PREDICTION OF SWELLING BEHAVIOR OF ADDIS ABABA EXPANSIVE SOIL USING SIMPLE MATHEMATICAL MODEL

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ABSTRACT

Damage due to soil swelling is very noticeable in wide spectrum of structures such as roads, buildings, canal linings, landfill liners, etc. To evaluate severity of swelling, an accurate assessment of the swell potential is required. The main reason of swelling behavior is water absorption of soil mass in time. And the time required for completion of swelling is relatively long. In view of that several attempts have been made by researchers to obtain time-swell relationships for expansive soils. In this study a simple hyperbolic mathematical model is used to predict the swelling behavior of an expansive soil from Addis Ababa. The main parameters that are needed to run the model are the applied pressure and initial dry density. The other parameters of the model including the initial slope of the swell-time curve, the final slope, the reference swell, and the peak swell were investigated experimentally. Based on the experimental results, empirical relationships were developed for determination of these parameters as functions of the applied pressure and initial dry density. The model prediction were compared with the experimental results and showed good agreements.

***Key words**: Expansive soil, mathematical modeling, swelling potential, swelling pressure.*

INTRODUCTION

In the capital, Addis Ababa, it has been noticed that expansive soils cover large parts of the city where recent constructions are carried out. During the past decades, rapid expansion of the city and the growth of population due to migration from different parts of the country led to the construction of various structures, particularly low-cost buildings in the suburbs. The existence of expansive soils in the city has caused structural damages on light buildings, asphalt pavement, and buried utility lines. Therefore it becomes necessary for soil engineers in their day-to-day practice to predict the total probable heave under given loading conditions of buildings founded on these soils.

The main reason of swelling behavior is water absorption of soil mass in time. Seed [1] reported that considerable time is required for complete swelling to take place. Blight [2]; Tsytovich and Zaratsky [3]; and Myslivec [4] have presented various swelling-time relationships. Thurairajah [5] has obtained a linear relationship for the variation of swelling with square root of time. Komorink and Zeitlen [6] have noted that the vertical movement follows a time function similar to that of consolidation. Srinivasan [7] has reported that the time required for the water to enter into a swelling clay system is a function of the permeability of soil, hydraulic gradient, soil structure, etc. An attempt was made by V.Dakshanamurthy [8] to represent the swelling-time relationship by a hyperbolic equation.

A versatile mathematical model presented by Richard and Abbott [9] has been used to represent the stress-strain spectrum of different types of concrete as well as triaxially confined soils. This model was used by Almusallam and Alsayed [10] to capture the complete stress-strain curves (hardening and softening parts) for normal, high strength and light weight concrete tested under various loading conditions. Al-Karni [11] extended the application of the model to predict the stress-strain curve of consolidated drained triaxial test on granular soil. Recently Abdullah I. Al-Mhaidib [12] used this mathematical model to predict the swelling behavior of an expansive soil from Saudi Arabia. The results of the model was compared with the experimental results and found to be in a good agreement.

The prime objective of this research program is to predict the swelling behavior of expansive soil found in Addis Ababa using simple mathematical model presented by Richard and Abott [9]. The main advantage of this model is that only one parameter is needed to run the model. The main parameters that are needed to run the model are the applied pressure and initial dry density. The other parameters of the model including the initial slope of the swell-time curve, the final slope, the reference swell, and the peak swell were investigated experimentally. Based on the experimental results, empirical relationships were developed for determination of these parameters as functions of the applied pressure and initial dry density.

EXPERIMENTAL STUDY

Material

Sampling sites were selected within the city of Addis Ababa based on information from previous studies. Within the selected areas random sampling strategy was employed. Three soil samples one from Akaki-Kality sub city and the others two from Bole sub city are used in this study. The average geotechnical properties of investigated soils are shown in Table 1.

Liquid limit, plastic limit, specific gravity, and particle size were all determined in accordance with ASTM test procedures. According to the Unified Soil Classification system, theses soils are classified as inorganic clay of high plasticity (CH).

Sample Preparation

To ensure uniformity of testing, block samples were dried, pulverized into a powder in batches and screened through sieve No. 40. The batches were thoroughly mixed to obtain a uniform bulk sample. Individual samples were oven-dried for about 24 hours and then thoroughly mixed to the chosen molding moisture content.

