https://dx.doi.org/10.4314/WOJAST.v15i1.33

ISSN: 2141 – 3290
www.wojast.com

CASSON FLUID FLOW WITH HEAT GENERATION AND RADIATION OVER A NON-LINEAR STRETCHING SHEET

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ABSTRACT

This study investigates Casson fluid flow with heat generation and radiation over a non-linear stretching sheet. The resulting partial differential equations are converted to a system of ordinary differential equations using similarity transformation and solved numerically using shooting technique with fourth order Runge-Kutta method. The effect of radiation, heat source, and Casson parameters on the momentum and thermal boundary layers are examined through graphs using MATLAB. From the graphs it is seen that increase in the values of heat source and Prandtl number parameter increases the temperature profiles. The momentum boundary layer thickness decreases while the rate of heat transfer increases with increasing non-linear parameter. The temperature and thermal boundary layer thickness decreases with increasing radiation parameter values.

Keywords: Casson fluid, heat generation, radiation, stretching sheet

INTRODUCTION

In recent years, researchers in the fields of science and engineering have focused on analyzing the flow caused by sheets that are either continuously or periodically stretched. This is due to the various practical applications in different engineering fields, such as the extrusion of polymer sheets from a dye, the cooling of metal plates in a cooling bath, and the movement of heat-treated materials between feed and wind-up rolls or on a conveyor belt. The flow past a stretching plate was derived by Crane (1970). Crane (1970) looked into the steady flow of a non-compressible, viscous fluid over an elastic sheet, where the flow was caused by the sheet being stretched within its own plane, with a velocity that varied linearly with the distance from a fixed point. Wang (1984) studied the three-dimensional flow due to a stretching flat surface. Andersson (1992) analysed MHD flow of a viscoelastic fluid past a stretching surface. Previous studies have primarily focused on linear stretching sheets, but it is not always the case that the stretching sheet is linear. Gupta and Gupta (1977) examined heat and mass transfer on a stretching sheet with suction or blowing. Vajravelu (2001) studied viscous flow over a nonlinearly stretching sheet. Raptis and Perdikis (2006) investigated viscous flow over a nonlinearly stretching sheet in the presence of a chemical reaction and magnetic field. Viscous flow and heat transfer over a nonlinear stretching sheet was performed by Cortell (2007). Recently, Pramanic (2014) studied Casson fluid flow and heat transfer past an exponentially porous stretching surface in the presence of thermal radiation. He found that increasing values of the Casson parameter suppress the velocity field but enhance the temperature. Bhattacharyya (2013) studied the MHD Stagnation-Point Flow of Casson Fluid and Heat Transfer over a stretching sheet with thermal radiation. Mabood (2015) analysed MHD boundary layer flow and heat transfer of nanofluid over a nonlinear stretching sheet. Ekang *et al*. (2021) studied MHD heat and mass flow of nanofluid over a nonlinear permeable stretching sheet.

Casson fluid is a type of non-Newtonian fluid that behaves like solid elastic, with a yield shear stress present in its consecutive equation. Mustafa (2011) examined unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate. Hayat (2009) studied the effect of thermal radiation on the flow of a second-grade fluid. Senge *et al*. (2020) investigated the influence of radiation on magneto-hydrodynamics flow over an exponentially stretching sheet embedded in a thermally stratified porous medium in the presence of heat source.

Radiation influence on boundary layer flow is also essential because it is used in engineering, physics, and industrial sectors such as, polymer processing, gas-cooled nuclear reactors, design of furnace, and space technologies such as aerodynamics rockets, power plants for interplanetary flights, propulsion systems, missiles, and space crafts that function at extreme temperatures. As a result, the effects of thermal radiation cannot be neglected in such mechanisms. Due to heat, fluids fluctuate in their viscosity, which may vary depending on the rate of deformation, and certain fluids have an elastic component in nature, which is considered as non-fluids. In general, heat generation can change the temperature of the fluid and alter its viscosity. A rise in temperature can cause the fluid to become less viscous and more fluid, making it easier to flow. On the other hand, heat generation can also cause the fluid to become more viscous and less fluid, making it harder to flow. Additionally, the heat generation could cause the fluid to reach a higher yield stress, requiring more stress to begin flowing.

In this work, Casson fluid flow with heat generation and radiation over a non-linear stretching sheet is examined. The governing partial differential equations are converted to a system of non-linear ordinary differential equations using the similarity transformation and then solved numerically using shooting technique with fourth order Runge-Kutta method. The effect of the controlling parameters on the fluid velocity and temperature distributions have been demonstrated graphically and discussed. A comparative study is also presented.

