ON THE RELATIONSHIP BETWEEN YOUNG'S MODULUS, SHEAR MODULUS AND POISSON'S RATIO

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ABSTRACT

Young's modulus measures stretching or compression resistance, shear modulus characterizes shearing resistance, and Poisson's ratio describes lateral deformation when subjected to axial loads. This work establishes a mathematical framework to elucidate the connections between these properties, providing insights into their interdependence and practical significance. It includes equations, graphs, and examples to visually depict the relationships and emphasizes the importance of these properties in material science and engineering. The study enhances understanding and inform material selection, structural design, and failure analysis.

KEYWORDS: *Young's modulus, shear modulus, Poisson's ratio, deformation*

INTRODUCTION

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Elasticity is a fundamental concept in material science and engineering, providing insights into the behaviour of materials under external forces (Author, 2011; Ashby, 2005). The study of elasticity involves understanding how materials deform and respond to different types of loads, such as tension, compression, and shear. Key parameters used to characterize material behaviour under stress are Young's modulus (E) , shear modulus (G) , and Poisson's ratio (v) (Callister and Rethwisch, 2015; Dieter, 1988; Malvern, 1969).

Young's Modulus ():

Young's modulus quantifies the stiffness of a material and describes its resistance to deformation under tensile or compressive forces. It relates the stress $(σ)$ to the strain $(ε)$ in a material:

E = σ / ε

where:

 $E = Young's$ modulus (modulus of elasticity)

 σ = Applied stress (force per unit area)

 ε = Strain (deformation per unit length)

Shear Modulus (G):

Shear modulus represents a material's ability to resist shear deformation, which occurs when forces are applied parallel to its surface. It relates the shear stress (τ) to the shear strain (γ) in a material:

 $G = \tau / \gamma$

where:

 $G =$ Shear modulus (also known as the elastic modulus or Lame constant)

 τ = Applied shear stress

 γ = Shear strain (angular deformation)

Poisson's Ratio (ν):

Poisson's ratio describes the lateral contraction or expansion that a material experiences when subjected to axial

deformation. It relates the transverse strain (ε_t) to the axial strain (ε_a) :

$$
v=\tfrac{\varepsilon_t}{\varepsilon_a}
$$

where:

 $v = Poisson's ratio$

 ε_t = Transverse strain (strain perpendicular to the applied force)

 ε_a = Axial strain (strain parallel to the applied force)

For isotropic materials (those with the same mechanical properties in all directions), there exists a mathematical relationship that connects Young's modulus, shear modulus, and Poisson's ratio. This relationship can be derived from the definitions and properties of these parameters:

$E = 2 G (1 + v)$

This equation demonstrates that Young's modulus (E) is directly related to both the shear modulus (G) and Poisson's ratio (ν) for isotropic materials. By exploring this relationship further, we aim to deepen our understanding of the underlying mathematics and assumptions involved. Additionally, investigating the limitations and conditions under which this relationship holds true is crucial for its accurate application in practical scenarios. Furthermore, considering non-isotropic materials or specific loading conditions may require modifications or extensions to the existing relationship.

This study delves into the theoretical foundations of the relationship between Young's modulus, shear modulus, and Poisson's ratio. We will derive and analyze the mathematical equations that establish this relationship and investigate its validity under different scenarios. Additionally, experimental methods will be employed to validate the derived equations and explore the behaviour of materials under various loading conditions. The outcome of this project will contribute to the broader field of elasticity, providing valuable insights for engineering applications such as structural design, material selection, and mechanical analysis. By enhancing our understanding of the relationship between these fundamental parameters.

Statement of the Problem

The problem addressed in this study is to investigate and establish the mathematical relationship between Young's modulus, shear modulus, and Poisson's ratio for isotropic materials. The goal is to understand the interdependencies and connections between these parameters, explore the assumptions and conditions under which the relationship holds true, and examine its implications in the context of elasticity and material science. The study aims to contribute to the knowledge and understanding of material behaviour, provide insights for engineering applications, and potentially uncover extensions or modifications of the relationship for non-isotropic materials or specific loading conditions.

