ISSN: 2141 – 3290
www.wojast.org

Supported by tetfund
TERTIARY EDUCATION TRUST FUND

DETERMINATION OF LAME'S PARAMETERS (λ and μ) FROM COMPRESSIONAL AND SHEAR WAVE VELOCITY FOR UYO AND ENVIRON, SOUTHEASTERN NIGERIA

AKANKPO, A.O.[A]*, ESSIEN, U. E. [B],
UMOREN, E. B. [A] AND GEORGE, A. M.[C]

[a]Department of Physics, University of Uyo, Uyo, Nigeria
[b]Department of Science Technology, Akwa Ibom State Polytechnic, Ikot Osurua, Nigeria
[c]Department of Physics, University of Calabar, Calabar, Nigeria
Corresponding Author: akaninyeneakankpo@uniuyo.edu.ng

ABSTRACT

In this work, compressional and shear wave velocity values were determined for the estimation of Lame's first and second constants (λ, μ) for Uyo and environ, which helped in the determination of elastic parameters of top soil as well as the degree of stability of engineering foundations. The study area lies between latitudes $4^{o}45'$ and $5^{o}15'$ N and between longitudes $7^{o}45'$ and $8^{o}30'$ E in the Niger Delta region of Southern Nigeria. A 24-channel signal enhancement seismograph was used in generating seismic waves. The mechanically generated seismic disturbances sensed by the geophones were received and recorded by a seismograph cascaded with the geophones. Shear modulus (μ) values ranged from 0.21×10^8 to 0.63×10^8 N/m^2 with an average of 0.43×10^8 N/m^2 for layer 1 and 0.78×10^8 to 2.55×10^8 N/m^2 with an average of 1.40×10^8 N/m^2 for layer 2. Also, λ values ranged from 0.20×10^8 to 0.58×10^8 N/m^2 with an average of 0.40×10^8 N/m^2 for layer 1 and 0.73×10^8 to 2.36×10^8 N/m^2 with an average of 1.30×10^8 N/m^2 for layer 2. The results indicate that the topsoil under study can support load that is being subjected to shear stress, provided the materials within the layer are well compressed.

KEYWORDS: Uyo, Wave Velocity, Seismic, Layer, Foundation

INTRODUCTION

The Lame's parameters (λ and μ) are two material-dependent quantities in continuum mechanics which arises in stress-strain relationships. They are sometimes referred to as the Lame's first parameter (λ) and Lame's second parameter (μ) (Weisstein, 2015). Kazemi (2012) opined that λ is sensitive to the fluid within the rock fabric, whereas μ is sensitive to the rock matrix. The Lame's constant (λ) and shear modulus (μ) form the two most important parameters in the study of fluids μ and reservoir rock. The constants λ and μ arise in strain- stress relationships. They are given in terms of other solid properties as:

$$\mu = \frac{E}{2(1+\sigma)} \qquad (1)$$

$$\lambda = \frac{\sigma E}{(1+\sigma)(1-2\sigma)} \qquad (2)$$

where E is Young's modulus and σ is the Poisson ratio (Akankpo and Essien, 2015).

New understanding into the original rock properties is offered by the conversion of velocity measurements to Lame's moduli parameters. These parameters allow for enhanced identification of reservoir zones (Ezeh, 2014). Several authors had worked on and provided the theory and concept for extracting Lame's parameters (Burianyk, 2000; Goodway, 2001. According to Sokolnikov (1972), Lame's constants (λ, μ) of homogeneous elastic material were always determined using conventional methods like mechanical testing, which was destructive. This led to the search for an alternative method by measuring the longitudinal and transversal velocity, which is known as Non-Destructive Evaluation (NDE) (Forsythe *et al*., 1977). In this work, compressional and shear wave velocity values were estimated for the determination of Lame's parameters for the area. This helped in determining the elastic parameters of top soil and the degree of stability of engineering foundation.

LOCATION AND GEOLOGY OF UYO AND ENVIRON

The study area lies between latitudes $4^{0}45'$ and $5^{0}15'$ N and between longitudes $7^{o}45'$ and $8^{o}30'$ E in the Niger Delta region of Southern Nigeria. It is situated in an equatorial climatic region, having two major seasons: the rainy season and dry season (Evans *et al*., 2010). Due to the current global climate changes, there are shifts in both the upper and lower boundaries of these climatic conditions (Farauta, *et al*., 2012, Akpabio and Ukaegbu, 2009).