METHODOLOGY

Mathematical Formulation

Consider the flow of an incompressible viscous fluid through a flat sheet intersected by plane $y = 0$. The flow is restricted to $y > 0$. Two equal and opposing forces are applied along the x axis to stretch the wall while keeping the origin fixed. The standard Casson fluid's rheological equation of state for an isotropic and incompressible flow is

$$\tau_{ij} = \begin{cases} \left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right) 2e_{ij} \; ; & \pi > \pi_c \\ \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right) 2e_{ij} \; ; & \pi < \pi_c \end{cases}$$

where $\pi = e_{ij}\, e_{ji}$, and e_{ij} is the (i, j)th component of the deformation rate, defined as

$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$. π is the product of the deformation rate with itself, π_c is the critical value of the product of the component of the rate of strain tensor with itself, μ_B is the plastic dynamic viscosity of casson fluid and p_y is the yield stress of the fluid. The positive x coordinate is measured along the sheet and the positive y coordinate is measured perpendicular to the sheet.

We make the following assumptions. The heat source and radiation are imposed at the plate surface, the temperature of the fluid at the surfaced raised to T_w, is assumed to be greater than the ambient temperature of the fluid, T_∞.

The temperature, T_w at the surface is given by

$T_w(x) = T_\infty + (T_w - T_\infty)\theta$
where T_∞ is a constant.

From these assumptions the governing equations are obtained as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where u and v are the velocity components in the x and y directions, respectively, ϑ is the kinematic viscosity, ρ is the fluid density,

$\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the parameter of the Casson fluid, κ is the thermal diffusivity of the fluid, Q_0 is dimensional heat generation, κ^* is the absorption coefficient, σ^* = Stefan-Boltzmann constant q_r is radiative heat flux, T is temperature.

Using Rosseland approximation for radiation,

$$q_r = \frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y} \tag{4}$$

By assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series and expanding T^4 about T_∞ and neglecting terms of higher orders, we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{5}$$

Substituting the partial derivative with respect to T of equation (5) in equation (4) the rate of change of radioactive heat flux in the y-axis direction becomes

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3\kappa^*}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

Therefore equation (3) becomes;

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) + \frac{16\sigma^* T_\infty^3}{3\rho c_p \kappa^*}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

With initial and boundary conditions:

$$u_w = U = cx^n, \; v = 0, \; T = T_w, \text{ at } y = 0 \tag{8}$$

$$u \to 0, \; T \to T_\infty, y \to \infty$$

Here, c (c > 0) is a parameter related to the surface stretching speed, T_w is the uniform temperature at the sheet, T_∞ is the free stream temperature, and n is the power index related to the surface stretching sheet.

Now we introduce the stream function $\varphi(x, y)$, define by;

$$u = \frac{\partial \varphi}{\partial y} \qquad v = -\frac{\partial \varphi}{\partial x} \tag{9}$$

These automatically satisfy continuity equation. Next introduce the similarity transformations;

$$\varphi = \frac{cx^{\frac{n+1}{2}}}{\sqrt{\frac{c(n+1)}{2\vartheta}}} f(\eta); \quad \eta = y\sqrt{\frac{c(n+1)}{2\vartheta}}\, x^{\frac{n+1}{2}}; \quad \frac{T - T_\infty}{T_w - T_\infty} = \theta(\eta)$$

$$u = cx^n f'(\eta); \qquad v = -\sqrt{c\vartheta\frac{(n+1)}{2}}\, x^{\frac{n-1}{2}}\left[f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right] \tag{10}$$

Substituting equations (9) and (10) into equations (2) to (7), the governing equations reduce to:

$$\left(1 + \frac{1}{\beta}\right)f''' + ff'' - \frac{2n}{n+1}f'^2 = 0 \tag{11}$$

$$\frac{1}{p_r}\left(1 + \frac{4}{3}R\right)\theta'' + f\theta' + \frac{2}{n+1}Q\theta = 0 \tag{12}$$

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The transformed boundary conditions are:

$$f'(0) = 1, f(0) = 0, \theta(0) = 1, \ at\ \eta = 0,$$

$$f'(\eta) \to 0, \ \theta(\eta) \to 0, \ as\ \eta \to \infty, \qquad (13)$$

where the prime denotes differentiation with respect to η and the dimensionless parameters are as follows:

$Pr = \frac{\vartheta}{k}$ is the Prandtl number.

