes were observed by Reissner [8] probably for the first time. However, in a more rigorous treatment of the system shown in Fig. 1(a), the spring and dashpot coefficients of Eq. (3) are dependent on a number of factors including the mode and frequency of excitation, foundation size, foundation shape, foundation embedment depth, the dynamic behavior of the soil, soil heterogeneity and stratification. These coefficients, commonly termed as *impedance* *functions*, for they relate the dynamic forces to the ensuing deformations, are now available in the literature for a wide range of conditions after decades of research works. They have already made their way into design codes starting from the late 1970s and have been greatly enhanced in recent years [7,9-11].

Reverting to the mechanical model of Fig. 1(d), its equation of motion given by Eq. (2) for zero mass takes the following form for any degree of freedom considered:

$$C\dot{w}(t) + Kw(t) = P_o e^{i\omega t} \tag{4}$$

The subscript of the deformation is dropped for brevity reasons, and the harmonic load is represented in its complex form for purposes of generality and convenience. The trial solution to this differential equation should also be complex, which after substitution and solving for the complex-valued impedance function, which by definition is the ratio of the load to the response, gives

$$P(t)/w(t) = \bar{K} = K + i\omega C \tag{5}$$

On the other hand, the complex-valued impedance functions obtained from rigorous mathematical treatments of the semi-infinite continuum are often presented in the literature in the following form [7-12]:

$$\bar{K} = K_s\left[\alpha(\omega) + i\omega a_0 \beta(\omega)\right] \tag{6}$$

where, K_s is the static spring stiffness; a_0 is a dimensionless frequency parameter given by $a_0 = \omega R/V_s$; $\alpha(\omega)$ and $\beta(\omega)$ are frequency-dependent dynamic impedance coefficients (also known as dynamic modifiers); and V_s is the shear wave velocity of the continuum. By equating Eqs. (5) and (6), one obtains the following important relationships for the real-valued, frequency-dependent coefficients of the massless spring-dashpot model in Fig. 1(d):

$$K = K_s \alpha(\omega); \qquad C = K_s \frac{R}{V_s} \beta(\omega) \tag{7}$$

As indicated above, the impedance coefficients, $\alpha(\omega)$ and $\beta(\omega)$, are nowadays available for various foundation and soil conditions.

A circular foundation on the surface of the homogenous viscoelastic half-space is the most basic and most important case. As far as the influence of foundation shape is concerned, studies have shown that use of an equivalent circular foundation is satisfactory for foundations of arbitrary shape provided that the aspect ratio of the encompassing rectangle of the foundation plan does not exceed 4:1. For other cases, suggested modifications are available [10]. We shall focus on circular foundations.

The static spring coefficients in Eq. (7) for a circular foundation are given by the following expressions for the most important modes of foundation motion of horizontal translation and rocking, respectively, in seismic design [7,10]:

$$K_{sh} = \frac{8}{2-\nu} G R_h; \qquad K_{s\theta} = \frac{8}{3(1-\nu)} G R_\theta^{\,3} \tag{8}$$

Note that the radii in the two cases are different for non-circular foundations and are determined by equating the area, A, and moment of inertia, I_θ, for rocking motion of the actual foundation to those of the equivalent circular foundation. Thus,

$$R_h = \sqrt{A/\pi}; \qquad R_\theta = \sqrt[4]{4I_\theta/\pi} \tag{9}$$

The corresponding impedance coefficients for a surface circular foundation were originally established by Vleletsos and his co-workers [12,13] and Luco and Westmann [14] independently of each other and are given in Fig. 2 as functions of the frequence parameter, a_0, corresponding to two values of hysteretic damping, β.