Table 1: Average Geotechnical Properties of the tested soils

Parameter	Kality	Bole High School	Close to Bole Air port
Depth, m	2.3	2	1.9
Sand content (grain size 2mm to 5□m), %	6.10	3.8	2.8
Silt content (grain size 75□m□□to 2□m), %	17.41	17.56	15.4
Clay content (grain size <2□m)	76.49	78.64	81.8
Liquid limit, LL, %	103	107	112.4
Plastic limit, PL, %	36	38	40
Plasticity index, PI, %	67	69	72.4
Specific gravity of solids,	2.73	2.76	2.8

They were then stored in air-tight plastic bags and sealed in plastic wraps to avoid loss in moisture content, and were allowed to cure at room temperature for 24 hours to achieve uniform distribution of moisture content. A prescribed weight of the wet soil, producing a required dry unit weight, was pressed into a mold and then dynamically compacted using small hand held tamper.

Immediately after preparation, the 50 mm-diameter and 20 mm-thick samples were transferred to the oedometer and the swell tests were performed.

Swell Test

The direct measurement of volume change that occurs when a soil undergoes heave is usually obtained by the use of the conventional one dimensional oedometer. The oedometer testing technique is the oldest and the most commonly used method for testing expansive soils [13]. The amount of swell measured in an oedometer test is dependent on test procedure (i.e. stress path and soaking conditions). The American society for testing and materials (ASTM) standardized the swell oedometer test procedures under the designation No. D4546. This standard describes three alternate oedometer testing procedures. Free swell, constant volume swell, and swell overburden tests to evaluate the amount of volume change. Sorochan [14] suggested that the swelling pressure should be determined using the loaded swell method since this method most closely reflects the process undergone by the soil in the field. It was therefore chosen and the tests were conducted according to ASTMD4546. The sample was put into the oedometer and subjected to seating pressure of 7 kPa for a period of 5 minutes. Then, deformation dial gauge was adjusted for the initial zero reading, and after 5 minute the specified vertical pressure was applied. Water was introduced to the specimen after 5 minutes of applying the vertical pressure. Thereafter, the expansion of the specimen was allowed until the rate of expansion reaches a negligible value, at which the test was terminated.

Proposed Model

The mathematical form of the model presented by Richard and Abbott, 1975 may be rewritten [12] to relate the percentage of swell (S) of expansive soil to time (T) as follows

$$S = \frac{(k-k_p)T}{\left[1+\left(\frac{(k-k_p)T}{S_o}\right)^n\right]^{\frac{1}{n}}} + k_p T \qquad (1)$$

Where S is the percentage of swell that corresponds to time T, k is the initial slope of the curve; k_p is the final slope of curve, S_o is a reference swell and n is a curve-shape parameter.

The value of n can be expressed as follows:

$$n = \frac{\ln m}{\ln\left[\frac{S_1}{S_o} - \frac{k_p}{k - k_p}\right]} \qquad (2)$$

Where m is a constant and S_1 can be calculated from:

(3)

Where S_p is the peak swell, T_p is the corresponding time and T_1 can be obtained from:

$$T_1 = \frac{S_o}{k - k_p} \qquad (4)$$

Evaluation of the Model Parameters

The parameters that needed to be determined in order to execute the model in Eq. (1) are: the initial slope of the curve, k; the final slope of curve, k_p; the reference swell, S_o; and the peak swell, S_p. From the vertical swell *vs.* time plot one can determine the parameters of the model in the manner as indicated in Fig. 1.

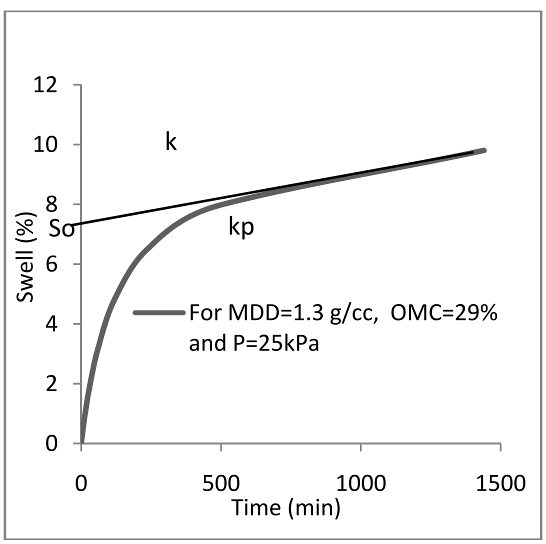


Figure 1 Illustration of Parameter determination for Kality soil

By way of the above geometrical construction the following parameters have been determined approximately. From Fig. 1:-

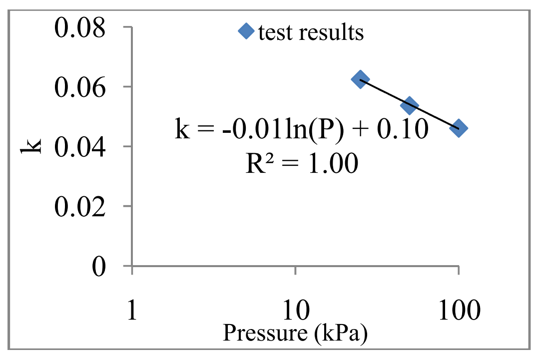
- Initials slope of the curve, k= (1.88-0.038)/ (30-0.5) =0.0624
- Final slope of the curve, kp= (9.8-8.0)/(1440-520)=0.00195
- The reference swell, So= 7.38
- Measured peak swell, Sp=9.8

