

A finite velocity simulation of sedimentation behaviour of flocculating particles – A real-time model evaluation

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ABSTRACT

A mechanistic velocity model is developed to simulate the behaviour of flocculating colloidal particles in turbid water. The current model is based on one-dimensional mass transport in the vertical direction as an integrated form of the model derived by Ramatsoma and Chirwa. The percentile removal model achieved more accurate simulation of physical experimental data than known models such as the Ozer's model and San's model. In this study, an integrated velocity form was used to estimate flocculent settling velocity of fine suspended particles under near quiescent conditions. Model closeness to experimental measurements was determined as a function of the sum of squares error (SSE) between model data and experimental data. The proposed velocity model offers a distinctive advantage over the interpolated-isopercentile based models which are prone to numerical errors during interpolation. The results contribute towards the ultimate goal of achieving full automation of the design of gravitational particle separation devices for water and wastewater treatment.

Keywords: flocculation model, semi-empirical system, velocity integrated model, continuity model

NOMENCLATURE

α	fitting parameter in San's model (dimensionless)
α_i	the model fitting parameters in Özer's model (dimensionless) ($i = 1, 2, \text{ and } 3$)
β	fitting parameter in San's model (dimensionless)
a	fitting parameter in Piro and co-workers' model and Je and Chang's model (dimensionless)
b	fitting parameter in Piro and co-workers' model and Je and Chang's model (dimensionless)
D	depth travelled by particle during settling (L)
D_z	dispersion coefficient in the vertical direction (L^2T^{-1})
h, H	ordinate representing depth
k	fitting parameter in San's model (dimensionless)
n	fitting parameter of the power law model and exponential model (dimensionless)
P	percentage removal $(1 - X_j/X_o) \times 100$
$P^{\text{''}}$	percentage remaining in suspension $(X_j/X_o) \times 100$
r_i	semi-empirical optimisable parameters of Ramatsoma and Chirwa's model ($i = 1, 2, \text{ and } 3$)
t	time of settling (T)
T	ordinate representing time
U_z	vertical water velocity (LT^{-1})
v	settling velocity (LT^{-1})
V_s	flocculent settling velocity (LT^{-1})
x	solids concentration in the power law model and exponential model (ML^{-3})
X_j	suspended solids concentration in layer j (ML^{-3})
X_o	initial concentration in the column (ML^{-3})
z, Z	vertical distance (L)

INTRODUCTION

In water and wastewater treatment facilities, the mass transport and behaviour of fine-grained cohesive sediments is influenced mainly by flocculation effects and nominal settling velocities of particles. Hence the understanding of batch settling processes of flocs is fundamental for effective thickener/clarifier design and control. The behaviour of flocculating particles and settling trajectories of individual particles is very complex. Flocculation effects and velocities are usually investigated using jar tests to establish dose, and settling columns to evaluate the flocculation behaviour.

Constant spatial and temporal variations and fluctuating initial conditions in physical sedimentation systems result in difficulty and uncertainty in the predictions of medium- and long-term behaviour of the settling particles (Xu et al., 2008; Mikkelsen et al., 2005). During the design of sedimentation tanks, data from settling columns are interpreted by a graphical technique. Firstly, samples are collected from different column depths at different times and are analysed for total suspended solids concentration (TSS). The batch settling data is then utilised to compute iso-percentage removal profiles as a function of time and depth. From the graphs, one can predict or calculate the removal efficiency, overflow rate and settling velocities of particles. However, it is common practice in industry to manually construct the isopercentile curves due to their non-linear nature. This practice is tedious, inaccurate, and susceptible to human error.

There is also the problem of irreproducibility since no two different technicians can construct exactly the same curves. When using currently proposed models to fit the percentage removal data, some fit the data well only at short retention times and others tend to violate the physical meaning of settling data profiles (Je and Kim, 2002). The use of inaccurate settling equations could lead to significant errors of aggregate properties.

This article evaluates a model based mainly on the prediction of the settling velocities derived as the integrated form

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of the rule-based model. Equipped with a robust velocity model, the analysis could then analytically revert back to the isopercentile trajectory analysis without the need for analysis of the physical data from a settling column. This approach will introduce accuracy and reproducibility, and will save time in the sedimentation zone and clarifier designs.

THEORETICAL BACKGROUND

Existing models for iso-percentile removal profiles

Modelling of the particle settling processes relies on the understanding of the dynamics of the flocculation system, variability of sediment concentration and composition, floc size, microbiological activity, salinity and temperature (Khelifa and Hill, 2006). Most current models assume a one-dimensional flow of particles in space and are therefore modelled by a set of linear differential equations. However, the true nature of settling particles is highly nonlinear and is influenced by fluctuating initial conditions.