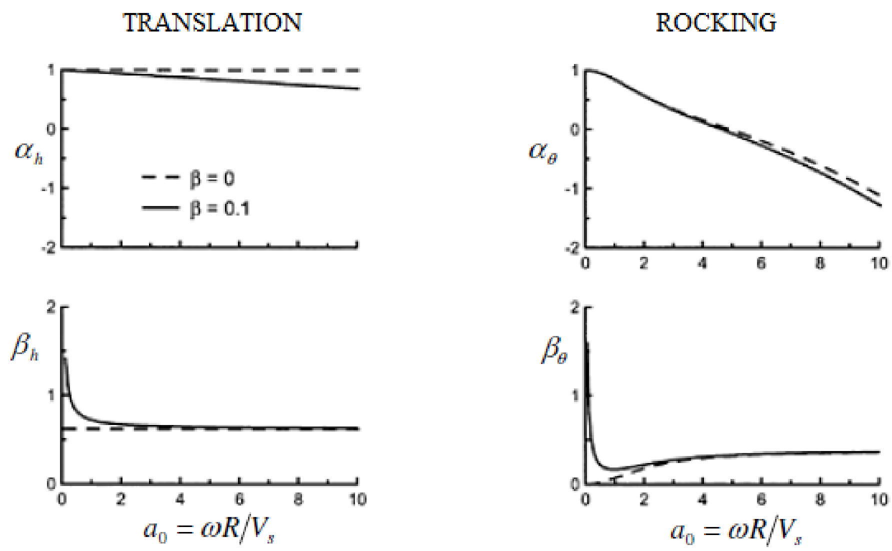


Figure 2 Dynamic impedance coefficients for horizontal translation and rocking corresponding to two values of hysteretic damping (After [13])

The variation of the impedance functions with frequency in this figure is fairly steady over a significant range of the frequency parameter, a_0, such that closed-form relations can be easily fitted to them. However, the functions are generally more erratic for other cases, especiallly for layered soils.

For a given problem of specific conditions, the appropriate impedance functions can be used in order to establish the dynamic spring and dashpot coefficients as per Eq. (7). Important factors to be further accounted for include foundation embedment depth, foundation depth, foundation flexibility, soil layering and increase in stiffness of soil with depth. Available literature should be consulted to this end [4, 7, 10, 11].

INFLUENCE OF INERTIAL SSI ON DESIGN SPECTRA

In the most general 3D case, a single mass oscillator fixed at its base acquires six more degrees of freedom (DOF) when the base is released. The additional DOFs consist of a translation in each direction of the three Cartesian coordinate axes and a rotation around each of them.

For excitation due to an upward propagating seismic shear waves, inclusion of the two DOFs at the base consisting of the horizontal and rocking motions is sufficient for planar analysis. This condition is depicted in Fig. 3 for a superstructure represented by a SDOF model, in which the complex-valued springs are introduced at the base in each of the horizontal and rotational DOFs. Accordingly, the planar system now has three degrees of freedom. This representation is equivalent to a real-valued spring and dashpot arranged in parallel for each DOF. The height h refers to the height of the roof in the case of a single-story building and to the centroid of the inertial forces associated with the fundamental mode in the case of a multi-story building which is commonly taken as $0.7h$ [3, 4].

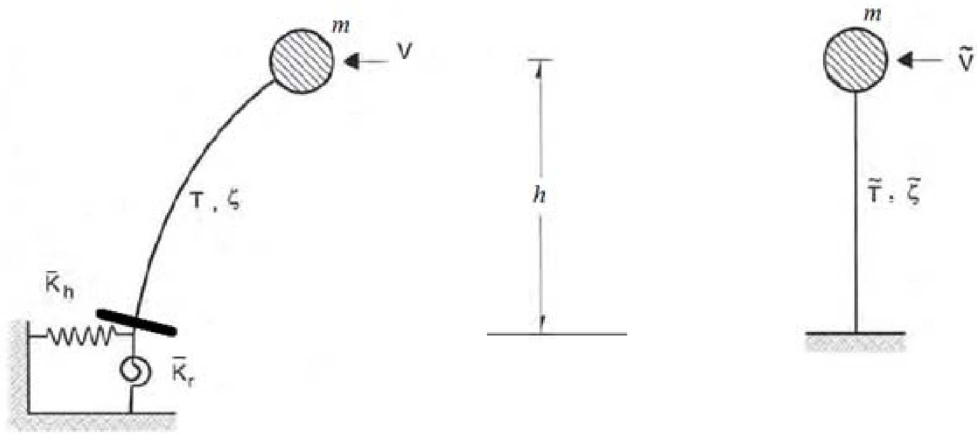


Figure 3 (a) A SDOF structural model with a flexible-base; (b) A replacement SDOF model

In time-history analysis (THA), the frequency dependence of the foundation parameters and the nature of the system damping make flexible-base structures more difficult to analyse than fixed-base structures. Such systems are termed as *non-classically damped systems* and can be tackled using specially tailored closed-form or iterative analysis methods [7,15-16].

