A MATHEMATICAL MODEL ON SEQUENTIAL DISCOUNT TRANSFER IN A THREE-LEVEL CHANNEL

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ABSTRACT

Although there are some game-theoretic price discount models, three-level game-theoretic sequential price discount models have not been considered. This work considers a manufacturer-distributor-retailer channel on price discount using game theory. The manufacturer is considered to be the channel leader; the distributor is the first follower, while the retailer is the second follower. It considers a situation where the manufacturer gives price discount to the distributor, who in-turn is expected to also provide price discount to the consumer. The work models the players' payoffs using balance equations involving price, discount rate and linear demand function. The work considers four scenarios: neither of the players gives discount; only one player gives discount; two players give discount; and all-three players give discount. For each scenario the work obtained the prices and payoffs of each player. It shows that giving of discount reduces a player's payoff, and that instead of all the players engaging in price discount, a player can do that for the entire channel. However, a sharing formula must be reached to ensure that the player that provides the discount is not short-changed.

INTRODUCTION

Discount can be considered as the price deduction from the actual price of a product or commodity. Discounts are usually given to cover-up the cost that may be incurred while performing some supply channel functions which may include warehousing, holding, advertising, product promotion, etc.

Researches on price discount can be generally categorized into empirically based works and mathematical models from which we have game-theoretic models. Bhatti (2018) observed that discount is not influential enough to affect the intention of end-users to purchase a product. He arrived at this conclusion from a study on the effect of discount, social media and product promotion on consumers' intention to purchase a product.

On the contrary, Choi and Chen (2019) observed that the daily sales of the applications are positively affected by discount and bundling. They arrived at this conclusion in their consideration of the effect of product bundling and discount on game-as-a-service applications. Another consideration of the purchase intentions of consumers was carried out by Sheehan *et al*. (2019). They worked on how different sizes of price discount can influence end-users purchase intentions, and observed that giving low discount at the beginning of a shopping visit on the internet is more effective. Another work on discount was done by Ya- Chiu *et al.* (2021). They compared price discounts between third-party sellers and established Walmart sellers' overtime. Their result revealed that the average discount for third-party sellers is lower when compared with that of Walmart sellers. Mncube (2014) examined the claim that a law on discount remedy is beneficial by looking at the design and effectiveness of the remedy via a comparative method. In an examination of the negative effect of post-purchase, Luo and Lee (2018) suggested optimal promotion formats for alleviating end-users' negative perception.

While data-based empirical research work are very important, mathematical models are helpful in the use of available data. Luo *et al.* (2014) noted that subsidy ceiling is a comparatively more effective strategy for a manufacturer whose cost of production is high, while price discount is better for a manufacturer whose cost of production is low.

Considering the effect of price discount on end-user perception Lee and Chen-Yu (2018) built a model and noted that end-users perceive high discount product as low quality and vice versa. Li *et al.* (2018) used optimal control to suggest dynamic discount pricing in word-of-mouth marketing model. They examined the effect of different market factors on the expected optimal net profit. In an effort to maximize marketing profit amidst competing market players, Chen *et al*. (2019) designed a dynamic discount pricing method which is effective in competitive marketing. Feng *et al.* (2019) considered pricing strategy for a firm's product associated with presale by using the behaviours of two distinct end-users. They showed that the retailer should adopt skimming price strategy when the proportion of end-users is low and willingness for immediate patronage is below the threshold, otherwise he should adopt a penetrating price strategy.

Game-theoretic models are very useful in modelling supply channels. Examples can be found in Xie and Wei (2009), Ezimadu and Nwozo (2018) and Ezimadu and Nwozo (2019). Game-theoretic price discount models are quite scarce. In an effort to determine the optimal interchange fee rate of an issuer, the discount rate of an acquirer, and the retail price of the acquirer in a credit card network, Guo *et al*. (2012) considered two game-theoretic settings: a non-cooperative game setting and a cooperative game setting involving the three players. Considering service provision, price and discount Sadjadi *et al*. (2018) used Stackelberg game to model interactions between two manufacturers and one retailer in a supply channel. In a study of channel coordination Zare *et al*. (2018) used game theory to model price discount strategy in a supplier-two retailers channel with uncertain demand and yield. Noreen *et al.* (2018) used a two-stage game to model a situation in which a cellular base station encourages transmitters using its network to render services to cellular users. In exchange, the cellular base station provides an interference price discount to the device-to-device users.