The initial slope of the curve, k; the final slope of curve, kp; the reference swell, So; and the peak swell, Sp could be expressed as functions of the applied pressure (P) and dry density (DD) by a linear equation of the general form

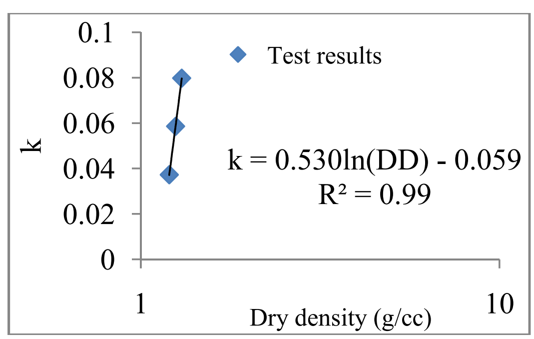
$$y = a + b\ \ln c \qquad (5)$$

where y may represent the parameters k, kp, So and Sp , while c may represent the applied pressure P and the dry density, DD.

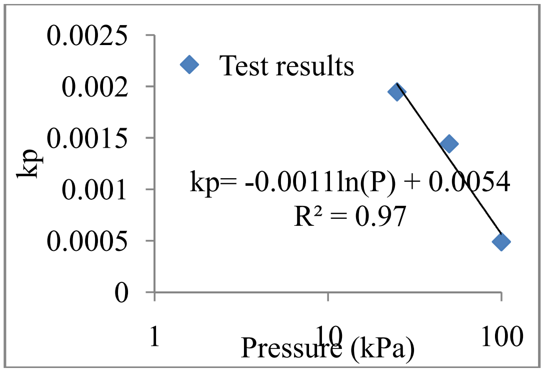
To test the validity of the proposed relationship, the test results from Kality and Bole High school are presented in Figs. 2 and 3 respectively.



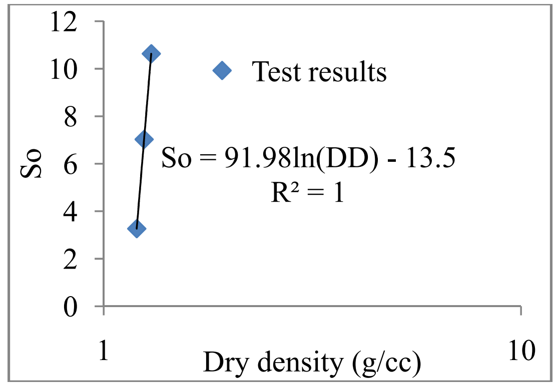
(a) Slope k for OMC=29 % and MDD=1.3 g/cc



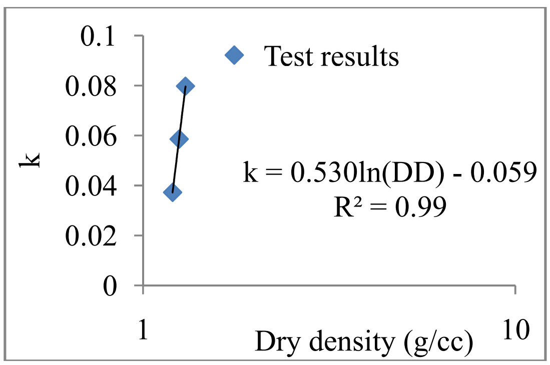
(b) Slope k for OMC=29 % and P=7 kPa



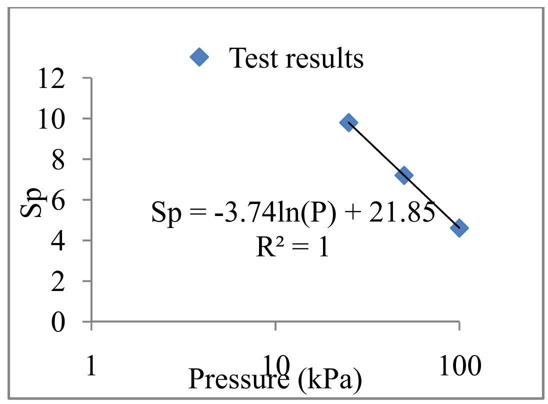
(c) Slope kp for OMC=29 % and MDD=1.3 g/cc