Several analytical systems have been suggested for simulation of iso-percentile concentration or removal profiles. Any suggested model may pass or fail on mathematical grounds or inconsistency with physical systems. According to Je and Kim (2002), a model can fit and predict flocculent settling profiles, only if it does not violate the physical meanings of flocculent settling curves. These conditions have been re-stated in this paper under Ramatsoma and Chirwa's modelling review (Eqs. (15)-(19)).

Available iso-percentiles removal models from literature include Ozer's model (1994), Sans' model (1989), Berthouex and Stevens (1982), rule-based model by Montgomery (1979), a combination of the rule-based model with regression by Hayes (1992) and the power law model by Piro and coworkers (2011). Amongst the four that were reviewed by Je and Kim (2002), only the Berthouex and Stevens (1982) model violated the criterion of constraint for settling particles and could not fit data at long settling times.

The best simulation results have so far been obtained using the three empirical models below, i.e. Özer's (1994) model, San's (1989) model and lately Piro and coworkers' (2011) model. Ozer's (1994) analytical approach was based on the observation of rainfall intensity duration frequency. The nonlinear relationship between the rainfall intensity, duration, and frequency of occurrence is viewed as a multiplicative model. Hence the author utilised this analogy to model the relationship between concentration percentage, P^* , remaining in suspension with time and column depth as follows:

$$P^* = \alpha_1 D^{\alpha_2} t^{\alpha_3} \quad (1)$$

$$P^* = \alpha_1 D^{\alpha_2} t^{\alpha_3} - 1 \quad (2)$$

where:

- P^* = percentage remaining $(X_i/X_o) \times 100$
- X_o = initial concentration in the column (ML^{-3})
- D = depth travelled by particle during settling (L)
- t = time of settling (T)

The parameters $\alpha_1, \alpha_2, \alpha_3$ = fitting parameters for the iso-percentile lines. Sans' (1989) expressed the per cent total suspended solids (TSS) remaining in tank, P^* , as a function of settling time (T) and depth D (L) in a quiescent settling column (Eq. (3)).

$$P^* = \frac{t^\beta}{\alpha D^k + t^\beta} \quad (3)$$

where:

$\alpha, \beta,$ and k = optimisable parameters.

San's and Özer's models have been shown to comply with the assumptions stated by Je and Kim (2002).

Piro et al. (2011) developed the simplest settling model presented by a power law function (Eq. (4)), where the removal of particles is expressed as a function of time at a particular distance from the top of a water column.

$$h = \bar{a} t^{\bar{b}} \quad (4)$$

where:

h is the depth (L), t (T) is the residence time
 \bar{a} and \bar{b} are empirical parameters.

If a derivative of distance is taken against time, the term decays to the velocity term (Eq. (5)).

$$\frac{dh}{dt} = abt^{b-1} \quad (5)$$

Based on this power law, the settling of the particles is discrete (Type I settling) if $\bar{b} = 1$ and is flocculent if $\bar{b} > 1$ and \bar{a} is the velocity term (LT^{-1}).

Existing settling velocity models

During settling, suspended particles coalesce with smaller solids to form larger flocs. Settling velocities of cohesive sediments are not uniquely related to particle sizes as in Stokes' law because the average density of a floc and surface area properties of the particles are time variant and highly irregular. The transport rate depends on floc size, floc density, fractal dimension and gravity (Lee et al., 2002).

In earlier studies by Cho et al. (1993), the so called 'solid flux' concept for settler calculation was introduced. So far, 2 empirical models have been utilised successfully for solid flux settler design. These include the power law model (Eq. (6)) and the exponential model (Eq. (7)):

$$v = kx^{-n} \quad (6)$$

$$v = k \exp(-nx) \quad (7)$$

where:

k, x and n are the maximum settling velocity, solids concentration and model parameter, respectively

The exponential model is reasonable in dilute concentrations but is more complicated in designing a settler. The power law model on the other hand becomes infinite in a dilute concentration range.

Je and Chang (2004) derived a simple velocity formula (Eq. (8)) based on Ozer's flocculation model and a one-dimensional continuity equation. The equation could predict the transport of re-suspended sediments introduced by dredging operations. The same analogy was followed in this paper, but with a more reliable flocculation model.

$$V_s = \frac{-aZ}{(b+1) \cdot T} \quad (8)$$

where:

Z , T , a and b are the settling column depth, time and parameters, respectively

The Ramatsoma and Chirwa Model reviewed

Ramatsoma and Chirwa's semi-empirical model was based on the original assumptions by Özer (1994) to arrive at an analogous model with optimisable parameters r_1 , r_2 , r_3 (Ramatsoma and Chirwa, 2012).

$$H_i = H_{\max} + r_1 P^2 t^{r_2} - r_3 P^2 t \quad (9)$$

where:

r_1 , r_2 and r_3 are semi-empirical optimisable parameters
 H_i = height of sampling points
 H_{\max} = design height of proposed tank
 P = percentage removal $(1 - X_i/X_o) \times 100$.