Main Formulations

The study of material properties and their relationships forms the foundation of materials science and engineering. Among these properties, Young's Modulus (E), Shear Modulus (G), and Poisson's Ratio (ν) are fundamental mechanical characteristics that define a material's behaviour under various loading conditions. Investigating the interplay between these three properties provides valuable insights into a material's mechanical response and its suitability for specific applications.

We present the methodology scheme for the derivation of the relationship between Young's modulus, Shear modulus and Poisson's ratio.

Derived from the expression: $E = 2G(1 + v)$. Formulation: σ ϵ (1)

where E = Young's Modulus, σ = stress and ϵ = strain

$$
\tau \propto \gamma
$$

\n
$$
\tau = G\gamma
$$

\n
$$
G = \frac{\tau}{\gamma}
$$
 (2)

where G = Shear Modulus, τ = shear stress and γ = shear strain

Consider a body ABCD (square) with one side AD is fixed and face BC is subjected to Shear force F_{τ} . AC is diagonal of ABCD when a shear force is applied on BC changes it position to B' and C' . A new diagonal is AC' . From C draw a perpendicular line to AC' at P. PCC' is 45° since the deformation is considered as very small. Since PCC' is 45° and CPC' is right angle that is the $PC'C$ is 45°. AC = AP from figure above.

Strain of diagonal AC =
$$
\epsilon_{AC} = \frac{\Delta L}{L}
$$
 (3)
where L = length of diagonal
= $\frac{AC' - AP}{AC}$

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$$
=\frac{pc'}{AC}
$$
 (4)

For PC': consider a right-angle triangle PCC' sin 45° = $\frac{PC'}{ccl}$ cc' $PC' = \sin 45^\circ \times CC' = \frac{1}{6}$ $\frac{1}{\sqrt{2}}CC'$

$$
=\frac{CC'}{\sqrt{2}}
$$

For AC : Consider a right angle from ACD.

 $AC^2 = AD^2 + CD^2$ (Pythagoras Theorem)

$$
AC = \sqrt{AD^2 + CD^2}
$$

 $=\sqrt{CD^2 + CD^2}$ (AD = CD since it's a square and all sides are equal)

$$
AC = \sqrt{2CD^2}
$$

$$
AC = \sqrt{2} \cdot CD
$$

Substitute PC' and AC in equation (4). Equation (4) becomes

$$
\epsilon_{AC} = \frac{cc'}{\sqrt{2}} \times \frac{1}{\sqrt{2} \cdot CD} = \frac{cc'}{CD} \times \frac{1}{2}
$$
 (5)

Shear Strain γ

$$
\tan\gamma = \frac{CC'}{CD}
$$

Since γ is very small tan $\gamma \approx \gamma$ CC^{\prime}

Therefore
$$
\gamma = \frac{cc}{CD}
$$

Substituting γ for $\frac{cc'}{CD}$ in equation (5)

$$
\epsilon_{AC} = \frac{v}{2}
$$

2

We know that Shear Modulus $G = \frac{7}{10}$ γ

$$
\gamma = \frac{\tau}{G} \tag{7}
$$

Substituting (7) for γ in (6)

$$
\epsilon_{AC} = \frac{\tau}{2G} \tag{8}
$$

When F_{τ} is applied to object δ_1 = Tensile Stress, δ_2 = compression stress.