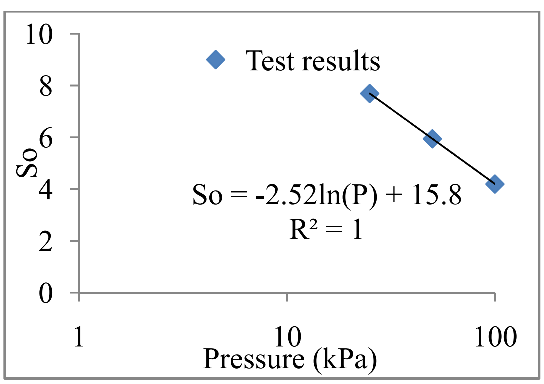
(f) So for OMC=29 % and P=7 kPa



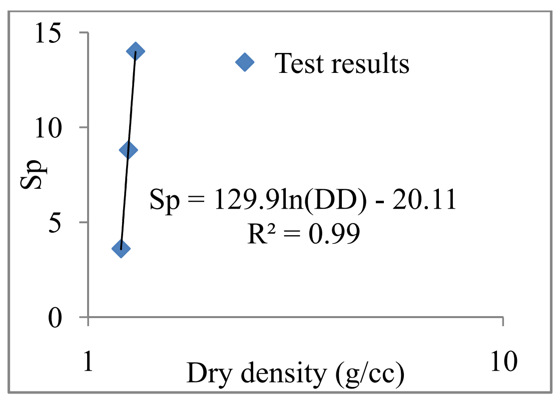
(d) Slope kp for OMC=29 % and P=7kPa



(g) Sp for OMC=29 % and MDD=1.3 g/cc

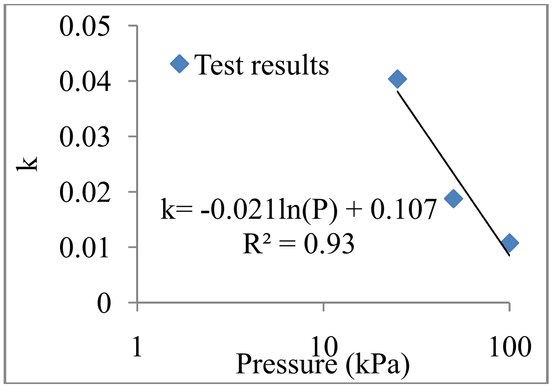


(e) So for OMC=29 % and MDD=1.3 g/cc

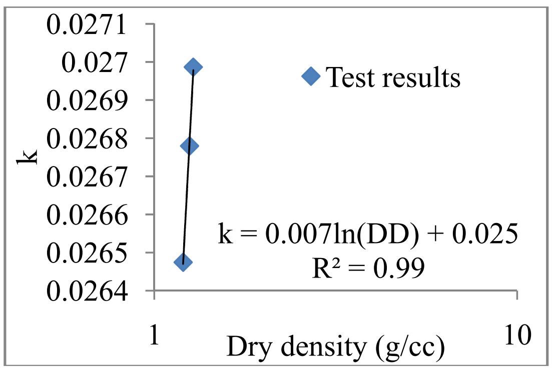


(h) Sp for OMC=29 % and P=7 kPa

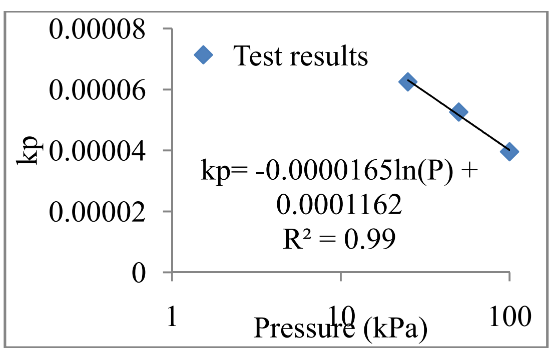
Figure 2 Variations of model parameters for Kality soil against applied pressure (25, 50 and100 kPa) and dry density (1.2, 1.25 and 1.3 g/cc)



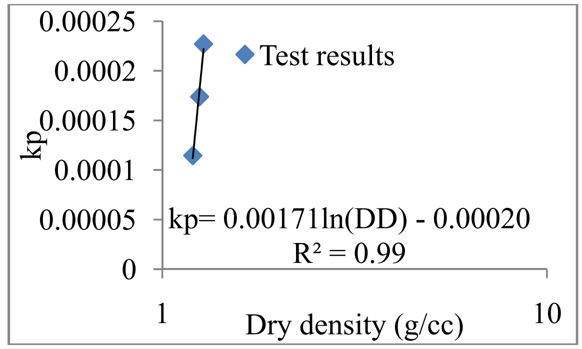
(a) Slope k for OMC=39 % and MDD=1.28 g/cc