$Q = \frac{Q_0 x}{u_w \rho c_p}$ is the heat source;

$R = \frac{4\sigma^* T_\infty^3}{3k\kappa^*}$ is the radiation parameter;

Hence the dimensionless form of Skin friction (C_f) and the Local Nusselt number (Nu_x) are given by;

$$C_f Re_x^{\frac{1}{2}} = \left(1 + \frac{1}{\beta}\right) f''(0); \qquad \frac{Nu_x}{Re_x^{\frac{1}{2}}}\left(\frac{2}{n+1}\right)^{\frac{1}{2}} = -\theta'(0) \quad (14)$$

where $Re_x = \frac{x u_w}{\vartheta}$ is the local Reynolds number.

Method of solution

The dimensionless equations (9) and (10) subject to the boundary conditions (11) are non-linear equations, hence solving numerically, we convert it to a system of equations by setting the following:

Let:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix} = \begin{pmatrix} \eta \\ f \\ f' \\ f'' \\ \theta \\ \theta' \end{pmatrix} \qquad (15)$$

where

$$f''' = \left(\frac{\beta}{1+\beta}\right)\{-y_2 y_4 + \frac{2n}{n+1} {y'_3}^2\}$$

$$\theta'' = \left(\frac{3Pr}{3+4R}\right)\{-y_2 y_4 - 2Q y_5\}$$

Satisfying

$$\begin{pmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \\ y_5(0) \\ y_6(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ A_1 \\ 1 \\ A_2 \end{pmatrix} \qquad (16)$$

where A_1 and A_2 are guessed such that

$$\begin{pmatrix} y_3(\infty) \\ y_5(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad as\ \eta \to \infty \qquad (17)$$

These equations are solved numerically using shooting technique with fourth order Runge-Kutta (R-K 4) method. The boundary condition as $\eta \to \infty$ was replaced by a finite value in accordance with standard practice in the boundary layer analysis. We set $\eta_\infty = 10$.

RESULTS AND DISCUSSION

The Effects of the dimensionless governing parameters namely: Radiation parameter (R), Prandtl number (P_r), Casson Parameter (β), Heat source parameter (Q) and the nonlinear parameter (n) on the velocity and temperature distribution profiles are analyzed numerically using the R-K 4 method stated in the previous section. To do this we set the following properties to be constant. The numerical values were generated and MATLAB was used to plot the following graphs by varying the fluid properties with basics at $p_r = 0.7, \ \beta = 2, \ R = 1.2, \ Q = 1, \ n = 1$.

In order to analyze the results, numerical computations have been carried out using the R-K 4 method for various values of Casson parameter β, nonlinear stretching parameter n, Prandtl number p_r, heat source Q and radiation parameter R. For illustrations of the results, the numerical results are plotted in Figs. 1-5.

Figure 1a presents the influence of variation of casson parameter (β) on velocity profiles of fluid.The velocity profile decreases as the casson parameter (β) values increase. The Casson parameter produces a resistance in the fluid flow and consequently the boundary layer thickness decreases for higher value of Casson parameter.

Figure 1b shows the influence of variation of casson parameter (β) on temperature profile of the fluid. It is observed that as the values of the casson parameter (β) increases, the temperature increases. There is a slight increase in the thermal boundary layer thickness as the Casson parameter increases.

Figure 2a shows the influence of variation of the non-linear parameter (n) on velocity profiles of the fluid. The velocity decreases as the non-linear parameter (n) values increase. As a result, the momentum boundary layer thickness decreases with increasing non-linear stretching parameter.

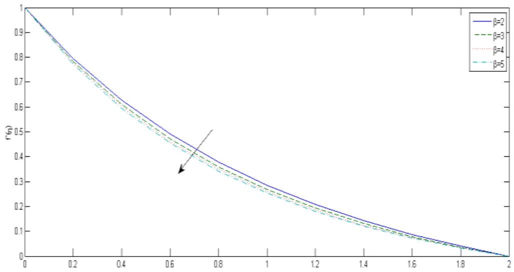

Figure 1a. Influence of β on $f'(\eta)$

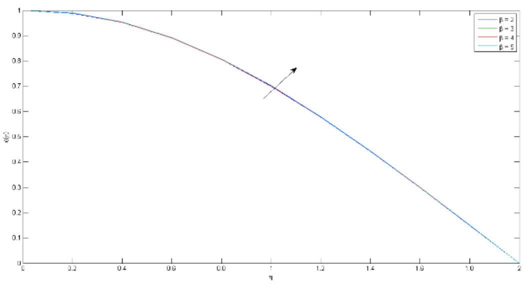

Figure 1b Influence of β on $\theta(\eta)$

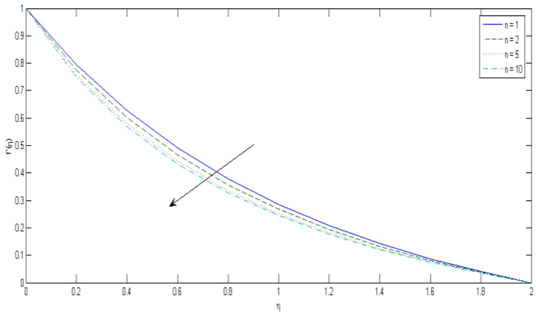

Figure 2a. Influence of n on $f'(\eta)$

Figure 2b presents the influence of variation of the non-linear parameter (n) on temperature profiles of fluid. It shows that the temperature increases as the non-linear parameter (n) values increase. This implies that the rate of heat transfer increases with increasing non-linear parameter.