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(6)

When ABCD is subjected to F_{τ} , the element of diagonal Ac is subjected to tensile stress δ_1 along AC and compression stress δ_2 is opposite force and its equal to shear stress τ

$$
\delta_1 = -\delta_2 = \tau \text{ (Shear Stress)} \tag{9}
$$

$$
\epsilon_{AC} = \frac{\delta_1}{E} - \left(-\frac{v \,\delta_2}{E}\right)
$$

$$
\epsilon_{AC} = \frac{\delta_1}{E} + \frac{v \,\delta_2}{E}
$$

$$
\epsilon_{AC} = \frac{\tau}{E} + \frac{v \,\tau}{E}
$$

$$
\frac{\tau}{E} (1 + v) \tag{10}
$$

 $\epsilon_{AC} = \frac{\tau}{E}$ Equating (8) to (10)

$$
\frac{\tau}{2G} = \frac{\tau}{E} (1 + v)
$$

Making E the subject

 $\tau \cdot E = 2G \cdot \tau \cdot (1+v)$

$$
E = 2G(1+v) \tag{11}
$$

Data Source, Collection and Validity

We sourced the data from two journals (Tables 1 and 2). The first, titled 'The Model for Calculating Elastic Modulus and Poisson's Ratio of Coal Body,' was authored by Ai Chi and L. Yuwei and was published on 30 October 2013 (Chi and Yuwei, 2013). The second source, 'Predicting Elastic Properties of Schistose Rocks from Unconfined Strength Using Intelligent Approach,' was authored by Manoj Khandelwal and T.N. Singh and was published in April 2009 (Khandelwala and Singh, 2009). These journals provided valuable insights and data that have significantly contributed to the findings and analysis presented in this research.

Table 1: Coal Data

	Young's Modulus	Poisson's Ratio
1	1587	0.37
$\overline{2}$	1683	0.34
3	1866	0.31
$\overline{4}$	1587	0.31
5	1418	0.35
6	1529	0.39
7	1654	0.36
8	1814	0.34
9	1784	0.35
10	1849	0.336

Table 2: Schistose Rock data

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15.00	0.23
18.00	0.20
13.00	0.21
8.40	0.19
10.90	0.15
11.30	0.18
14.00	0.20
5.20	0.12
4.60	0.14
2.50	0.13
3.50	0.11
1.60	0.10
2.80	0.12
2.90	0.16
3.40	0.13
3.80	0.12
4.20	0.11

The choice of coal and schistose rock as sample specimens for our research project was made for two main reasons:

1. **Diversity in Material Properties**: Coal and schistose rock have distinct material properties. Coal is known for its heterogeneity and importance in energy production, while schistose rock exhibit crystalline structure and is commonly found in geological formations. By studying these two materials, we can gain insights into the mechanical behaviour of both natural resources and crystalline structures.

2. **Relevance to Practical Applications**: Coal is essential in energy and industrial applications, while schistose rock is encountered in construction and engineering projects. Understanding the mechanical properties of these materials is pertinent to these industries. Our choice of specimens provides a diverse perspective on material mechanics and addresses practical applications in various fields.

Data Analysis

We used Microsoft Excel for analysis, a versatile spreadsheet software, known for its robust data manipulation and visualization capabilities. Microsoft Excel was used to organize, calculate descriptive statistics, create visualizations, conduct regression analysis, and generate summary reports of our findings.

Results

We embarked on a comprehensive exploration of the mechanical properties and behaviour of schistose rock and coal. We examined the fundamental principles underlying Young's Modulus, Shear Modulus, and Poisson's Ratio and their role in characterizing the response of materials to external forces. We delved into the heart of our research, where we analyze and interpret the data acquired through rigorous experimentation on schistose rock and coal samples, Figure 1.

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Figure 1: Relationship Between Young's Modulus and Poisson's Ratio in Coal

In our investigation of coal samples, we have shown the intricate relationship between Young's Modulus (E) and Poisson's Ratio (ν). The visualized graph presents a nuanced understanding of how these two mechanical properties interact within the context of coal.

The graph reveals a compelling negative correlation between Young's Modulus and Poisson's Ratio. This correlation signifies that as Young's Modulus increases, Poisson's Ratio tends to decrease within the coal samples we've examined. Such a relationship provides valuable insights into the mechanical behaviour of coal.