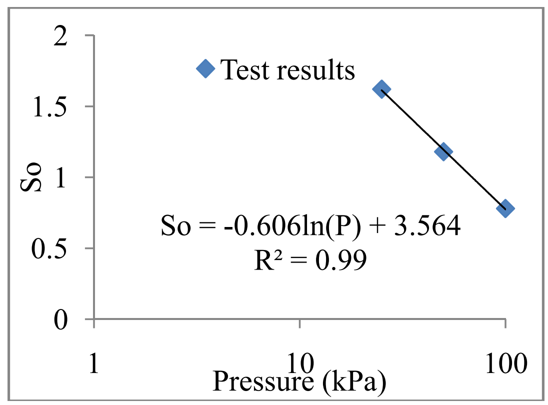
(b) Slope k for OMC=39 % and P=7 kPa



(c) Slope kp for OMC=39 % and MDD=1.28 g/cc



(d) Slope kp for OMC=39 % and P=7 kPa



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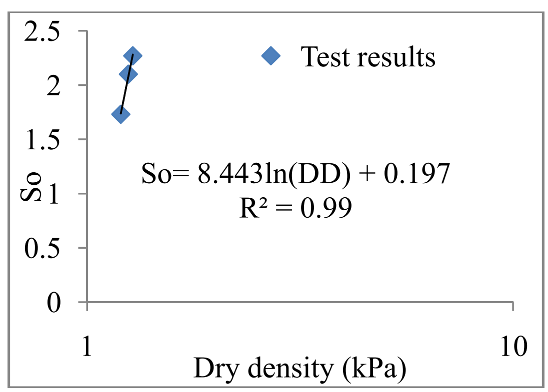
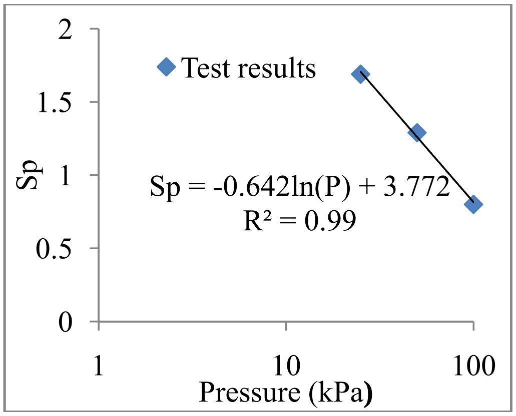
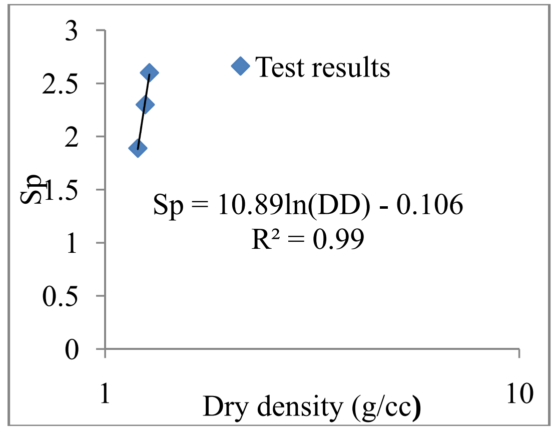


f) So for OMC=39 % and P=7 kPa



g) Sp for OMC=39 % and MDD=1.28 g/cc



(h) Sp for OMC=39 % and P=7 kPa

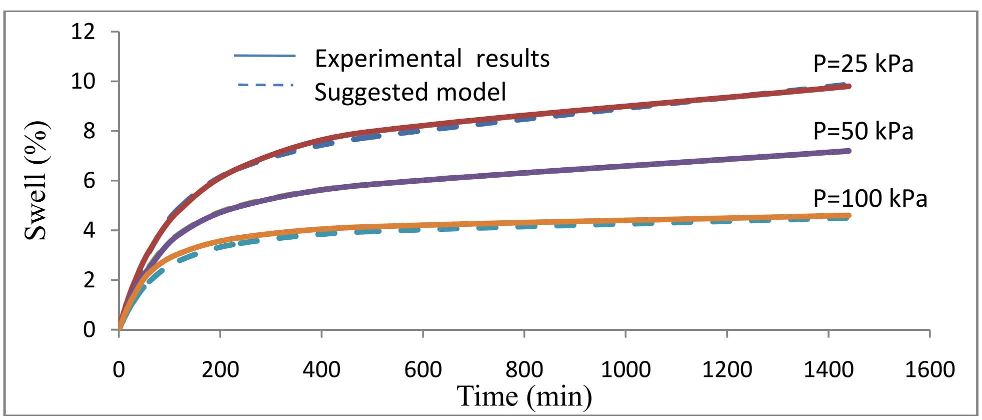
Figure 3 Variations of model parameters for Bole High school soil against applied pressure (25, 50 and100 kPa) and dry density (1.2, 1.25 and 1.28 g/cc)