This work considers a channel involving a manufacturer, distributor and a retailer in which the manufacturer sells his product to the consumer through the distributor who in-turn sells through the retailer. It examines possible transfer of discount from the manufacturer to the consumer through the distributor and retailer.

In essence we consider a price-discount game-theoretic model in which the manufacturer reimburses the distributor a fraction of the wholesale price spent on the product; the distributor reimburses the retailer a fraction, and the retailer reimburses the consumer a fraction of the price of the product.

The Model

The Demand Function

The retailer, distributor and manufacturer's decision variables are the prices P_R, P_D and P_M respectively. We assume that the manufacturer's products are normal goods so that the inverse relationship between price and demand holds. Thus, we employ a linearly decreasing price-demand function

$$f(P_R) = 1 - \theta P_R \tag{1}$$

where θ is a positive constant which represents the rate at which demand changes with price. We note that demand function of this form has been employed in Ma *et al.* (2013), He *et al.* (2014), Jeuland and Shugan (1988) and Ezimadu (2019a).

Players' Optimization Problem

From Ezimadu (2019a) we have that the players' payoffs can be expressed as

$$Payoff = Price\ Margin \times Demand - Expenditure. \tag{2}$$

We will model the decisions as a Stackelberg game in which the manufacturer is the channel leader, with the distributor as the first follower, and the retailer as the last follower. First, the manufacturer informs the distributor of his wholesale price P_M and discount rate ϕ. Next, the distributor informs the retailer of his price P_D and discount rate α. Thus, based on the distributor's available information, the retailer seeks to maximize his payoff Π_R. Such a sequential transfer of funds in the form of reimbursements was first considered by Ezimadu (2019b).

Now, the retailer's price margin is $P_R - P_D$. Since it is expected that he will be given back a fraction of the distributor's price in the form of price discount αP_D, and at the same time he is expected to give price discount λP_R to the consumer, we express his expenditure as $\lambda P_R - \alpha P_D$. Thus from (1) and (2) we have that the retailer seeks to

$$\max_{P_R>0} \Pi_R = (P_R - P_D)(1-\theta P_R) - \lambda P_R + \alpha P_D \tag{3}$$

That is, the retailer maximizes his payoff based on available information.

Similarly, the distributor's margin is given by $P_D - P_M$. He is expected to be given a fraction of the manufacturer's wholesale price as price discount ϕP_M, and also to give a fraction of his price to the retailer as price discount αP_D. As such his expenditure can be expressed as $\alpha P_D - \phi P_M$. Thus From (1) and (2) we have that the distributor aims to

$$\max_{P_D>0} \Pi_D = (P_D - P_M)(1-\theta P_R) - \alpha P_D + \phi P_M \tag{4}$$

Further, the manufacturer's price margin is $P_M - P_C$, where P_C is the manufacturer's production cost. The manufacturer is expected to give a fraction of his wholesale price as price discount ϕP_M to the distributor. From (1) and (2) we have that the manufacturer's goal is to

$$\max_{P_M>0} \Pi_M = (P_M - P_C)(1-\theta P_R) - \phi P_M. \tag{5}$$

This type of three-level game-theoretic model was first considered by Ezimadu (2016), with transfer of support modelled by Ezimadu (2019b). We will proceed in the analysis using backward induction by first solving the problem of the retailer.

The Game Scenarios

Total Provision of Discount

In this section we start by considering a situation where all the players engage in giving discount. We refer to this as total provision of discount.