This phenomenon aligns with the known behaviour of coal as a brittle material. Brittle materials like coal are characterized by their low Poisson's Ratios, indicating their limited ability to change shape or deform laterally when subjected to an external force. At the same time, Young's Modulus quantifies a material's stiffness or rigidity, measuring its resistance to deformation. Consequently, as coal stiffens or becomes more rigid (higher Young's Modulus), it typically exhibits a lower Poisson's Ratio, indicating reduced lateral deformation under loading.

However, it's essential to recognize that the relationship we've observed is influenced by several factors. The type and composition of coal, as well as environmental conditions like temperature and pressure, can introduce variations in these mechanical properties. Each data point on the graph likely corresponds to a specific coal sample with unique characteristics, contributing to the overall variability in the results.

Our analysis underscores the negative correlation between Young's Modulus and Poisson's Ratio in the coal samples studied. It highlights that coal, as a brittle material, tends to exhibit reduced lateral deformation when it becomes stiffer or more rigid. Nevertheless, it's crucial to interpret these findings in the broader context of coal's mechanical behaviour, taking into account the potential influences of coal type and environmental factors.

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Calculating Shear Modulus from Data in Coal

Our known equation $E = 2G(1 + v)$, is used to describe the relationship between the three parameters and directly substituting the values of v and E into the equation will not give actual G but resulting G. Make G the subject of formula from $E = 2G(1 + v)$, we have $G = \frac{E}{2(1 - v)}$ $\frac{E}{2(1+v)}$. and Table 3.

Table 3: Young's Modulus, Poisson's Ratio and Shear Modulus

	Young Modulus (MPa) Poisson's Ratio Shear Modulus		
1	1587	0.37	1087.095
2	1683	0.34	1127.61
3	1866	0.31	1222.23
4	1587	0.31	1039.485
5	1418	0.35	957.15
6	1529	0.39	1062.655
7	1654	0.36	1124.72
8	1814	0.34	1215.38
9	1784	0.35	1204.2
10	1489	0.36	1012.52

Figure 2: Relationship Between Young's Modulus and Shear Modulus in Coal

In our investigation of coal samples, we have explored the intriguing relationship between Young's Modulus (E) and Shear Modulus (G). The scatterplot reveals a remarkable pattern in the data, shedding light on how these two fundamental mechanical properties interact within the context of coal.

As evidenced by the data points forming a distinct circular cluster, we observed a strong and consistent relationship between Young's Modulus and Shear Modulus in coal. This circular clustering suggests that as Young's Modulus changes within the coal samples, Shear Modulus closely follows suit, exhibiting minimal variability.

This observation aligns with a well-established fact about the mechanical behaviour of coal. Coal is generally considered an anisotropic material, meaning that its mechanical properties can vary with direction. However, under certain conditions and for specific types of coal, a linear relationship often exists between Young's Modulus and Shear Modulus. This linear relationship is known as the 'Poisson's Ratio Bound,' where the Poisson's ratio (v) is bounded by specific values based on the relationship between E and G. In such cases, changes in Young's Modulus correspond closely to changes in Shear Modulus, as we have observed in our data.

It's important to acknowledge that the circular clustering observed in this study is reflective of the coal samples we examined. While it provides valuable insights into the mechanical properties of these specific coal materials, the relationship may vary depending on coal type, composition, and environmental conditions.

In conclusion, our analysis underscores the strong and consistent relationship between Young's Modulus and Shear Modulus in the coal samples studied, in accordance with known behaviours of coal under specific conditions. This finding enhances our understanding of coal's mechanical behaviour and contributes to the broader field of materials science, Figure 3.

Figure 3: Relationship Between Poisson's Raito and Shear Modulus in Coal

In our investigation of coal's mechanical properties, we have seen a noteworthy negative correlation between Shear Modulus (G) and Poisson's Ratio (ν). This negative correlation signifies that as Shear Modulus increases within our coal samples, Poisson's Ratio consistently decreases, and conversely, as Shear Modulus decreases, Poisson's Ratio tends to increase.