On rearranging Eq. (9) and writing it with the full range of parameters:

$$P = \left(\frac{-D}{r_1 t^{r_2} - r_3 t} \right)^{0.5} \quad (10)$$

where the following parameter constraints apply within the meaningful physical space with real mass concentration:

$$D = H_{\max} - H_i \quad (11)$$

$$r_1 < 0 \quad (12)$$

$$r_2 > 0 \quad (13)$$

$$0 \leq r_3 \sim \frac{0.1}{r_2} \quad (14)$$

The right hand side of Eq. (14) was used as a first guess of the value of r_3 during parameter optimisation. Eq. (10) is then utilised to construct iso-percentile removal profiles for the settling process. Ramatsoma and Chirwa ensured that the model adheres to the physical meanings of a settling particle as stated by Je and Kim (2002).

Physical conditions considered during the simulation of sedimentation tanks should satisfy the following conditions at any layer j , time t and depth D : (i) the settling velocity should always be positive, (ii) TSS concentration remaining should increase with depth, (iii) TSS concentration remaining should decrease with time, and (iv) variation of TSS concentration remaining should not increase with depth. These conditions are represented by the following equations, Eqs. (15)–(19).

$$X_j = f(t, D) \quad (15)$$

$$\frac{\partial D}{\partial t} \geq 0 \quad (16)$$

$$\frac{\partial X_j}{\partial D} \geq 0 \quad (17)$$

$$\frac{\partial X_j}{\partial t} \leq 0 \quad (18)$$

$$\frac{\partial^2 X}{\partial D^2} \leq 0 \quad (19)$$

where:

X_j = suspended solids concentration (TSS) (ML^{-3})
 D = distance travelled by particle (L)
 t = time of travel (T)

Proposed flocculent velocity model derivation

A general sediment transport equation in a quiescent settling column is described by a one-dimensional continuity equation, Eq. (20) (Je and Chang, 2004).

$$\frac{\partial C}{\partial t} = \frac{\partial(D_z \partial C)}{\partial Z^2} - \frac{\partial(U_z C)}{\partial Z} + \frac{\partial(V_s C)}{\partial Z} \quad (20)$$

where:

t = settling time (h)
 Z = settling column depth (m)
 U_z = vertical water velocity (m/h)
 V_s = flocculent settling velocity (m/h)
 D_z = dispersion coefficient in the vertical direction (m^2/h)

In a batch column with quiescent flow, the continuity equation can be further simplified by applying the following assumptions:

- There is no vertical flow passing through the settling column
- There is no vertical dispersion within the settling column
- Since there is no bottom re-suspension, additional source terms do not apply

Using the above assumptions Eq. (20) is simplified to Eq. (21):

$$\frac{\partial C}{\partial t} = \frac{\partial(V_s C)}{\partial Z} \quad (21)$$

Eq. (21) is then integrated with respect to depth, z , to give Eq. (22):

$$V_s C = - \int \frac{\partial C}{\partial t} dz \quad (22)$$

On substituting Ramatsoma and Chirwa's flocculation model into Eq. (22):

$$V_s \left(\frac{-D}{r_1 t^{r_2} - r_3 t} \right)^{0.5} = - \int \frac{\partial}{\partial t} \left(\frac{-D}{r_1 t^{r_2} - r_3 t} \right)^{0.5} dz \quad (23)$$

Integrating Eq. (23) gives the settling velocity equation as a function of time and depth:

$$V_s = - \frac{D^{1.5}}{3} (r_1 r_2 t^{r_2-1} - r_3) \left(\frac{1}{r_1 t^{r_2} - r_3 t} \right)^{1.5} \quad (24)$$

This Eq. (24) becomes important in determining the velocity of particles that are 100% removed at given residence times and tank depth.

MATERIALS AND METHODS

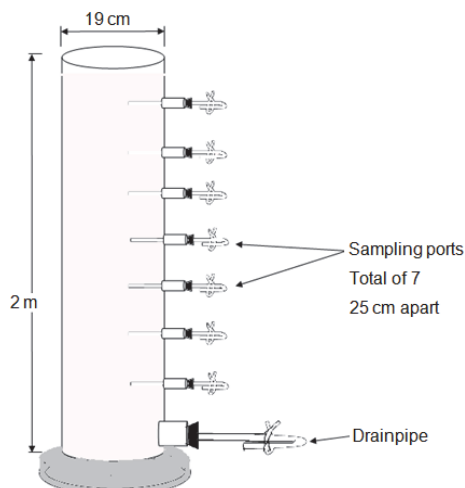
Standard batch settling column

Figure 1 shows a simple batch column that was used in the collection of physical data. A 100 l mixing tank was used together with a stirrer to mix water with a ferric chloride ($FeCl_3$) coagulant (Experiments 1–5) and an aluminium sulphate ($Al_2(SO_4)_3$) coagulant (Experiments 6–7) and dry clay soil (Table 1). A 5 l beaker was used to transfer the dilute suspension from the mixing tank to the column (200 cm \times 19 cm diameter). Sampling bottles were used to withdraw samples from the column at any given time.