In contrast to THA, in response spectrum and pseudo-static analyses, SSI is accounted for by dealing with an equivalent SDOF system as shown in Fig. 3(b) with modified parameters to account for the foundation flexibility instead of the system in Fig. 3(a). This was proposed by Veletsos and Meek [17], who found that the maximum displacement of the mass in Fig. 3(a) can be sufficiently accurately predicted by the replacement SDOF system in Fig. 3(b) with a modified natural period of $\tilde{T}$ and a modified damping ratio of $\tilde{\zeta}$. These modified parameters are called *flexible-base* parameters and have the convenience of enabling the engineer to use the conventional code-specified seismic design spectra as usual.

Veletsos and Meek [17] found out that the flexible-base period may be determined from

$$\frac{\tilde{T}}{T} = \sqrt{1 + \frac{k}{k_h} + \frac{kh^2}{k_\theta}} \tag{10}$$

The fixed-base period is given by $T = 2\pi/\sqrt{k/m}$, where k is the stiffness of the structure and m is its mass. According to Eq. (10), the flexible-base period, $\tilde{T}$, is always larger than the fixed-base period and increases with decreasing stiffness of the foundation. Measured period lengthening of up to 50% are reported in the literature [18]. Note that the period ratio is dependent on frequency (or period) because of the frequency-dependent foundation stiffnesses. It is, however, sufficient to establish the stiffnesses for the fundamental frequency/period of the fixed-base system [18].

The effective flexible-base damping, $\tilde{\zeta}$, is contributed from both the structural viscous damping, ζ, and the foundation damping, $\tilde{\zeta}_0$, consisting generally of radiation and material damping components. Veletsos and Nair [19] came up with the following relationship based on equivalence of the two oscillators in Fig. 3:

$$\tilde{\zeta} = \tilde{\zeta}_0 + \frac{\zeta}{\left(\tilde{T}/T\right)^3} \tag{11}$$

The plots of the effective system damping of Eq. (11) against the period ratio are given in Fig. 4 for the commonly assumed fixed-base structural damping (FBSD) of 5% and a number of foundation damping (FD) values ranging from 3% to 20%. Such ranges of foundation damping ratios are reported in the past [18].

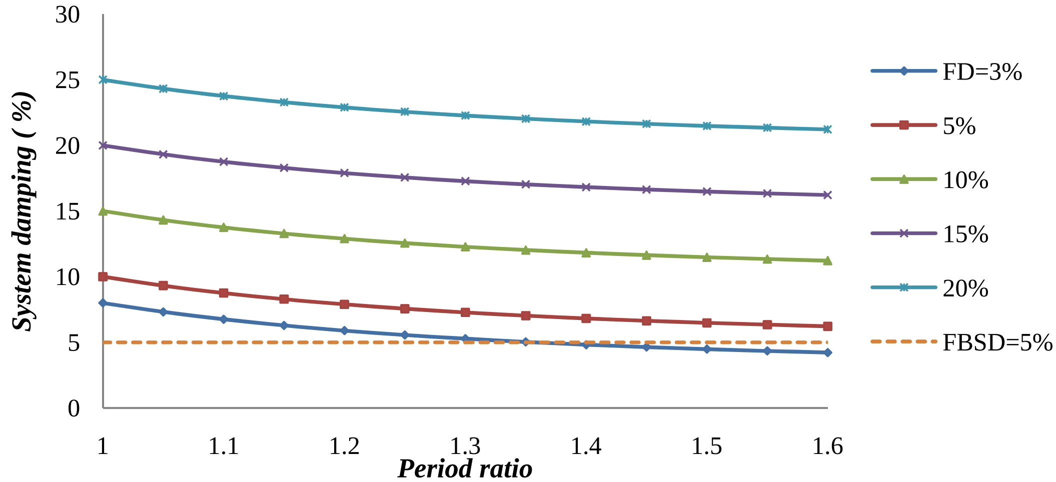


Figure 4 Variation of the system damping with $\tilde{T}/T$ for different foundation damping

The plots show that the overall effective damping of the flexible-base system is usually larger than the fixed-base damping (FBSD= 5%) with the exception of the rare case of the foundation damping itself being much smaller than 5%, and the period ratio is large. For any given foundation damping, the system damping gradually decreases with increasing period ratio.