Uyo is located in the Benin Formation (Coastal Plain Sands (CPS)) and Alluvium environments of the Niger Delta region of Southern Nigeria (Figure 1). 1. The Formation covers over 80% of the study area and beneath it is the parallic Agbada Formation (Reijers *et al*., 1997; Nganje *et al*., 2007). The engineering foundations in the area concentrate within the Benin Formation. The Formation comprises fine-medium-coarse sands and gravels which are poorly sorted (Essien and Akankpo, 2013; Essien, *et al*., 2014). The area is generally porous, permeable and usually interrupted by clay-sand sequence at various depths (Okwueze, 1991a; Ekweme & Onyeagoda, 1985).

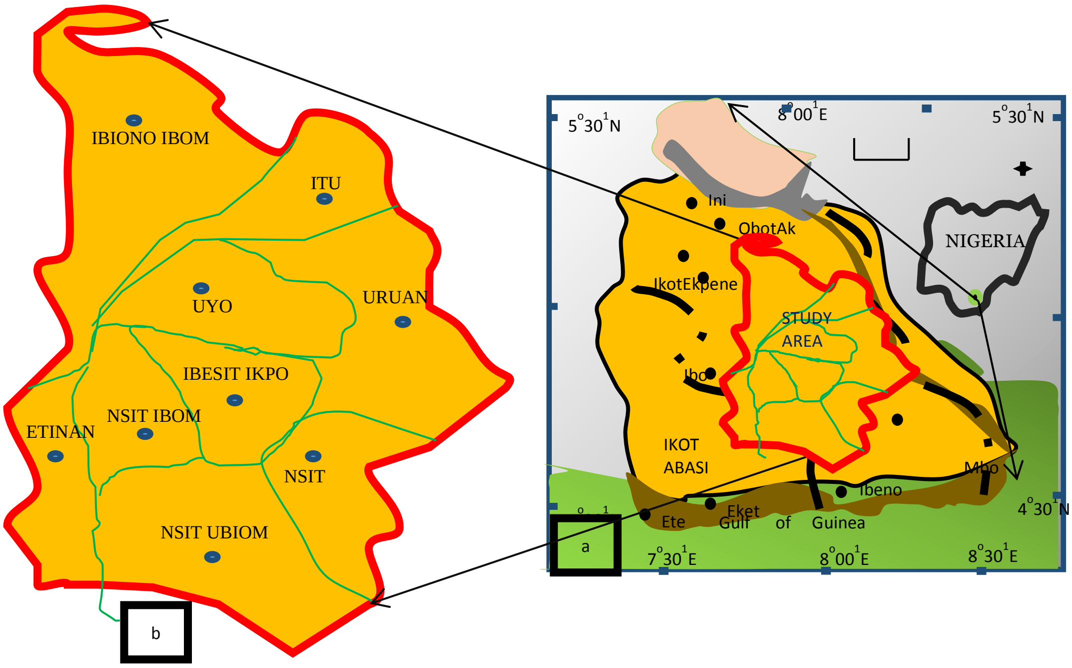

Fig. 1: Map showing (a) the study area location and general geology of Akwa Ibom State of Nigeria (b) Map showing the central Uyo district that the study area situates

.

MATERIALS AND METHOD

A 24 - channel signal enhancement seismograph was used in generating seismic wave. The electromagnetic geophone which were in direct contact with the earth, transformed the seismic energy generated by the source to electrical voltage which is a function of velocity. The mechanically generated seismic disturbances sensed by the geophones were received and recorded by a seismograph cascaded with the geophones (Kearey and Brooks, 1991; Reynolds, 1997, Akpabio and Adeniran 2023).

The generated energy penetrated into the subsurface and refracted off at various interfaces corresponding to the geological boundaries and consequently returned to the surface at later time to be picked up by the geophone (Kearey and Brooks, 1991). The seismic wave received by the geophone was converted into electrical pulse and was amplified by the preamplifier. The generated waves and energy penetrated into the subsurface and are retracted off at various interfaces corresponding to geological boundaries and consequently returned to the surface at a later time to be picked up by the geophones (Kumar and Kumar, 2003, Obianwu, *et al*, 2019).