Now, maximizing (3) with respect to P_R we have

$$\frac{\partial \Pi_R}{\partial P_R} = (P_R - P_D)(-\theta) + (1-\theta P_R) - \lambda = 0$$

$$\Rightarrow \qquad P_R = \frac{\theta P_D + 1 - \lambda}{2\theta}. \tag{6}$$

Using (6) in (4) we have

$$\max_{P_D>0} \Pi_D = (P_D - P_M)\left(1 - \theta\frac{\theta P_D + 1 - \lambda}{2\theta}\right) - \alpha P_D + \phi P_M,$$

so that

$$\frac{\partial \Pi_D}{\partial P_D} = \frac{1}{2}\big((P_D - P_M)(-\theta) + (1 - \theta P_D + \lambda)\big) - \alpha = 0$$

$$\Rightarrow \qquad P_D = \frac{\theta P_M + \lambda - 2\alpha + 1}{2\theta}. \tag{7}$$

Thus using (7) in (6) we have

$$P_R = \frac{\theta\left(\frac{\theta P_M + \lambda - 2\alpha + 1}{2\theta}\right) + 1 - \lambda}{2\theta}$$

$$= \frac{\theta P_M - \lambda - 2\alpha + 3}{4\theta}, \tag{8}$$

Using (8) in (5) we have

$$\max_{P_M>0} \Pi_M = (P_M - P_C)\left(1 - \theta\frac{\theta P_M - \lambda - 2\alpha + 3}{4\theta}\right) - \phi P_M$$
$$= \frac{1}{4}(P_M - P_C)(1 - \theta P_M + \lambda + 2\alpha) - \phi P_M = 0.$$

Thus, maximizing with respect to P_M we have

$$\frac{\partial \Pi_M}{\partial P_M} = \frac{1}{4}[(P_M - P_C)(-\theta) + (1 - \theta P_M + \lambda + 2\alpha)] - \phi = 0$$
$$\Longrightarrow \quad P_M = \frac{\theta P_C + \lambda + 2\alpha - 4\phi + 1}{2\theta}. \tag{9}$$

Using (9) in (7) we have

$$\Longrightarrow \quad P_D = \frac{\theta\frac{\theta P_M + \lambda + 2\alpha - 4\phi + 1}{2\theta} + \lambda - 2\alpha + 1}{2\theta}$$
$$= \frac{\theta P_C + 3\lambda - 2\alpha - 4\phi + 3}{4\theta}. \tag{10}$$

Using (9) in (8) we have

$$P_R = \frac{\theta P_C - \lambda - 2\alpha - 4\phi + 7}{8\theta}. \tag{11}$$

Thus using (9), (10) and (11) in (3) we have

$$\Pi_R = \left(\frac{\theta P_C - \lambda - 2\alpha - 4\phi + 7}{8\theta} - \frac{\theta P_C + 3\lambda - 2\alpha - 4\phi + 3}{4\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda - 2\alpha - 4\phi + 7}{8\theta}\right) - \lambda\frac{\theta P_C - \lambda - 2\alpha - 4\phi + 7}{8\theta} + \alpha\frac{\theta P_C + 3\lambda - 2\alpha - 4\phi + 3}{4\theta}$$
$$= \frac{1}{64\theta}[\theta^2 P_C^2 + 12\alpha\theta P_C - 2\lambda\theta P_c - 8\phi\theta P_C + 8\lambda\phi + 52\alpha\lambda - 2\theta P_C - 48\alpha\phi + 16\phi^2 + \lambda^2 - 28\alpha^2 + 8\phi + 52\alpha - 62\lambda + 1], \tag{12}$$

$$\Pi_D = \left(\frac{\theta P_C + 3\lambda - 2\alpha - 4\phi + 3}{4\theta} - \frac{\theta P_C + \lambda + 2\alpha - 4\phi + 1}{2\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda - 2\alpha - 4\phi + 7}{8\theta}\right) - \alpha\frac{\theta P_C + 3\lambda - 2\alpha - 4\phi + 3}{4\theta} + \phi\frac{\theta P_C + \lambda + 2\alpha - 4\phi + 1}{2\theta}$$
$$= \frac{1}{32\theta}[\theta^2 P_C^2 + 8\phi\theta P_C - 2\lambda\theta P_C - 4\alpha\theta P_C + 24\phi\lambda + 48\phi\alpha - 2\theta P_C - 28\alpha\lambda + 4\alpha^2 + \lambda^2 - 48\phi^2 + 2\lambda + 24\phi - 28\alpha + 1] \tag{13}$$