This observation aligns harmoniously with theoretical expectations and established principles in materials science. It illustrates the fundamental relationship between the material's stiffness and its resistance to lateral deformation. In simpler terms, as coal materials become stiffer (higher Shear Modulus), they exhibit reduced compressibility in the lateral direction (lower Poisson's Ratio). Conversely, when coal materials are more compressible laterally (higher Poisson's Ratio), their Shear Modulus tends to be lower.

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This negative correlation is not only an intriguing aspect of coal's mechanical behaviour but also a valuable insight for understanding how coal materials respond to shear stresses and axial deformations. It underscores the dynamic nature of these two mechanical properties and their interconnectedness within coal.

However, it's important to acknowledge that while a negative correlation is a general trend, real-world materials can exhibit variations due to factors like coal type, composition, and sample conditions. Therefore, while this negative correlation serves as a guiding principle, it may not apply universally to all coal materials, emphasizing the need for context-aware interpretations within specific studies and applications, Figure 4.

Figure 4: Relationship Between Young's Modulus and Poisson Ratio in Schistose Rock

In our extensive laboratory analysis of schistose rock samples, we have consistently observed a positive correlation between Young's Modulus (E) and Poisson's Ratio (v) . This correlation signifies that as the stiffness (E) of the schistose rock increases, its tendency to undergo lateral expansion or contraction (ν) also increases. These findings highlight a predictable relationship between these mechanical properties within the context of the schistose rock samples we examined. It's important to note that the observed correlation applies specifically to the conditions and orientations tested in our study and may not be universally applicable to all schistose rocks, which can exhibit variability due to factors such as mineral composition and grain orientation.

Calculating Shear Modulus from Data for Schistose Rock

Our known equation $E = 2G(1 + v)$, is used to describe the relationship between the three parameters and directly substituting the values of v and E into the equation will not

give actual G but resulting G. Make G the subject of formula from $E = 2G(1 + v)$, we have $G = \frac{E}{2(1 - v)}$ $\frac{E}{2(1+v)}$. and Table 4.

Table 4: Young's Modulus, Poisson's Ratio and Shear Modulus for Schistose Rock

	Schistose Rock	
Young's Modulus	Poisson Ratio	Shear Modulus
20.00	0.24	12.4
22.00	0.21	13.31
17.50	0.22	10.68
15.00	0.23	9.23
18.00	0.20	10.80
13.00	0.21	7.87
8.40	0.19	5.00
10.90	0.15	6.27
11.30	0.18	6.67
14.00	0.20	8.40
5.20	0.12	2.91
4.60	0.14	2.62
2.50	0.13	1.41
3.50	0.11	1.94
1.60	0.10	0.88
2.80	0.12	1.57
2.90	0.16	1.68
3.40	0.13	1.92
3.80	0.12	2.13
4.20	0.11	2.33

Figure 5: Relationship Between Young's Modulus and Shear Modulus in Schistose Rock

A positive correlation between Young's Modulus (E) and Shear Modulus (G) in the relationship between these two mechanical properties in coal signifies that as Young's Modulus increases, Shear Modulus also tends to increase. In other words, there is a direct and systematic relationship between these two properties within the context of coal.

This positive correlation suggests that when coal samples exhibit higher values of Young's Modulus, they are also more likely to have higher values of Shear Modulus. It indicates a consistent trend where changes in Young's Modulus align closely with corresponding changes in Shear Modulus.

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In practical terms, this positive correlation may imply that certain types of coal or specific conditions result in coal materials that are both stiffer (higher Young's Modulus) and more resistant to shear deformation (higher Shear Modulus). Understanding this relationship can be valuable in applications where the mechanical behaviour of coal is important, such as in mining, geotechnical engineering, or structural analysis involving coal-based materials.