RESULTS AND DISCUSSION

The values of the first and second Lame's constants are shown in Tables 1 and 2. Shear modulus (μ) values ranged from 0.21 × 10^8 to 0.63 × 10^8 N/m^2 with an average of 0.43 × 10^8 N/m^2 for layer 1 and 0.78 × 10^8 to 2.55 × 10^8 N/m^2 with an average of 1.40 × 10^8 N/m^2 for layer 2. Also, λ values ranged from 0.20 × 10^8 to 0.58 × 10^8 N/m^2 with an average of 0.40 × 10^8 N/m^2 for layer 1 and 0.73 × 10^8 to 2.36 × 10^8 N/m^2 with an average of 1.30 × 10^8 N/m^2 for layer 2. The values of Lame's constants (μ, λ) in this work testify that the bearing pressure of the foundation layers is nearly isotropic and consequently not susceptible to deformation. Lame's first constant and shear modulus (the second Lame's constant) are linearly related in this study (Tables 1 and 2). Using 3D contour maps in Figures 2(a, b) and 3(a, b), the distributions of the first and second Lame's parameters in the study area were examined.

The geoelastic parameters of the engineering foundation fall within dense sands and gravels as well as silty sands (Sawangsuriya 2012). Soils with high arenaceous formations have a higher bearing capacity than soil with high argillaceous materials (Atat *et al*., 2013). The range indicates that the topsoil under study can support load that is being subjected to shear stress, provided the materials within the layer are well compressed. The considered foundation layers are cohesionless, gritty and therefore not susceptible to creep, erosion and failures provided proper compaction is done during road construction. The Ultimate Bearing Capacity depends on the soil type, moisture content, compaction and the amount of uniformity of the formation. The higher values of these constants increase the cohesion of the topsoil.

Table 1: Summary of Lame's Parameters (μ, λ)

Location Name	**Number**	**Latitude (^{0})**	**Longitude (^{o})**	**Elevation (m)**	**Layer**	**V_p (m/s)**	**V_s (m/s)**	**μ x 10^8 (N/m^2)**	**λ x 10^8 (N/m^2)**
Etinan	1	5.9833	7.8500	67.00	L1	285.0	128.8	0.37	0.34
					L2	556.9	252.9	1.41	1.30
	2	4.9500	7.8333	61.00	L1	327.5	148.2	0.48	0.45
					L2	503.4	228.5	1.15	1.06
	3	4.8333	7.8510	31.00	L1	317.0	143.4	0.45	0.42
					L2	495.1	224.7	1.11	1.03
Nsit Ibom	1	4.8166	7.8330	36.00	L1	350.0	158.5	0.55	0.51
					L2	656.3	298.3	1.96	1.81
	2	4.8667	7.9167	46.00	L1	350.0	158.5	0.55	0.51
					L2	603.7	274.3	1.66	1.53
	3	4.8510	7.9000	43.00	L1	291.0	131.6	0.38	0.35
					L2	745.3	338.9	2.53	2.33
Nsit Ubium	1	4.7833	7.9000	34.00	L1	285.5	129.1	0.48	0.34
					L2	429.2	194.7	1.23	0.77
	2	4.7833	7.9166	49.00	L1	326.0	147.6	0.48	0.54
					L2	519.9	236.0	1.23	1.13
	3	4.8167	7.9667	37.00	L1	269.0	121.5	0.33	0.30
					L2	483.5	219.4	1.06	0.98
Ibesikpo	1	4.8500	7.9667	49.00	L1	350.5	158.7	0.55	0.51
					L2	621.0	282.1	1.75	1.62
	2	4.9000	7.9833	133.00	L1	218.0	98.3	0.21	0.20
					L2	518.9	235.6	1.22	1.13
	3	4.9500	7.9667	72.00	L1	325.0	147.1	0.48	0.44
					L2	507.5	230.4	1.17	1.08
Uruan	1	4.9167	8.0167	52.00	L1	295.0	133.4	0.39	0.36
					L2	653.1	296.8	1.94	1.79
	2	4.9167	8.0333	52.00	L1	334.5	151.4	0.50	0.47
					L2	749.3	340.7	2.55	2.36
	3	4.9500	8.0000	57.00	L1	306.5	138.7	0.42	0.39
					L2	509.4	231.2	1.18	1.09
Nsit Atai	1	4.8667	8.0500	45.00	L1	313.0	141.6	0.44	0.41
					L2	558.4	253.6	1.41	1.31
	2	4.8000	8.0667	37.00	L1	302.5	136.8	0.41	0.38
					L2	501.3	227.5	1.14	1.05
	3	4..8333	8.0333	31.00	L1	303.0	137.1	0.41	0.38
					L2	464.7	210.9	0.98	0.91
Uyo	1	4.9833	8.0000	50.00	L1	372.5	168.8	0.63	0.58
					L2	558.3	253.6	1.41	1.31
	2	5.0000	7.9500	82.00	L1	341.0	154.4	0.52	0.49
					L2	533.8	242.4	1.29	1.20
	3	5.0333	7.9167	67.00	L1	319.5	144.6	0.46	0.43
					L2	560.8	254.7	1.43	1.32
Itu	1	5.0500	7.9167	65.00	L1	272.5	123.1	0.33	0.31
					L2	569.9	258.9	1.47	1.36
	2	5.0667	7.9167	68.00	L1	301.0	136.1	0.41	0.38
					L2	538.4	244.5	1.31	1.22
	3	5.1000	7.9500	57.00	L1	268.5	121.3	0.32	0.30
					L2	614.7	279.3	1.72	1.59
Ibiono Ibom	1	5.1833	7.9000	66.00	L1	237.5	107.2	0.25	0.24
					L2	416.5	188.9	0.78	0.73
	2	5.1832	7.8667	63.00	L1	278.5	125.9	0.35	0.32
					L2	485.2	220.2	1.07	0.99
	3	5.2000	7.8500	72.00	L1	331.0	149.8	0.49	0.46
					L2	492.2	223.4	1.10	1.02