and

$$\Pi_M = \left(\frac{\theta P_C + \lambda + 2\alpha - 4\phi + 1}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C - \lambda - 2\alpha - 4\phi + 7}{8\theta}\right) - \phi\frac{\theta P_C + \lambda + 2\alpha - 4\phi + 1}{2\theta}$$
$$= \frac{1}{16\theta}[\theta^2 P_C^2 - 2\phi\lambda P_C - 4\alpha\theta P_C - 8\phi\theta P_C + 4\alpha\lambda - 2\theta P_C - 8\phi\lambda - 16\phi\alpha + 4\alpha^2 + 16\phi^2 + \lambda^2 + 2\lambda + 4\alpha - 8\phi + 1] \tag{14}$$

respectively. Thus:

Proposition 3.1 Suppose that all-three players give discount, then the retailer, the distributor and the manufacturer's prices are given by (11), (10) and (9), and their payoffs are given by (12), (13) and (14) respectively.

Provision of Discount by the Retailer

In this section we consider a situation where only the retailer gives discount (in this case to the consumer). This means that $\alpha = \phi = 0$. Thus (11), (10) and (9) become

$$P_R = \frac{\theta P_C - \lambda + 7}{8\theta}, \tag{15}$$
$$P_D = \frac{\theta P_C + 3\lambda + 3}{4\theta} \tag{16}$$

and

$$P_M = \frac{\theta P_C + \lambda + 1}{2\theta} \tag{17}$$

respectively.

Using (15), (16) and (17) in (3), (4) and (5) we have

$$\Pi_R = \left(\frac{\theta P_C - \lambda + 7}{8\theta} - \frac{\theta P_C + 3\lambda + 3}{4\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda + 7}{8\theta}\right) - \lambda\frac{\theta P_C - \lambda + 7}{8\theta}$$
$$= \frac{\theta^2 P_C^2 - 2\lambda\theta P_C - 2\theta P_C + \lambda^2 - 62\lambda + 1}{64\theta}, \tag{18}$$

$$\Pi_D = \left(\frac{\theta P_C + 3\lambda + 3}{4\theta} - \frac{\theta P_C + \lambda + 1}{2\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda + 7}{8\theta}\right)$$
$$= \frac{\theta^2 P_C^2 - 2\lambda P_C Q - 2\theta P_C + \lambda^2 + 2\lambda + 1}{32\theta}, \tag{19}$$

and

$$\Pi_M = \left(\frac{\theta P_C + \lambda + 1}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C - \lambda + 7}{8\theta}\right)$$
$$= \frac{\theta^2 P_C^2 - 2\lambda\theta P_C - 2\theta P_C + \lambda^2 + 2\lambda + 1}{16\theta} \tag{20}$$

respectively. Thus:

Proposition 3.2 Suppose that only the retailer gives discount, then the retailer, the distributor and the manufacturer's prices are given by (15), (16) and (17), and their payoffs are given by (18), (19) and (20) respectively.

Provision of Discount by the Distributor

This section deals with the provision of discount by only the distributor to the retailer. Thus, we have that $\lambda = \phi = 0$. Thus (11), (10) and (9) become

$$P_R = \frac{\theta P_C - 2\alpha + 7}{8\theta}, \quad (21)$$

$$P_D = \frac{\theta P_C - 2\alpha + 3}{4\theta} \quad (22)$$

and

$$P_M = \frac{\theta P_C + 2\alpha + 1}{2\theta} \quad (23)$$

respectively.