The influence of the lengthened period and the modified damping on a smoothened response spectrum is qualitatively shown in Fig. 5. The figure shows that for a fixed-base period of up to around 0.3s, SSI has the effect of increasing the spectral response of the structure. However, for the most common case of building structures having a fundamental natural period larger than about 0.3s, SSI has the effect of reducing the spectral response and thereby reducing the design base shear force (compare ordinates of the two curves corresponding to the pairs of T and $\tilde{T}$ on either sides of $T \approx 0.3s$).

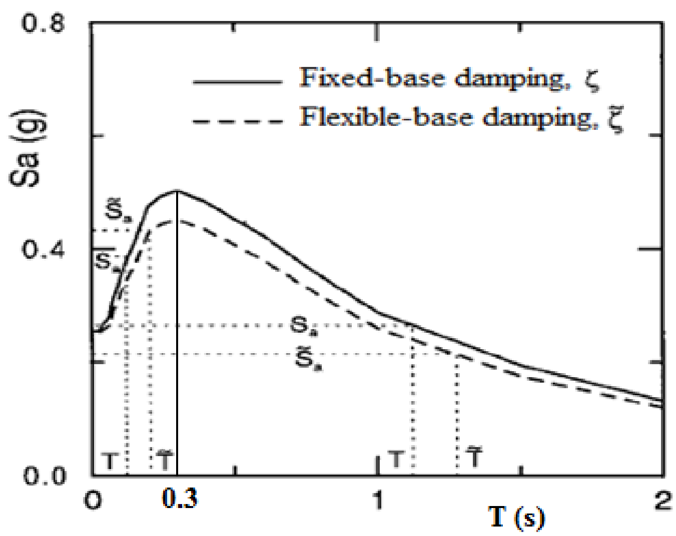


Figure 5 Schematic representation of influence of SSI on design spectra (adapted from Stewart et al [4])

A more helpful insight into the influence of SSI on code-specified design spectra can be obtained by considering the EC 8 [20] Type 1 design spectra specified for five different site soil classes as shown in Fig. 6 for a structural damping ratio of 5%, whereby Site Class A represents rock site. The various site soil classes are defined in the code [20]. The amplification potential of the site soils is evident from the curves. These spectra are likely to be directly adoptedby EBCS, which is currently under revision.

Let us further consider the two soft site soil classes of C and D characterized by an average shear-wave velocity of 180 to 360 m/s and less than 180 m/s, respectively, over the upper 30m depth in accordance with EC 8 [20]. The corresponding design spectra for the two site classes are presented separately in Figs. 7(a) and 7(b) together with the spectrum for Site Class A – rock site.

For Site Class C, the foundation damping ratio including both material (hysteretic) and geometric damping is conservatively estimated to reach up to 10% for the purpose of this study. The corresponding period lengthening of the SDOF system due to SSI can also be reasonably estimated to reach 10%.