The incessant failures of road in the study area can be attributed to the poor compaction of the foundation layers during construction and not due to the geomaterials as the materials are elastic and can withstand pressure under load. This study identifies that the subgrades in this locality have reliable bearing pressure/capacity to carry the heavy duty equipment witnessed especially in the urban region of the study area if and only if proper compaction has taken place during road design. Generally, higher values of Lame's constants are obtained in the southern part of the study area in layer 1. This suggests that the northern parts and other locations need more compaction than the southern part. In layer 2, the counties within the region have higher values of Lame's constants while other parts in the north have relatively lower values of Lame's constants, which in this case are still considered to support good engineering foundation when sufficiently and evenly compacted

Table 2: The minimum, maximum and mean values of Lame's parameters

Layer	μ x 10^8 (N/m^2)			λ x 10^8 (N/m^2)		
	Minimum	Maximum	Mean	Minimum	Maximum	Mean
L1	0.21	0.63	0.43	0.20	0.58	0.40
L2	0.78	2.55	1.30	0.73	2.36	1.30

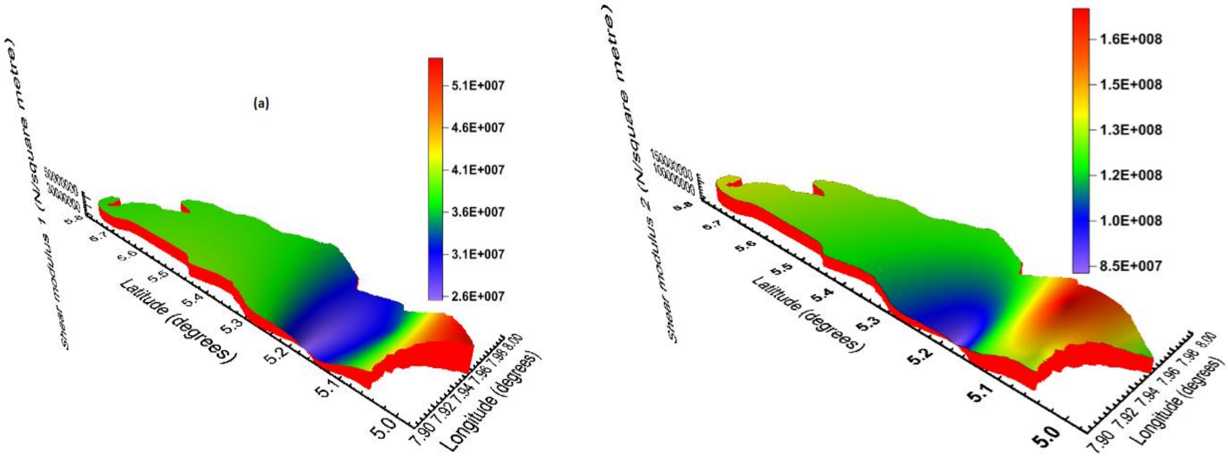

Figure 2: 3-D blanked contour map of (a) layer 1 shear modulus and (b) layer 2 shear modulus in the study area