Using (21), (22) and (23) in (3), (4) and (5) we have

$$\Pi_R = \left(\frac{\theta P_C - 2\alpha + 7}{8\theta} - \frac{\theta P_C - 2\alpha + 3}{4\theta}\right)\left(1 - \theta\frac{\theta P_C - 2\alpha + 7}{8\theta}\right) + \alpha\frac{\theta P_C - 2\alpha + 3}{4\theta} = \frac{\theta^2 P_C^2 + 12\alpha\theta P_C - 2\theta P_C - 28\alpha^2 + 52\alpha + 1}{64\theta}, \quad (24)$$

$$\Pi_D = \left(\frac{\theta P_C - 2\alpha + 3}{4\theta} - \frac{\theta P_C + 2\alpha + 1}{2\theta}\right)\left(1 - \theta\frac{\theta P_C - 2\alpha + 7}{8\theta}\right) - \alpha\frac{\theta P_C - 2\alpha + 3}{4\theta} = \frac{\theta^2 P_C^2 - 4\alpha\theta P_C - 2\theta P_C + 4\alpha^2 - 28\alpha + 1}{32\theta} \quad (25)$$

and

$$\Pi_M = \left(\frac{\theta P_C + 2\alpha + 1}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C - 2\alpha + 7}{8\theta}\right) = \frac{\theta^2 P_C - 4\alpha\theta P_C - 2\theta P_C + 4\alpha^2 + 4\alpha + 1}{16\theta} \quad (26)$$

respectively. Thus:

Proposition 3.3 Suppose that only the distributor gives discount, then the retailer, the distributor and the manufacturer's prices are given by (21), (22) and (23), and their payoffs are given by (24), (25) and (26) respectively.

Provision of Discount by the Manufacturer

This section deals with the provision of discount by only the manufacturer to the distributor. Thus, we have that $\lambda = \alpha = 0$. Thus (11), (10) and (9) become

$$P_R = \frac{\theta P_C - 4\phi + 7}{8\theta}, \quad (27)$$

$$P_D = \frac{\theta P_C - 4\phi + 3}{4\theta} \quad (28)$$

and

$$P_M = \frac{\theta P_C - 4\phi + 1}{2\theta} \quad (29)$$

respectively.

Using (27), (28) and (29) in (3), (4) and (5) we have

$$\Pi_R = \left(\frac{\theta P_C - 4\phi + 7}{8\theta} - \frac{\theta P_C - 4\phi + 3}{4\theta}\right)\left(1 - \theta\frac{\theta P_C - 4\phi + 7}{8\theta}\right) = \frac{\theta^2 P_C - 8\phi\theta P_C - 2\theta P_C + 16\phi^2 + 8\phi + 1}{64\theta}, \quad (30)$$

$$\Pi_D = \left(\frac{\theta P_C - 4\phi + 3}{4\theta} - \frac{\theta P_C - 4\phi + 1}{2\theta}\right)\left(1 - \theta\frac{\theta P_C - 4\phi + 7}{8\theta}\right) + \phi\frac{\theta P_C - 4\phi + 1}{2\theta} = \frac{\theta^2 P_C^2 + 8\phi\theta P_C - 2\theta P_C - 48\phi^2 + 24\phi + 1}{34\theta} \quad (31)$$

and

$$\Pi_M = \left(\frac{\theta P_C - 4\phi + 1}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C - 4\phi + 7}{8\theta} - \phi\frac{\theta P_C - 4\phi + 1}{2\theta}\right) = \frac{\theta^2 P_C^2 - 8\phi\theta P_C - 2\theta P_C + 16\phi^2 - 8\phi + 1}{16\theta} \quad (32)$$

respectively. Thus:

Proposition 3.4 Suppose that only the distributor gives discount, then the retailer, the distributor and the manufacturer's prices are given by (27), (28) and (29), and their payoffs are given by (30), (31) and (32) respectively.

Provision of Discount by the Manufacturer and the Distributor

In this section we consider a situation where the manufacturer and the distributor give discount to the distributor and retailer respectively. Thus, we have that $\lambda = 0$, $\alpha > 0$, $\phi > 0$. Now, letting $\lambda = 0$ in (11), (10) and (9) we have

$$P_R = \frac{\theta P_C - 2\alpha - 4\phi + 7}{8\theta}, \quad (33)$$

$$P_D = \frac{\theta P_C - 2\alpha - 4\phi + 3}{4\theta} \quad (34)$$

and

$$P_M = \frac{\theta P_C + 2\alpha - 4\phi + 1}{2\theta} \quad (35)$$

respectively.