Simulation of dirty water

In the simulation of dirty water, 90 l of water was poured into

Figure 1
Settling column test



the mixing tank and approximately 400 g of dry clay soil was added to make the water dirty. The coagulant was then added and the stirrer was switched on for 2 min at 180 r/min. The colloidal suspension was then carefully transferred into the test column. Immediately after filling the column, 2 to 3 min were allowed before starting data collection. Samples from the 7 sampling ports were withdrawn at different intervals. Table 1 shows the experiments that were performed.

Samples were analysed by a HACH Turbidity Meter (Model 2100N, Hatch Company, Loveland, CO) and the recorded values in NTUs were converted to the mass concentration using the equation $\ln(\text{TSS}) = 1.5 \ln(\text{NTU}) + 0.15$, based on the 10-point calibration line from a TSS versus NTU data set ($R^2 = 0.97$) (Packman et al., 1999). The total suspended solids (TSS) were measured in the range 10–300 mg/l as dry weight of suspended solids trapped on a 0.45 μm micro-pore membrane filter (APHA, 2005).

Experiment No.	Coagulant concentration (mg/l)	Average initial solids concentration (NTU)
1	10	195
2	50	208
3	70	170
4	68	112
5	70	141
6	10	235
7	15	242

Numerical analysis and optimisation

Iso-percentile lines were calculated by fitting the model (Eq. (10)) to interpolated values of the chosen percentiles from experimental data collected from the settling columns. Data interpolation and curve fitting was done using the programming platform, Octave (GNU Octave Version 3.4.2, Free Software Foundation, Boston, MA). Individual iso-percentile removal profiles were calculated from the geometrical relationship between the percentage removal and column height at any particular time according to the model chosen.

RESULTS AND DISCUSSION

Evaluation of settling data

Suspended solids concentration data was collected from settling columns in experiments run for 80 min in a 2.0 m column. An example of the results from an optimum dose of 10 mg/l $\text{Al}_2(\text{SO}_4)_3$ is shown in Table 2 and the corresponding percentage removal data is shown in Table 3. The results collected were characteristic of most data from similar water samples.

Distance, H (m)	Concentration at different times						
	3 min	10 min	20 min	30 min	40 min	60 min	80 min
1.75	126	76	51	32	21	16	11
1.5	132	80	55	42	26	20	14
1.25	154	96	58	45	33	29	17
1	--	103	60	47	42	32	23
0.75	--	113	64	49	43	39	24
0.5	--	118	67	56	52	32	27
0.25	--	122	74	65	52	39	31

Distance, H (m)	Removal percentage (P) at different times						
	3 min	10 min	20 min	30 min	40 min	60 min	80 min
1.75	46.3	67.8	78.5	86.4	91.1	93.4	95.2
1.5	44.0	65.8	76.4	82.3	88.8	91.6	94.2
1.25	34.6	59.2	75.3	81.0	85.9	87.8	92.7
1	--	56.2	74.3	80.2	82.2	86.4	90.3
0.75	--	51.8	72.8	79.2	81.7	83.3	89.7
0.5	--	49.8	71.3	76.2	77.9	86.4	88.5
0.25	--	47.9	68.7	72.3	77.9	83.4	87.0

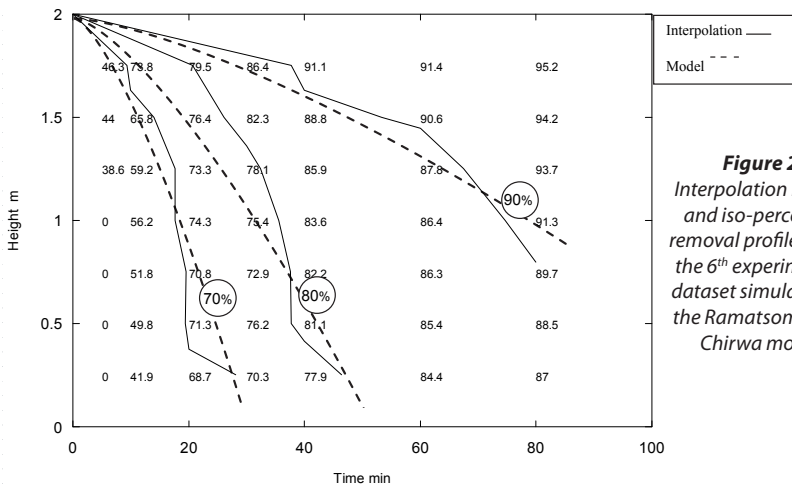


Figure 2
Interpolation results and iso-percentile removal profiles from the 6th experimental dataset simulated by the Ramatsoma and Chirwa model