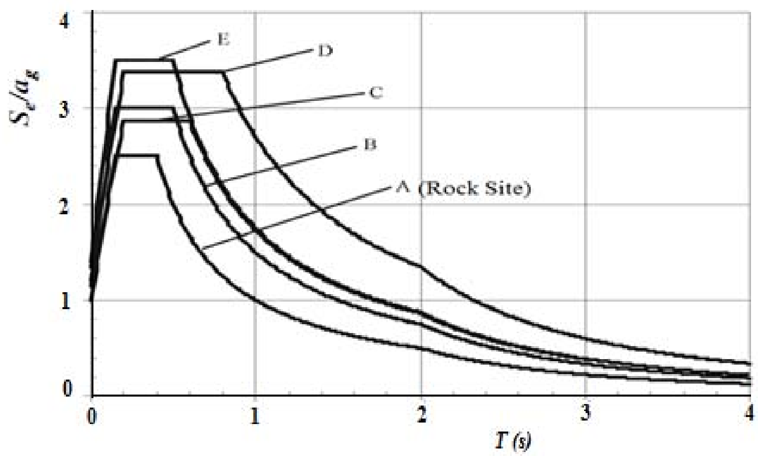
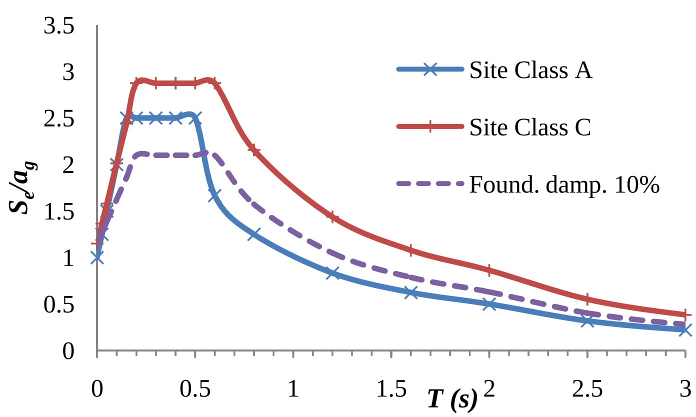
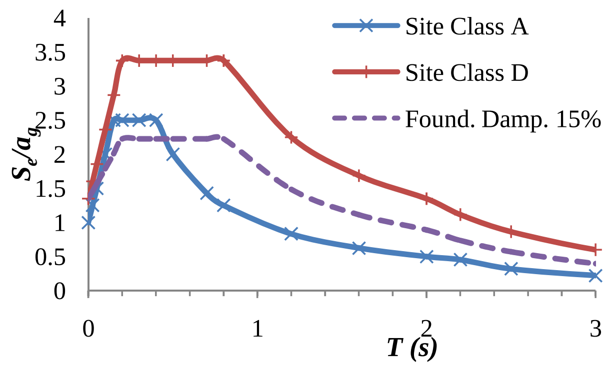


Figure 6 EC 8-2004 design spectra for different site conditions for a damping ratio of 5% (After [20])



(a)



(b)

Figure 7 Comparison of SSI-modified design spectra against EC 8 design spectra for (a) Site Class C; (b) Site Class D

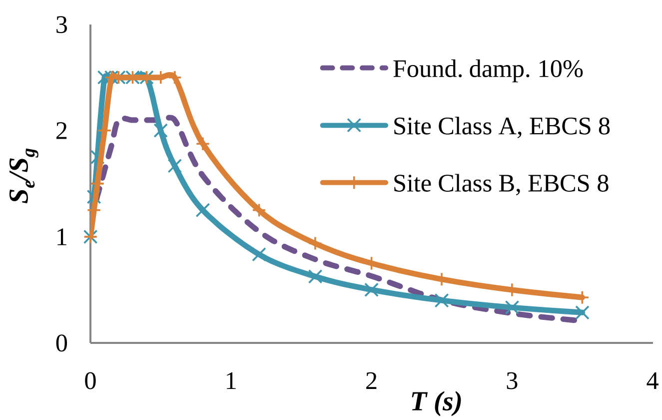
With the effective system damping calculated from Eq. (11) or read from Fig. 4 as 13.76%, the corresponding design spectral curve is determined as per appropriate provisions of EC 8 [20] by scaling down the site-dependent spectral curve using the following factor to account for the modified damping:

$$\eta = \sqrt{10/(5+\zeta)} \geq 0.55 \qquad (12)$$

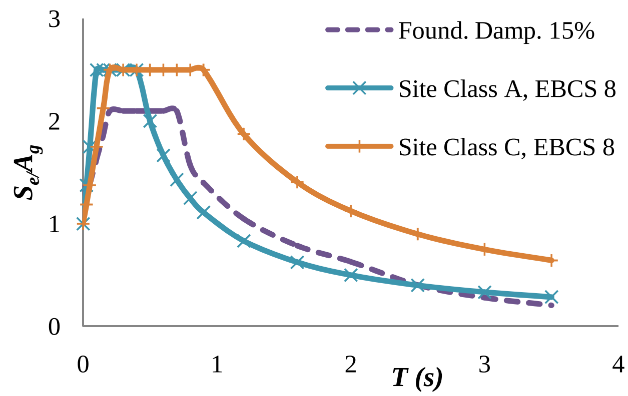
In Eq. (12), ζ is the effective system damping that accounts for both structural and foundation damping. The plot is shown in Fig. 7(a) as the dashed curve and indicates that a significant reduction in the design base shear up to 30% could be achieved for structures with a fundamental period larger than about 0.2 seconds, to which most building structures belong. Even though the SSI effect is more visible for rigid (short-period) structures, the influence on the more flexible long-period structures is also significant.