Using (33), (34) and (35) in (3), (4) and (5) we have

$$\Pi_R = \left(\frac{\theta P_C - 2\alpha - 4\phi + 7}{8\theta} - \frac{\theta P_C - 2\alpha - 4\phi + 3}{4\theta}\right)\left(1 - \theta\frac{\theta P_C - 2\alpha - 4\phi + 7}{8\theta}\right) + \alpha\frac{\theta P_C - 2\alpha - 4\phi + 3}{4\theta} = \frac{\theta^2 P_C^2 + 12\alpha\theta P_C - 8\phi\theta P_C - 2\theta P_C - 48\alpha\phi + 16\phi^2 - 28\alpha^2 + 8\phi + 52\alpha + 1}{64\theta}, \quad (36)$$

$$\Pi_D = \left(\frac{\theta P_C - 2\alpha - 4\phi + 3}{4\theta} - \frac{\theta P_C + 2\alpha - 4\phi + 1}{2\theta}\right)\left(1 - \theta\frac{\theta P_C - 2\alpha - 4\phi + 7}{8\theta}\right) - \alpha\frac{\theta P_C - 2\alpha - 4\phi + 3}{4\theta} + \phi\frac{\theta P_C + 2\alpha - 4\phi + 1}{2\theta} = \frac{\theta^2 P_C^2 + 8\phi\theta P_C - 4\alpha\theta P_C + 48\alpha\phi - 2\theta P_C + 4\alpha^2 - 48\,\phi^2 + 24\phi - 28\alpha + 1}{32\theta} \quad (37)$$

and

$$\Pi_M = \left(\frac{\theta P_C + 2\alpha - 4\phi + 1}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C - 2\alpha - 4\phi + 7}{8\theta} - \phi\frac{\theta P_C + 2\alpha - 4\phi + 1}{2\theta}\right) = \frac{\theta^2 P_C^2 - 4\alpha\theta P_C - 8\phi\theta P_C - 2\theta P_C - 16\alpha\phi + 4\alpha^2 + 16\phi^2 + 4\alpha - 8\phi + 1}{16\theta} \quad (38)$$

respectively. Thus:

Proposition 3.5 . Suppose that both the manufacturer and the distributor give discount, then the retailer, the distributor and the manufacturer's prices are given by (33), (34) and (35), and their payoffs are given by (36), (37) and (38) respectively.

Provision of Discount by the Retailer and the Manufacturer

In this section we consider a situation where the retailer and the manufacturer give discount to the consumer and the distributor respectively. Thus, we have that $\lambda > 0$, $\alpha = 0$, $\phi > 0$. Now, letting $\alpha = 0$ in (11), (10) and (9) we have

$$P_R = \frac{\theta P_C - \lambda - 4\phi + 7}{8\theta}, \quad (39)$$

$$P_D = \frac{\theta P_C + 3\lambda - 4\phi + 3}{4\theta} \quad (40)$$

and

$$P_M = \frac{\theta P_C + \lambda - 4\phi + 1}{2\theta} \quad (41)$$

respectively.

Using (39), (40) and (41) in (3), (4) and (5) we have

$$\Pi_R = \left(\frac{\theta P_C - \lambda - 4\phi + 7}{8\theta} - \frac{\theta P_C + 3\lambda - 4\phi + 3}{4\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda - 4\phi + 7}{8\theta}\right) - \lambda\frac{\theta P_C - \lambda - 4\phi + 7}{8\theta} = \frac{\theta^2 P_C^2 - 2\lambda\theta P_C - 8\phi\theta P_C + 8\lambda\phi - 2\theta P_C + 16\phi^2 + \lambda^2 + 8\phi - 62\lambda + 1}{64\theta}, \quad (42)$$