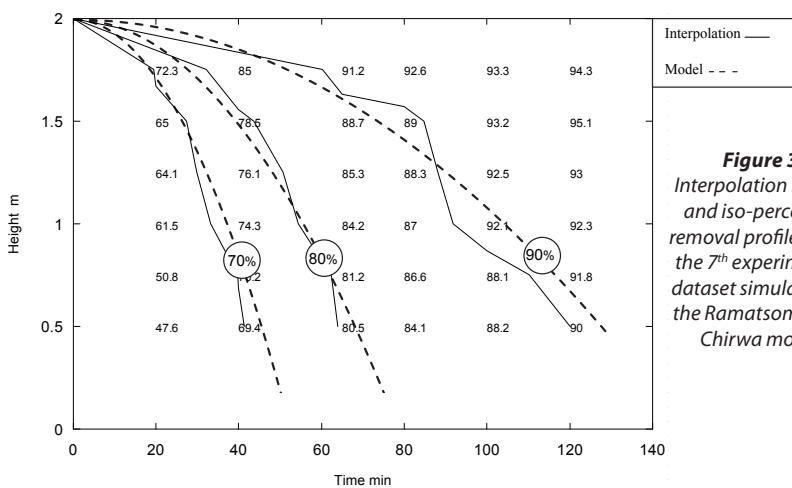


Figure 3
Interpolation results and iso-percentile removal profiles from the 7th experimental dataset simulated by the Ramatsoma and Chirwa model

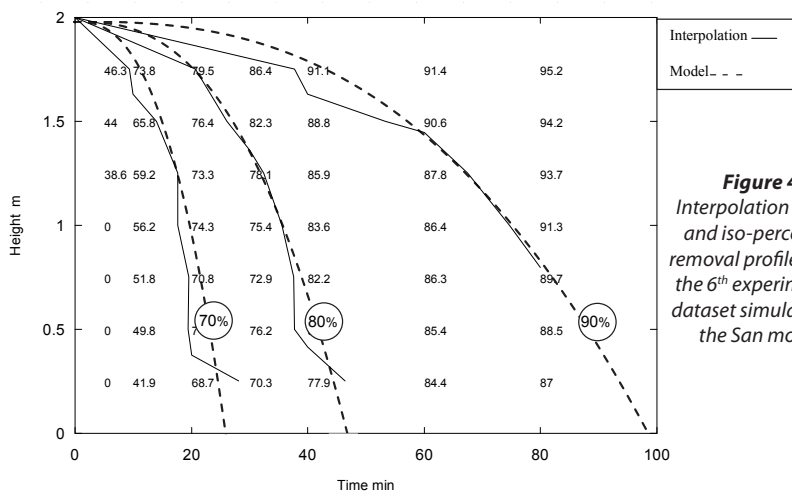


Figure 4
Interpolation results and iso-percentile removal profiles from the 6th experimental dataset simulated by the San model

Examples are found in a range of technical texts including Tchobanoglous et al. (2003), Reynolds and Richards (1996), Sawyer et al. (2003), and others. The conventional method of analysing these data is by calculating percentage removals (usually by spreadsheet), plotting interpolation points, followed by plotting smoothed iso-percentile lines using a model.

Interpolation and simulation

Before simulation of iso-percentile lines, a computational

algorithm was used to generate the raw percentage removal data points. Normally, this exercise is done by hand in the real industry context. The automated interpolation was conducted across the grid in the zero and 90° directions from a given data point. The accuracy of the interpolated data set is critical to the accuracy of the final percentiles since these provide the fixed scaffold points for parameter optimisation and therefore determine the degree of the overall model accuracy. For this reason, the improvement of the interpolation procedure is regarded in this study as the key to achieving the parameters of the highest possible accuracy for physical design of clarifiers.

The model presented in Eq. (9) and (10) was then used to provide the smooth lines to approximate the trajectory of constant percentage removal. The model was tested against data collected in the laboratory and previous data presented in textbooks. Representative data from practical data collected in the laboratory is presented for example in Figs. 2, 3, 4 and 5.

Typically, iso-percentile plots do not fit all areas of the data equally well. In practice, the accuracy of the interpolation is only seen after simulation of the whole data set and is evaluated holistically. This process can be improved by including 2 or more data points from the surrounding iso-percentile line and then establishing a point and using statistical means to improve accuracy.

In the given example data sets (Figs. 2-5), a reasonable fit of model data to interpolation (real) data is observed in Fig. 3 using Ramatsoma and Chirwa's model. However in Fig. 2, Ramatsoma and Chirwa's model underestimates the interpolated removal profiles at the top three column ports. The reason for this may be due to insufficient flocculation having occurred in the first few minutes for the particles to behave in the manner consistent with theoretical flocculent particles. In Fig. 4, Sans' model overestimates the 90% profile within the first 40 min. In Fig. 5, Sans' model also overestimates the 70%, 80% and 90% removal profiles, with the 90% profile fitting only between 60 min and 80 min. This inconsistency may be due to the limited ability of empirical models.