Calibration of the Proposed Model

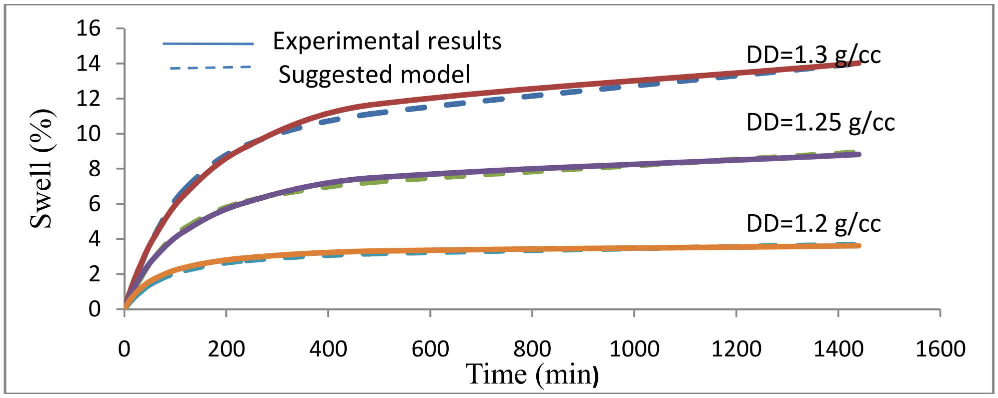
The model depends on the determination of the parameter *m* since the other parameters are evaluated directly from the tests results. This was carried out by testing the model for different values of *m* using the test results. In this study m is determined by minimizing the error between the predicted results using Eq. 1 and experimental results. The minimization of the error was achieved by changing the assumed value of m using spreadsheet application (solver tool). For soils brought from Kality and Bole High School area the best values that are applicable to all the applied pressure and dry density levels with marginal deviation were found to be when m=0.0689 and m=0.0162 respectively.

Comparison with Test Results

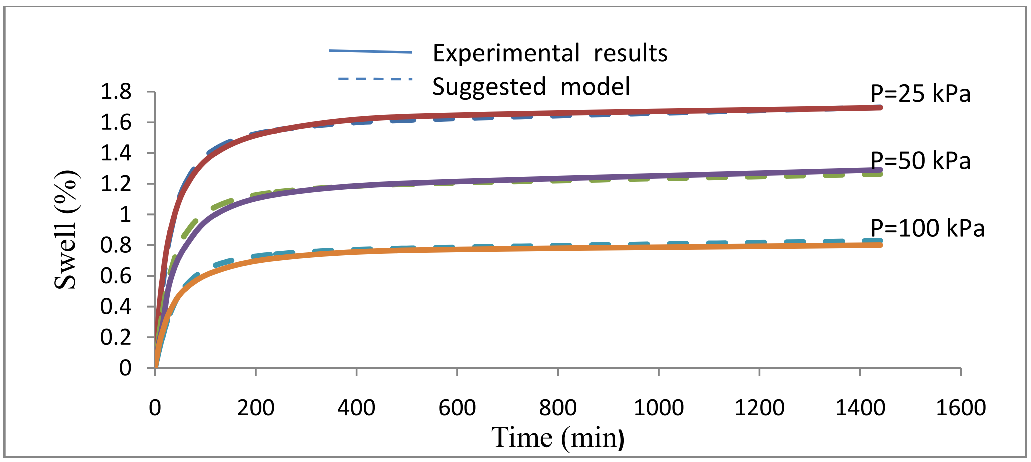
Figure 4 shows the experimental results of swell-time relationships as well as the curves generated by the proposed model. It can be seen from these figures (Fig. 4) that the curves generated by the proposed model are in good agreement with the experimental results.