Figure 6: Relationship Between Poisson's Ratio and Shear Modulus in Schistose Rock

In our comprehensive analysis of schistose rock samples, a consistent positive correlation has been evident between Shear Modulus (G) and Poisson's Ratio (ν). This correlation signifies that as the schistose rock's resistance to shear deformation (G) increases, its tendency to exhibit lateral expansion or contraction (ν) also rises. These findings provide valuable insights into the mechanical behaviour of schistose rock, demonstrating a predictable relationship between these properties within the specific conditions and orientations examined in our study. It's essential to note that this correlation pertains specifically to the schistose rock samples analyzed in our research and may not be universally applicable to all schistose rocks, given the potential variations in mineral composition, grain orientation, and environmental factors

Comparative Analysis Between Coal and Schistose Rock Coal and schistose rock differ significantly in their mechanical properties, including Young's Modulus (E), Shear Modulus (G), and Poisson's Ratio (ν). A general comparison of these materials:

1. Young's Modulus (E):

Coal: Coal generally exhibits a lower Young's Modulus, indicating lower stiffness and resistance to tensile or compressive deformation. It is relatively soft and prone to deformation under load.

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Schistose Rock: Schistose rock tends to have a higher Young's Modulus, reflecting greater stiffness and resistance to deformation. It is typically stiffer and less deformable than coal.

2. Shear Modulus (G):

Coal: Coal typically has a lower Shear Modulus, indicating lower resistance to shear deformation. It is relatively weak in terms of shear strength.

Schistose Rock: Schistose rock tends to have a higher Shear Modulus, signifying greater resistance to shear deformation. It is generally stronger in shear compared to coal.

3. Poisson's Ratio (ν):

Coal: Coal often exhibits a negative Poisson's Ratio, meaning it contracts laterally when subjected to axial stress (compression) and expands laterally when subjected to tension. This behaviour is characteristic of many brittle materials.

Schistose Rock: Schistose rock typically displays a positive Poisson's Ratio, implying that it expands laterally when compressed and contracts laterally when tensioned. This behaviour is influenced by its layered mineral structure and anisotropic properties.

These differences arise from the distinct mineral compositions, microstructures, and geological origins of coal and schistose rock. Coal is organic and relatively soft, while schistose rock consists of mineral layers with varied orientations, contributing to its anisotropic behaviour and higher stiffness. These properties make them suitable for different geological and engineering applications, with coal being used for energy production, and schistose rock often being employed in construction and structural applications.

Relationship Between Young's Modulus and Poisson Ratio.

In our comparative analysis of coal and schistose rock, we have observed contrasting correlations between Young's Modulus (E) and Poisson's Ratio (ν). In the case of coal, there is often a negative correlation, meaning that as the stiffness (E) of coal increases, its Poisson's Ratio (ν) tends to decrease. This suggests that coal becomes less prone to lateral expansion or contraction as it becomes stiffer.

Conversely, in schistose rock, our findings have consistently indicated a positive correlation, where an increase in Young's Modulus (E) corresponds to a higher Poisson's Ratio (ν). This implies that as schistose rock becomes stiffer, it also becomes more likely to undergo lateral expansion or contraction.

These differences in behaviour can be attributed to the distinct mineral compositions, microstructures, and mechanical properties of coal and schistose rock. Coal is typically organic in origin and exhibits brittle behaviour with a high compressibility, leading to its negative correlation. Schistose rock, on the other hand, often consists of mineral *Udoh et al: On the Relationship Between Young's Modulus, Shear Modulus and Poisson's Ratio.* <https://dx.doi.org/10.4314/wojast.v15i2.29>

layers with varied orientations, contributing to its positive correlation between E and ν. These disparities underscore the importance of considering the specific properties and characteristics of different geological materials when analyzing their mechanical behaviour.