Error analysis

In order to check the accuracy or performance of the model, the interpolated values were used as the real values and compared with the predicted model values. In an earlier study, Je and Kim (2002) observed better performance of mechanistic models over

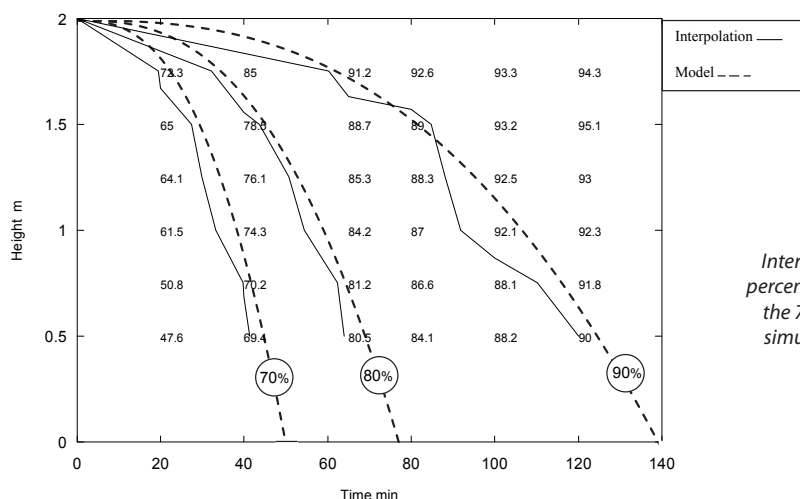


Figure 5
Interpolation results and iso-percentile removal profiles from the 7th experimental dataset simulated by the San model

empirical models under certain data ranges. The lack of fit of empirical models was later attributed to limited range as they failed to approximate data around the outer boundaries of the calibrated areas. Another semi-empirical model, the Berthouex and Stevens' model, was the second best for fitting the data but it was later rejected as it violates the flocculent sedimentation rules stated in Eqs. (18) and (19).

The fitting of the datasets was compared with San's model since it is consistent with the physical basis of flocculent sedimentation. It is also among the best fits of the literature-based

models, as concluded by Je and Kim (2002). As a result the proposed model is compared to San's model. The model fitness is determined by the statistical data presented in Table 4.

Visual inspection of the iso-percentile computations using the semi-empirical model derived by Ramatsoma and Chirwa achieves a reasonably accurate representation of the per cent removals. The accuracy of the model is shown in Table 4 for Ramatsoma and Chirwa's model versus San's model, and in Tables 5 and 6 for Ramatsoma and Chirwa's versus Özer's model. Comparison of the sum of squares error (SS_E) between

TABLE 4
Model performance compared against San's model using experimental data with 10 mg/ℓ $Al_2(SO_4)_3$ coagulant and literature data (Source: this study)

Data source	Ramatsoma and Chirwa's model				San's model			
	r_1	r_2	r_3	SS_E	a	b	k	SS_E
Experiment 6	-0.2	1.4	71	1.21	0.008	-0.06	0.23	1.95
Experiment 7	-0.009	2.0	118	0.74	1.91	0.55	-0.44	0.81
Reynolds & Richards (1996)	-0.027	1.5	540	0.30	0.043	-0.60	-0.23	0.20
Peavy et al. (1985:122)	-0.02	1.3	79	0.15	0.321	-0.30	-0.23	0.39

TABLE 5
Model performance compared against Özer's model using experimental data with 10 mg/ℓ $FeCl_3$ coagulant (From: Ramatsoma and Chirwa, 2012)

Experiment No.	Ramatsoma and Chirwa's model				Özer's model			
	r_1 ($\times 10^{-3}$)	r_2	r_3 ($\times 10^{-3}$)	SS_E	a_1	a_2	a_3	SS_E
1	-0.94	2.141	0.00328	3.62	4.1	0.45	-0.53	9.58
2	-0.7	2.09	0.000041	1.71	4.35	0.46	-0.512	3.23
3	-0.7501	2.2	0.02911	3.58	4.5	0.69	-0.56	8.59
4	-0.751	2.048	0.01681	1.71	4	0.5	-0.48	4.65
5	-1	2.4	15.7	3.52	2.8	0.65	-0.51	13.54

TABLE 6
Model performance compared to Özer's model using literature data (From: Ramatsoma and Chirwa, 2012)