$$\Pi_D = \left(\frac{\theta P_C + 3\lambda - 4\phi + 3}{4\theta} - \frac{\theta P_C + \lambda - 4\phi + 1}{2\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda - 4\phi + 7}{8\theta}\right) + \phi\frac{\theta P_C + \lambda - 4\phi + 1}{2\theta} = \frac{\theta^2 P_C^2 + 8\phi\theta P_C - 2h\theta P_C + 24\lambda\phi - 2\theta P_C + \lambda^2 - 48\phi^2 + 2\lambda + 24\phi + 1}{32\theta} \quad (43)$$

and

$$\Pi_M = \left(\frac{\theta P_C + \lambda - 4\phi + 1}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C - \lambda - 4\phi + 7}{8\theta}\right) - \phi\frac{\theta P_C + \lambda - 4\phi + 1}{2\theta} = \frac{\theta^2 P_C^2 - 2\lambda\theta P_C - 8\phi\theta P_C - 2\theta P_C - 8\lambda\phi + 16\phi^2 + \lambda^2 + 2\lambda - 8\phi + 1}{16\theta} \quad (44)$$

respectively.

Proposition 3.6 Suppose that both the retailer and the manufacturer give discount, then the retailer, the distributor and the manufacturer's prices are given by (39), (40) and (41), and their payoffs are given by (42), (43) and (44) respectively.

Provision of Discount by the Retailer and the Distributor

In this section we consider a situation where the retailer and the distributor give discount to the consumer and the retailer respectively. Thus, we have that $\lambda > 0$, $\alpha > 0$, $\phi = 0$. Now, letting $\phi = 0$ in (11), (10) and (9) we have

$$P_R = \frac{\theta P_C - \lambda - 2\alpha + 7}{8\theta}, \quad (45)$$

$$P_D = \frac{\theta P_C + 3\lambda - 2\alpha + 3}{4\theta} \quad (46)$$

and

$$P_M = \frac{\theta P_C + \lambda + 2\alpha + 1}{2\theta} \quad (47)$$

respectively.

Using (39), (40) and (41) in (3), (4) and (5) we have

$$\Pi_R = \left(\frac{\theta P_C - \lambda - 2\alpha + 7}{8\theta} - \frac{\theta P_C + 3\lambda - 2\alpha + 3}{4\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda - 2\alpha + 7}{8\theta}\right) - \lambda\frac{\theta P_C - \lambda - 2\alpha + 7}{8\theta} + \alpha\frac{\theta P_C + 3\lambda - 2\alpha + 3}{4\theta} = \frac{\theta^2 P_C^2 + 12\alpha\theta P_C - 2h\theta P_C + 52\alpha\lambda - 2\theta P_C + \lambda^2 - 28\alpha^2 + 52\alpha - 62\lambda + 1}{64\theta}, \quad (48)$$

$$\Pi_D = \left(\frac{\theta P_C + 3\lambda - 2\alpha + 3}{4\theta} - \frac{\theta P_C + \lambda + 2\alpha + 1}{2\theta}\right)\left(1 - \theta\frac{\theta P_C - \lambda - 2\alpha + 7}{8\theta}\right) - \alpha\frac{\theta P_C + 3\lambda - 2\alpha + 3}{4\theta} = \frac{\theta^2 P_C^2 - 2\lambda\theta P_C - 4\alpha\theta P_C - 2\theta P_C - 28\alpha\lambda + 4\alpha^2 + \lambda^2 + 2\lambda - 28\alpha + 1}{32\theta} \quad (49)$$

and

$$\Pi_M = \left(\frac{\theta P_C + \lambda + 2\alpha + 1}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C - \lambda - 2\alpha + 7}{8\theta}\right) = \frac{\theta^2 P_C^2 - 2\lambda\theta P_C - 4\alpha\theta P_C + 4\alpha\lambda - 2\theta P_C + 4\alpha^2 + \lambda^2 + 2\lambda + 4\alpha + 1}{16\theta} \quad (50)$$

respectively. Thus:

Proposition 3.7 . Suppose that both the retailer and the distributor give discount, then the retailer, the distributor and the manufacturer's prices are given by (45), (46) and (47), and their payoffs are given by (48), (49) and (50) respectively.