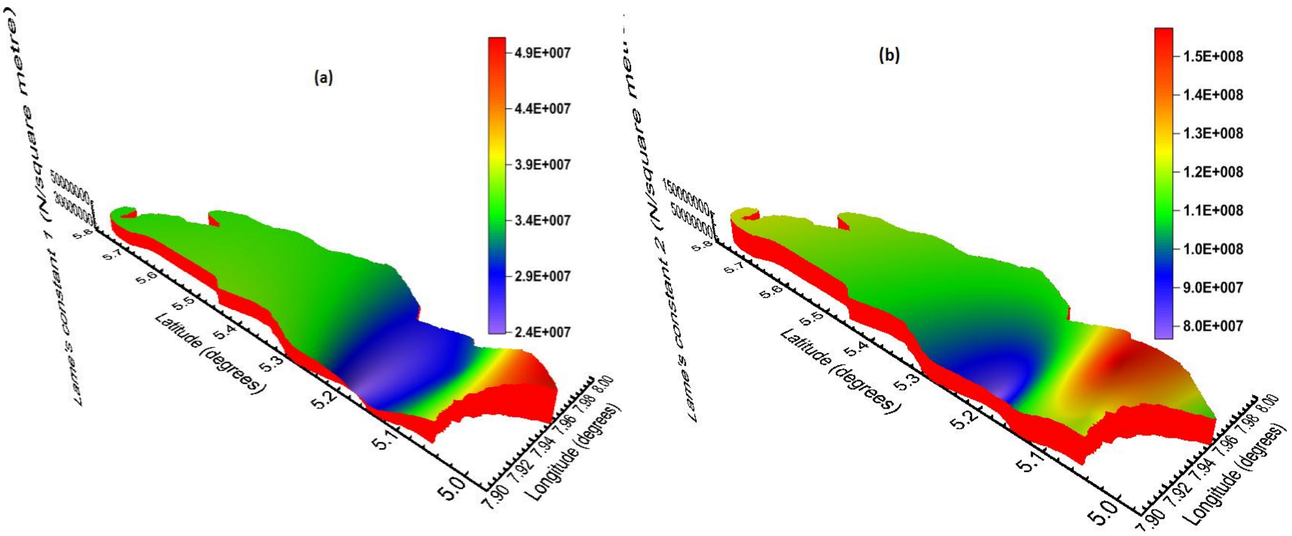

Figure 3: 3-D blanked contour map of (a) layer 1 Lame's second constant and (b) layer 2 Lame's second constant in the study area

CONCLUSION

The Lame's parameters (λ and μ) are two material-dependent quantities in continuum mechanics which arise in stress-strain relationships. Compressional and shear wave velocity values were estimated for the determination of Lame's parameters (λ and μ) for Uyo and environ. The values of Lame's constants (μ, λ) in this work testify that the bearing pressure of the foundation layers is nearly isotropic and consequently not susceptible to deformation. Lame's first constant and shear modulus (the second lame's constant) are linearly related in this study. The geoelastic parameters of the engineering foundation fall within dense sands and gravels as well as silty sands. Soils with high arenaceous formations have a higher bearing capacity than soil with high argillaceous materials.

REFERENCES

Akankpo. O. and Essien, U. E. (2015). Determination of Lame's Constants of Surface soils and Shallow Sediments from Seismic Wave Velocities. International Journal of Scientific & Engineering Research, 6 (12): 1052-1065.

Akpabio, Idara O. and Ukaegbu, V. U. (April 2009). Niger Delta Depositional Lithology Pattern Icfai *University Journal of Earth Sciences*, 3 (2). Available at SSRN: *http://ssrn.com/abstract =1394844*.