Data source	Ramatsoma and Chirwa's model				Özer's model			
	r_1 ($\times 10^{-3}$)	r_2	r_3 ($\times 10^{-3}$)	SS_E	a_1	a_2	a_3	SS_E
Je et al. (2007)	-1.3	1.64	71	1.31	2.439	0.55	-0.441	1.34
Reynolds and Richards (1996)	-0.76	2.101	118	1.49	1.91	0.55	-0.44	3.72
Berthouex and Stevens (1982)	-1.12	2.307	540	1.55	1.938	0.250	-0.633	0.85
Eckenfelder (1989)	-0.002	1.0009	79	6.16	2.365	0.46	-0.428	6.70

the fit from Eq. (10) and San's model (Table 4), shows that Ramatsoma and Chirwa's model performed better than San's model for the data produced in the laboratory and literature data from Peavy et al. (1985). For an unknown reason, San's model produced a better fit from literature data by Reynolds and Richards (1996). The initial concentration of suspended solids in the literature data by Reynolds and Richards (1996, p. 236) was slightly higher ($X_0 = 320 \text{ mg/l}$) than the initial concentration in the measured experimental data ($X_0 =$ approximately 112–242 mg/l). This probably affected compaction factors resulting in different settling behaviour.

Further analysis was conducted using Özer's model and Ramatsoma and Chirwa's model. To challenge the model, a different coagulant (FeCl_3) was used at lower solids concentration. Compared to Özer's model, Ramatsoma and Chirwa's model performed much better when evaluated against experimental data and literature data, with the exception of literature data from Berthouex and Stevens (1982), (Tables 5–6). In this case, the concentrations in the data from literature were very high (up to 800 mg/l). This again appears to have affected the behaviour of the settling particles such that the semi-empirical model fell out of range of some of the interpolated data points.

Model validity

The observed order of performance in fitting experimental data was Ramatsoma and Chirwa's model > San's Model > Özer's model. At higher initial concentrations, San's model outperformed the Ramatsoma and Chirwa model resulting in the reversal of performance order to San's model > Ramatsoma and Chirwa's model > Özer's model. Theoretically, a rule-based (mechanistic model) is envisaged to be suitable for highly variant conditions whereas most empirical models operate well within a narrow range of conditions. This expectation was confirmed in this study by the better performance of the semi-empirical Ramatsoma and Chirwa model in the dilute to medium concentration ranges. The observed results show promise that this model could be improved to simulate the full range of concentrations including the high concentration scenario.

Velocity model

According to Je and Chang (2004), sediment suspensions at higher initial concentration aggregate at a faster rate than those at low initial concentration. This is in line with the findings of this study. It is observed in Table 2 that the addition of the coagulant (within 3 min) resulted in the decrease of TSS concentration in the top part of the column and an increase of TSS concentration in the bottom part of the column. The same scenario applies for 10 min and 20 min up to 60 min. This result is due to more frequent inter-particle collisions within the column.

If one calculates the difference between the concentration at the lowest depth (0.25 m) and the one at the highest depth (1.75 m) for all the time frames, there is a general decrease in the amount of flocs (concentration) removed with time. The TSS concentration differences are 46 mg/l, 21 mg/l, 33 mg/l, 23 mg/l and 20 mg/l for 10 min, 20 min, 30 min, 40 min, 60 min and 80 min, respectively. The exception with the 20 min concentration may be due to floc break up. A larger amount of solids was removed within the first 10 min (46 mg/l), implying a high aggregation rate.

From the above reasoning, it is evident that as the floc formation increases down the column, the velocity also

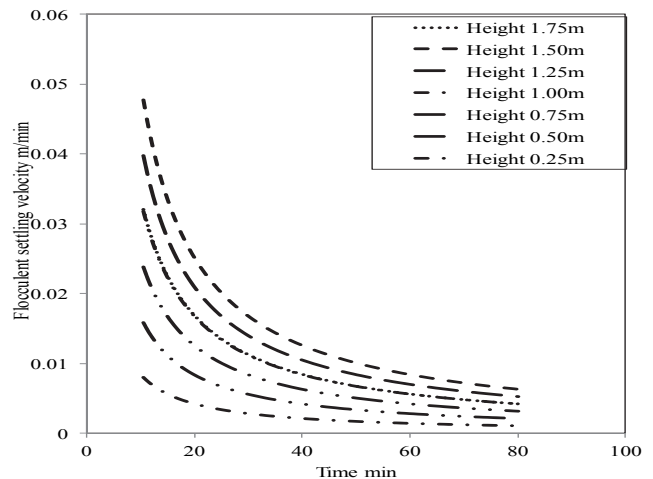


Figure 6
Settling velocity curves from the 6th experiment, coagulant dose ($10 \text{ mg/l Al}_2(\text{SO}_4)_3$), simulated by the proposed model