Similarly, a little larger foundation damping of up to 15% is assumed for the softer Site Class D with corresponding period lengthening of up to 15%. The effective damping determined in a similar manner as in Site Class C is 18.3% and resulted in the dashed curves shown in Fig. 7(b). In this case, a reduction in the design base shear of up to 35% seems attainable.

The modified spectral curves for the two site classes of EC8 are compared in Figure 8 with the corresponding site-dependent design spectra specified by the current EBCS 8 [6]. In Fig. 8(a), design spectra for Site classes A and B of EBCS 8 are compared with the EC 8 design spectrum for Site Class C modified for the effective damping, whereas in Fig. 8(b), the spectra for site classes A and C of EBCS 8 are compared against the EC 8 design spectrum for Site Class D modified for the corresponding effective damping.



(a)



(b)

Figure 8: Comparison of SSI-modified design spectra against EBCS 8 design spectra for (a) Site Class B; (b) Site Class C

It is interesting to note from the plots that, the design spectra, and thus the design base shear, as per EC 8 [20] modified for inertial SSI effects can even be significantly lower than the spectra specified by the less stringent EBCS 8 spectra for the corresponding soil classes over a significant range of fundamental period. This is particularly evident in Fig. 8(b) in which reductions by more than 50% are achieved for long-period structures. This shows that accounting for the influence of inertial SSI could even result in base shear forces significantly lower than those obtained from the site-dependent spectra of EBCS 8 currently in use. The reduction would have even been much larger, had the SSI provisions been directly applied on the site-dependent spectra of EBCS 8 instead of on the current EC 8 spectra. This can make the introduction of SSI provisions into the future issue of EBCS 8 an attractive option so that potential construction cost escalations associated with the adoption of the more demanding site-dependent spectra from current codes like EC8 are at least partially offset.

Larger scales of reductions can be expected for the much softer site soil classes of EC 8 [20]. In all cases, to be noted is the fact that the foundation damping and the period lengthening are the key factors that affect the amount of spectral reduction due to SSI. However, it should be recalled that the reductions demonstrated in the above plots are based on assumed ranges of foundation damping and period lengthening for the purpose of this study, even though these are based on reasonable engineering judgment. Hence, the actual gains must be strictly established on a case-by-case basis by the structural engineer, and no generalization is warranted based merely on the material presented in this paper. Furthermore, it should be pointed out that SSI has also the effect of increasing the lateral displacement thereby having an impact on P-Δ effect and the ductility requirement. This aspect, which has not been addressed in this work, should be appropriately taken care of by the implementation of pertinent provisions during the design process.

Nonetheless, the plots in Figs. 7and 8 demonstrate that, even though the net effect of the site soil could eventually be to amplify the rock-site spectra for some structures, this could be substantially offset by properly accounting for inertial SSI effects, which are mostly beneficial. If properly employed, this approach has the promising potential to lead to a significant financial saving.

CONCLUSIONS AND RECOMMENDATIONS

The material presented in this paper demonstrated that site soils have two major influences on the response of structures subjected to earthquake ground motions. The more widely recognized effect is to amplify the ground acceleration at the rock level as the seismic waves travel through the soil. This effect is obviously detrimental to structures founded on such soils, especially to those, the fundamental period of which matches or is closer to the predominant period of the site soil formation, due to resonance. The other important influence, which is the subject of this article and less known by many, is inertial SSI, which often has the beneficial effect of reducing design spectral values or base shear in the seismic design of a large class of building structures. It is observed that the effects of both site amplification and SSI increase with decreasing stiffness of the site soil so that they tend to balance each other, if not equally.

As per current knowledge, the amplification potential of site soils is much more than stipulated in old building codes like EBCS 8 [6]. The paper has shown that the cost implications due to site amplifications, which in some cases could be prohibitive, could be significantly offset if SSI provisions are accounted for in the analysis. The necessary procedures for SSI analysis are available in recent code provisions like in NEHRP 2003 [3]. Recent reports have shown that current code-specified relationships for computing the period lengthening, the effective damping and the reduction in base shear are calibrated using recorded and measured data so that these provisions would give reliable results. It is thus recommended that these updated provisions are incorporated into the ongoing revision of EBCS 8 [6], at least in form of nonobligatory requirements, for an eventual economical structural design. This will provide a room for the engineer to consider his options on a project-by-project basis.

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