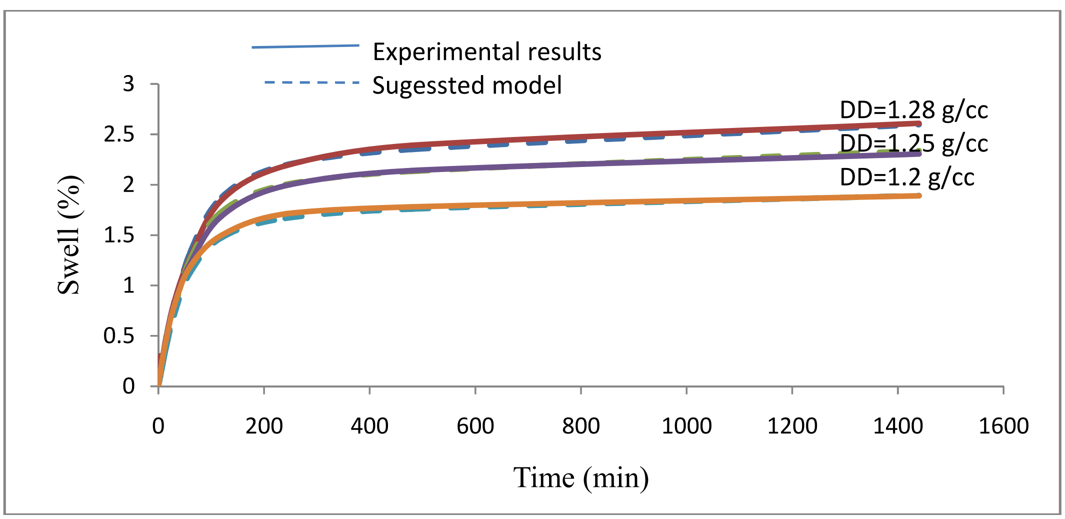
(a) OMC=29 % and MDD=1.3 g/cc (Kality)



(b) OMC=29 % and P=7 kPa (Kality)



(c) OMC=39 % and MDD=1.28 g/cc (Bole High School)



(d) OMC=39 % and P=7 kPa (Bole High School)

Figure 4 Time vs. Swell relationships for Kality and Bole High School Soils

Verification of the Model

To verify the proposed model, the predicted vertical swell developed for Kality and Bole High School soil were compared to the experimental result obtained for soil close to Bole Airport under similar testing conditions (Figs.5 and 6).

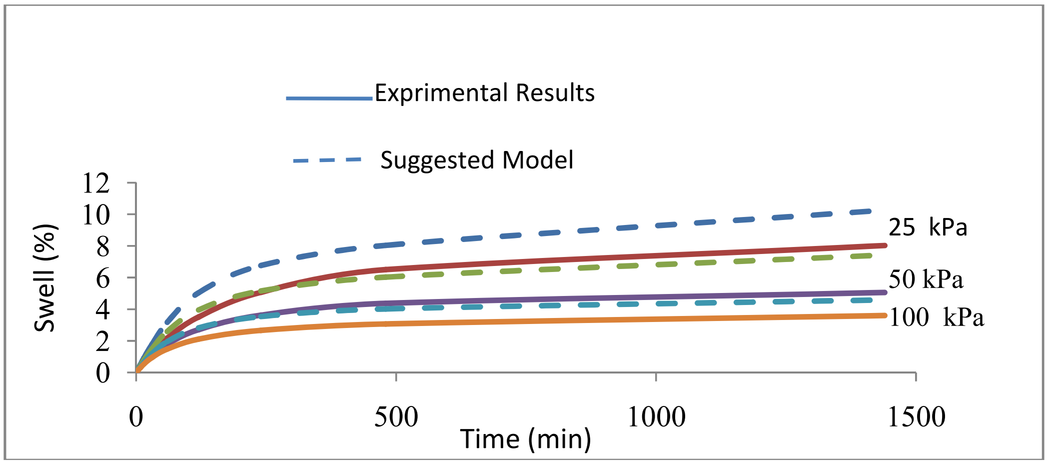


Figure 5 Swell vs. time relationship (MDD=1.3 g/cc and OMC=29 %), soil close to Bole air port.