Akpabio I. O.. and Adeniran O. A. (2023). Multichannel Frequency Selective Seismograph Graphical User Interface (GUI). International Journal of Creative Research Thoughts, IJCRT.org, ISSN: 2320 – 2882

Atat, J. G., Akpabio I. O. and George, N. J. (2013). Allowable Bearing Capacity for Shallow Foundation in Eket Local Government Area, Akwa Ibom State, Southern Nigeria. International Journal of Geosciences, 4(10): 1491 – 1500.

Burianyk, M. (2000). Amplitude-vs-offset and seismic rock property analysis: a Primer. CSEG Recorder, 25 (11): 4–14

Essien, U. E. and Akankpo, A. O. (2013). Compressional and Shear Wave Relief, Measurement in the Consolidated Topsoil in Eket, Southern Nigeria. Pacific Journal of Science and Technology, 14(1): 476-491.

Essien, U. E. Akankpo, A. O. and Igbokwe, M. U. (2014). Poisson's Ratio of Surface Soils and Shallow Sediment Determined from Seismic Compressional and Shear Wave Velocities. International Journal of Geosciences, 5: 1540-1546.

Evans, U. F., George, N. J., Akpan, A. E., Obot, I. B. and Ikot, A. N. (2010). A Study of Superficial Sediments and Aquifers in Parts of Uyo Local Government Area, Akwa Ibom State, Southern Nigeria, using Electrical Sounding Method. E-J Chem, 7(3): 1018–1022.

Ezeh, C. C. (2014). Using Lame's petrophysical parameters for fluid detection and lithology determination in parts of Niger delta. Global Journal of Geological Sciences, 13: 23-33.

Forsythe, G. E., Malcolm, M. and Moler, C. (1977). Computer Methods for Mathematical Computations. ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik, 59(2): 141-142.

Farauta, B. K., Egbule, C. L., Agwu, A. E., Idrisa, Y. L, and Onyekuru, N. A. (2012). Farmers' Adaptation Initiatives, the Impact of Climate Change on Agriculture in Northern Nigeria Journal of Agricultural Extension, 16: 132-144.

Goodway, B. (2001). AVO and Lame Constants for Rock Parameterization and Fluid Detection. CSEG Recorder, 26 (6): 39–60.

Kazemi, M. S. (2012). Determination of Lame' Parameter and LMR in one of the Reservoirs in South of Iran. Technical and Professional University, Tehran, Iran.

Kearey, P. and Brooks, M. (1991). An Introduction to Geophysical Exploration. 2nd Edition. London: Blackwell Scientific Publications Limited. Pp. 22 – 23 and 96 – 115.

Kumar, J. and Kumar, N. (2003). Seismic Bearing Capacity of Rough Footings on Slopes using Limit Equilibrium. Geotechnique, 53(3): 363 – 369.

Nganje, T. N., Edet, A. E. and Ekwere, S. J. (2007). Concentrations of Heavy Metals and Hydrocarbons in Groundwater near Petrol Stations and Mechanic Workshops in Calabar Metropolis, South-eastern Nigeria. Journal of Environmental Geosciences, 14(1): 15-29.

Obianwu, I. V., Akpabio, I.O., Adeniran. A. O., Akankpo, A. O. and Umoren, E. B. (2019). Design and Construction of a Multichannel Microcontroller Based Seismograph for Field and laboratory Use. SSRG International Journal of Applied Physics (SSRG-IJAP) – Volume 6 Issue 2

Okwueze, E. E. (1991a). Shear Wave Observation in Seismic Refraction Prospecting: Journal of Mapping and Geology, 24 (1 and 2): 61-65.

Reijers, T. J. A., Petters, S. W. and Nwajide, C. S. (1997). The Niger Delta Basin, in Selley, R.C., ed., African Basins--Sedimentary Basin of the World 3: Amsterdam. Elsevier Science, Pp. 151-172.

Reynolds, J. M. (1997). An Introduction to Applied Environmental Geophysics, New York: Wiley. p 780.

Sawangsuriya, A. (2012). Wave Process in Classical and New Solids, InTech, USA, ISBN, Pp. 978-953-51-6258-2.

Sokolnikov, I. V. (1972). Mathematical Theory of Elasticity. Nauka i Izkustvo, Sofia (in Bulgarian).

Weisstein, E. (2015). Eric Weisstein's World of Science, A Wolfram Web Resource