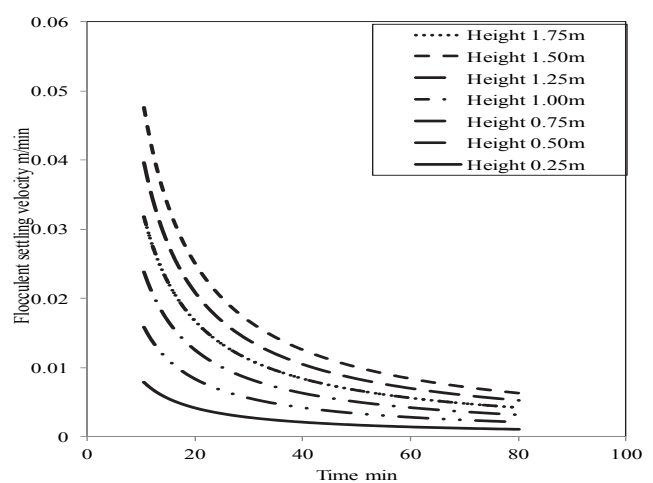


Figure 7
Settling velocity curves from the 7th experiment, coagulant dose ($15 \text{ mg/l Al}_2(\text{SO}_4)_3$), simulated by the proposed model

increases. However, if one considers the instantaneous velocity, it can be noticed that the velocity at 15 min and a depth of 1.75 m is higher than the one at the same depth but at a later time, i.e. 60 min. With time the velocity of the aggregates slows down to a lower value as the majority of larger aggregates quickly settled within the first 25 min. By that time the column looks less turbid. Also the effect of the coagulant dose becomes less pronounced, hence a low settling velocity is inevitable with time.

The same velocity pattern is observed from Fig. (6) to Fig. (9). For instance, in Fig. (6) the velocity at height of 1.75 m and 10.51 min is 0.0478 m/min and this decreased to 0.00629 m/min after 80 min. The same decline in velocity has been noticed even for literature data. The velocity for Reynolds and Richards (1996) data at a height of 1.75 m and 13.13 min is 0.0381 m/min and decreased to 0.005089 m/min after 99 min. It can be concluded that as the floc size increases due to increases in collision frequency as a result of Brownian motion and differential settling, the velocity also increases, but would decrease with settling time to a lower value as the column becomes

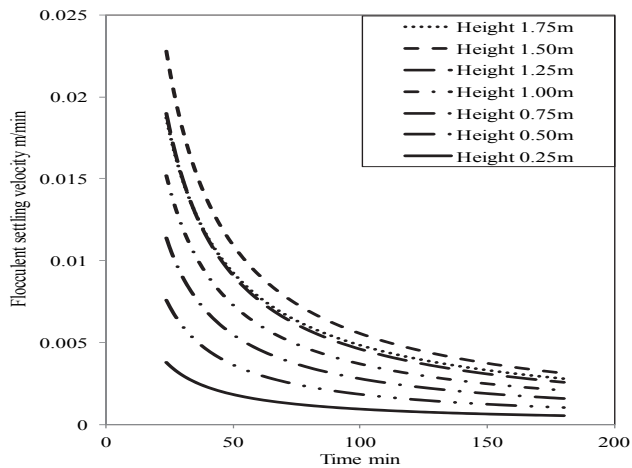


Figure 8

Settling velocity curves from literature data, Peavy et al (1985), simulated by the proposed model

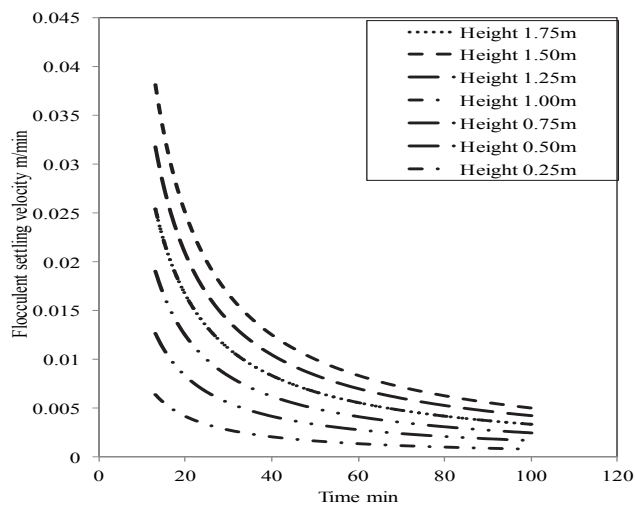


Figure 9

Settling velocity curves from literature data, Reynolds and Richards (1996), simulated by the proposed model

clearer. Hence the proposed settling velocity model can successfully determine the instantaneous velocity of flocs.

CONCLUSION

In this study, an improved model developed by Ramatsoma and Chirwa (2012) performed better than San's model and Özer's model in interpreting settling column data. Human errors due to interpolation by hand were minimised by using a reliable interpolation/optimisation algorithm to evaluate settling column data. Automation of data analysis will enable engineers and scientists to perform more advanced interpolation/optimisation schemes of settling column data to improve design accuracy. A velocity integrated equation emanating from the above model was also successfully used to determine instantaneous flocculent settling velocity of a settling particle, which is critical for design and control of sedimentation basins. The outcome was found to be in agreement with literature results in which the velocity increases due to increase in floc size. It was also noted that the velocity decreases with time as fewer and less heavy particles remain in suspension.

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