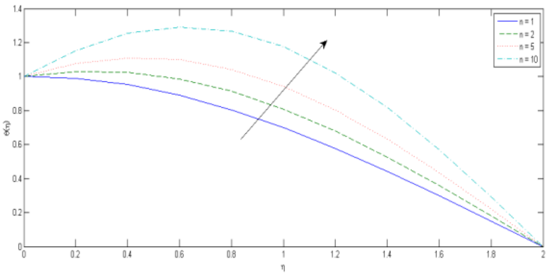

Figure 2b: Influence of n on $\theta(\eta)$

Figure 3 presents the influence of variation of the Prandtl number parameter (Pr) on temperature profiles of fluid. The temperature increases as the Prandtl number parameter (Pr) values increases. It is noted that increase in the prandtl number corresponds to a weaker thermal diffusivity and a thinner thermal boundary layer.

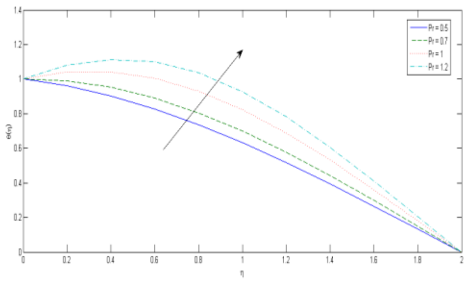

Figure 3 Influence of p_r on $\theta(\eta)$

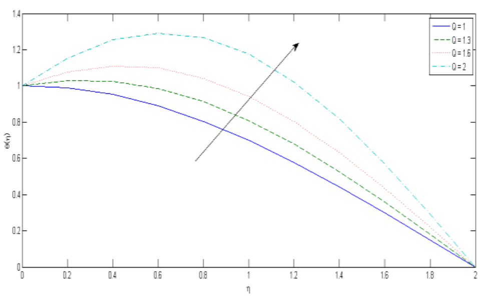

Figure 4 Influence of Q on $\theta(\eta)$

Figure 4 presents the influence of variation of the Heat source parameter (Q) on temperature profiles of fluid.The temperature increases as the Heat source parameter (Q) value increases. This is due to the fact that additional energy was generated and this brought about the increase in thickness of the thermal boundary layers.

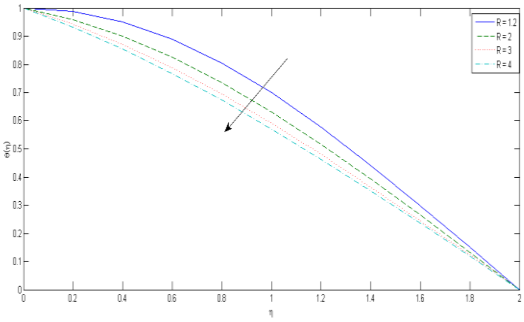

Figure 5 Influence of R on $\theta(\eta)$

Figure 5 presents the influence of variation of the radiation parameter (R) on temperature profiles of fluid. The temperature and thermal boundary layer thickness decreases with increasing radiation parameter (R) values.

Conclusion

The effects of the fluid parameters on the heat transfer are examined with the help of graphs. From the graphs, it is seen that increase in the values of Casson parameters decreases the velocity profiles and increases the temperature profiles. The momentum boundary layer thickness decreases while the rate of heat transfer increases with increasing nonlinear parameter. The temperature profile increases as the Prandtl number parameter (Pr) values increase. The temperature profile increases as the Heat source parameter (Q) values increase. The temperature and thermal boundary layer thickness decreases with increasing radiation parameter (R) values.

Notation

Dynamic viscosity of the Casson fluid: μ
Density of the fluid: ρ
Thermal conductivity of the fluid: k
Fluid temperature: T
Heat generation: Q
Velocity of the fluid along the x-axis: u
Velocity of the fluid along the y-axis: v
Ambient temperature: T_∞
Prandtl number: Pr
Reynolds number: Re
Nusselt number: Nu
Skin friction: C_f
∂ (partial derivative): used to represent partial derivatives in equations
∇ (nabla): used to represent the gradient operator in equations
Kinematic viscosity: ϑ

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