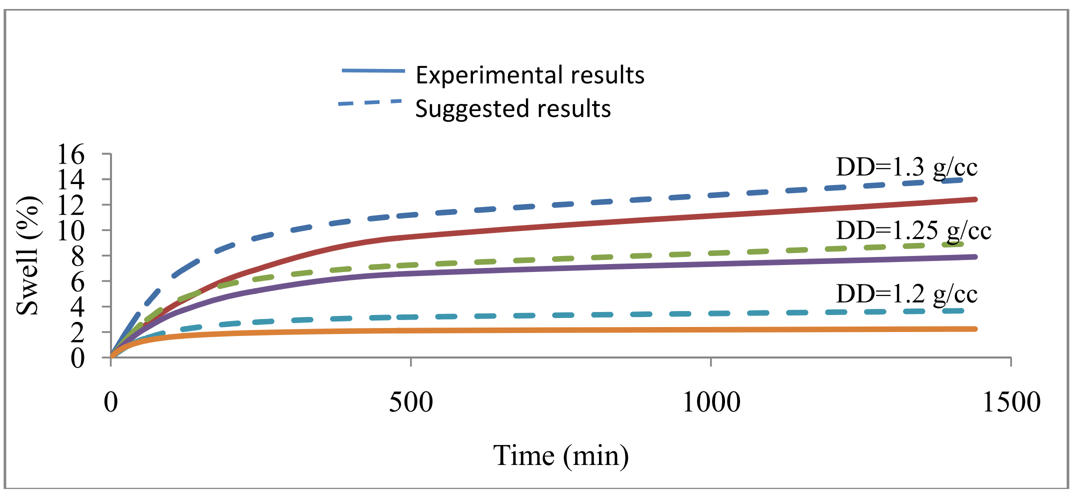


Figure 6 Swell vs. time relationship (OMC=29 % and P=7 kPa), soil close to Bole air port

Swelling pressure

Pidgeon [15] suggested that the linear relationship between the amount of swell and the logarithm of the applied stress could be accepted as a universal soil property. A number of research workers in various parts of the world have found the above relationship to apply to tests carried out on undisturbed or remolded specimens of both transported and residual soils.

The relationships between the maximum swell percentage and the applied overburden pressure for samples under different testing conditions are shown in Fig. 7. The swell pressures were determined by extrapolating the linear relationship (i.e. at 0 % vertical swell). The values of swell pressure for all samples are shown in Table 2.

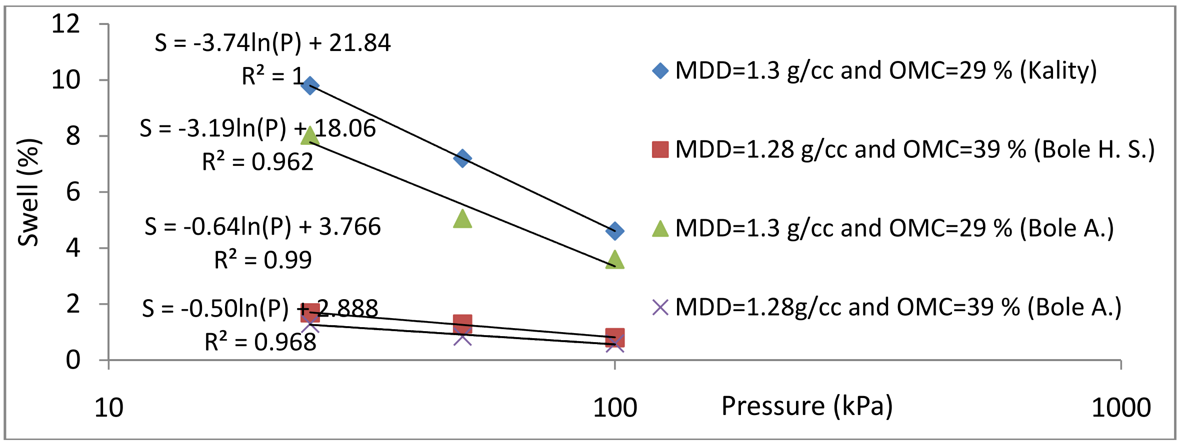


Figure 7 Relationship between swell percentage and overburden pressure

Table 2: Swell percentage under different overburden pressure

Test conditions	Vertical swell (%) 25 (kPa)	Vertical swell (%) 50 (kPa)	Vertical swell (%) 100 (kPa)	Empirical equation from Fig. 7	Swelling pressure (kPa)
MDD=1.3 g/cc and OMC=29 % (Kality)	9.8	7.2	4.61	S=-3.74ln (P)+21.84	343.6
MDD=1.28 g/cc and OMC=39 % (Bole H.S)	1.69	1.29	0.802	S=-0.64ln (P)+3.766	359.4
MDD=1.3 g/cc and OMC=29 %(Bole A.)	8.03	5.06	3.6	S=-3.19ln (P)+18.06	287.6
MDD=1.28 g/cc and OMC=39 % (Bole A.)	1.3	0.84	0.6	S=-0.5ln (P) +2. 88	322.5

CONCLUSIONS

In this study a simple hyperbolic mathematical model is used to generate swell-time curves of expansive soil from Addis Ababa. Two sets of swell tests were carried out.

1. Swell test at different levels of applied pressure keeping moisture content and initial dry density constant.
2. Swell test at different initial dry density keeping applied pressure and moisture content constant.

The unknown parameters are the applied pressure and initial dry density which are considered necessary to run the model. The other parameters of the model including the initial slope of the swell-time curve, the final slope, the reference swell, and the peak swell were investigated experimentally.

Based on the experimental results, empirical relationships were developed for determination of these parameters as functions of the applied pressure and initial dry density. The model prediction were compared with the experimental results and showed good agreements for all levels of applied pressure and initial dry density of the tested soil.

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