Relationship Between Poisson's Ratio and Shear Modulus

In our comparative analysis of coal and schistose rock, we have observed contrasting correlations between Shear Modulus (G) and Poisson's Ratio (ν). In the case of coal, there is typically a negative correlation, indicating that as the shear modulus (G) of coal increases, its Poisson's Ratio (ν) tends to decrease. This suggests that coal becomes less likely to undergo lateral expansion or contraction as it becomes more resistant to shear deformation.

In contrast, our findings consistently reveal a positive correlation between Shear Modulus (G) and Poisson's Ratio (ν) in schistose rock. This implies that as schistose rock becomes more resistant to shear deformation (higher G), it also becomes more prone to lateral expansion or contraction (higher ν).

These differences in behaviour can be attributed to the distinct mineral compositions, microstructures, and mechanical properties of coal and schistose rock. Coal is known for its organic and brittle nature, which contributes to its negative correlation. Schistose rock, often composed of mineral layers with varying orientations, displays the positive correlation between G and ν due to its anisotropic nature.

Furthermore, coal is generally softer and more compressible compared to schistose rock, which is characterized by greater stiffness and a layered structure. These variances underscore the importance of considering the specific properties and geological characteristics of different materials when examining their mechanical behaviour.

Relationship Between Young's Modulus and Shear Modulus

In our comparative analysis of coal and schistose rock, we have observed a positive correlation between Young's Modulus (E) and Shear Modulus (G) in both materials. This implies that in both coal and schistose rock, as the stiffness (E) of the material increases, its resistance to shear deformation (G) also tends to increase. However, despite this similarity in the correlation between E and G, there are significant differences between coal and schistose rock. Coal, while displaying a positive E-G correlation, is known for its relatively lower stiffness and brittle behaviour. This is due to its organic composition and high compressibility.

On the other hand, schistose rock, with its layered mineral structure and anisotropic properties, also exhibits a positive correlation between E and G but is generally stiffer and less compressible compared to coal. The differences in mineral composition, microstructure, and geological formation contribute to these variations in mechanical behaviour.

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In summary, while both coal and schistose rock show a positive correlation between Young's Modulus and Shear Modulus, their distinct properties and geological origins result in significant differences in their overall mechanical behaviour.

CONCLUSION

Through our comprehensive exploration of the relationships between Young's Modulus (E), Shear Modulus (G), and Poisson Ratio (ν), we have illuminated the intricate web of connections that govern the mechanical behaviour of materials, with a particular focus on coal. Our analysis has underscored the intrinsic relationships that exist between these parameters. Young's Modulus, as a measure of stiffness, significantly influences Shear Modulus, reflecting the material's resistance to shear deformation. Meanwhile, Poisson Ratio reveals the lateral deformation response as axial loads are applied. These relationships are integral to our understanding of how materials like coal respond to mechanical stresses.

In the context of coal, our analysis has revealed a consistent positive correlation between Young's Modulus and Shear Modulus. As Young's Modulus increases, Shear Modulus tends to increase, reflecting a direct relationship between these two mechanical properties. This aligns with theoretical expectations and known material behaviours. Understanding this relationship has practical implications, particularly in geotechnical engineering and materials science.

Our investigation has confirmed a negative correlation between Shear Modulus and Poisson Ratio in coal. This indicates that as Shear Modulus increases, Poisson Ratio tends to decrease, and vice versa. This observed trend aligns with the expected behaviour of materials under different types of deformation. Recognizing this negative correlation *Udoh et al: On the Relationship Between Young's Modulus, Shear Modulus and Poisson's Ratio.* <https://dx.doi.org/10.4314/wojast.v15i2.29>

is essential for a comprehensive understanding of coal's mechanical response to various stress conditions.

The relationships between these mechanical properties, as demonstrated within coal materials, offer valuable insights into the behaviour of this resource. Our findings are not only academically significant but also have practical applications. They can be harnessed in the selection and design of materials in engineering, mining, and construction industries.

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