DISCUSSION

We now discuss the results. To do this we chose the following values for the parameters. Let$\lambda = 0.1$, $\alpha = 0.2$, $\phi = 0.2$, $\theta = 0.05$ and $P_C = 0.2$. We allow the following representations:

$\lambda > 0, \alpha = 0, \phi = 0$ Only the retailer provides discount
$\lambda = 0, \alpha > 0, \phi = 0$ Only the distributor provides discount
$\lambda = 0, \alpha = 0, \phi > 0$ Only the distributor provides discount

$\lambda = 0, \alpha > 0, \phi > 0$ The manufacturer and distributor provide discount
$\lambda > 0, \alpha = 0, \phi > 0$ The manufacturer and retailer provide discount
$\lambda > 0, \alpha > 0, \phi = 0$ The distributor and retailer provide discount
$\lambda = 0, \alpha = 0, \phi = 0$ No player provides discount
$\lambda > 0, \alpha > 0, \phi > 0$ All the players provide discount

Further, we let $\Pi = \Pi_R + \Pi_D + \Pi_M$

Table 1 Optimal prices and payoffs for game scenarios where only one player gives discount

Game Scenarios	Prices			Payoffs			
	P_R	P_D	P_M	Π_R	Π_D	Π_M	Π
$\lambda > 0, \alpha = 0, \phi = 0$	17.275	16.550	11.100	0.000	0.743	1.485	2.228
$\lambda = 0, \alpha > 0, \phi = 0$	16.775	13.550	13.100	2.553	0.000	2.080	4.633
$\lambda = 0, \alpha = 0, \phi > 0$	15.525	11.050	2.100	1.001	2.423	0.005	3.429

Table 2 Optimal prices and payoffs for game scenarios where two players give discount

Game Scenarios	Prices			Payoffs			
	P_R	P_D	P_M	Π_R	Π_D	Π_M	Π
$\lambda = 0, \alpha > 0, \phi > 0$	14.775	9.550	5.100	2.796	0.750	0.260	3.808
$\lambda > 0, \alpha = 0, \phi > 0$	15.275	12.550	3.100	0.000	2.853	0.065	2.928
$\lambda > 0, \alpha > 0, \phi = 0$	16.525	15.050	14.100	0.861	0.000	2.415	3.276

Table 3 Optimal prices and payoffs for a game scenario where all-three players give discount

Game Scenario	Prices			Payoffs			
	P_R	P_D	P_M	Π_R	Π_D	Π_M	Π
$\lambda = 0, \alpha = 0, \phi = 0$	17.525	15.050	10.100	0.306	0.613	1.225	2.144

Table 4 Optimal Prices and Payoffs for a Game Scenario where All-three Players Do Not Give Discount

Game Scenario	Prices			Payoffs			
	P_R	P_D	P_M	Π_R	Π_D	Π_M	Π
$\lambda > 0, \alpha > 0, \phi > 0$	14.525	11.050	6.100	1.156	0.918	0.395	2.469

From Tables 1 and 4 we observe that the entire channel payoff is better-off with a single player providing discount than for all-three players or no player providing discount (Table 3). Clearly, while individual players' payoffs may not be easily comparable, total channel payoffs affirms that they are better-off with only one player providing discount. With the adoption of only one player providing discount, it will only remain for the channel members to agree on a channel sharing formula that ensures that the discount proving player is not short-changed since a player stands to have a very low payoff if he alone provides the discount (Table 1).

Further, looking at Table 2 we observe that when two players engage in discount provision their payoffs are much lower compared to that of the other player. However, the entire channel payoff is larger than that for both when there is no provision of discount by any of the players and when there is provision of discount by all the players. Thus, the players can adopt this scenario, and enter into a bargain on how to share the payoff to ensure that no player is short-changed. We also observe a similar trend by comparing Tables 2, 3 and 4.

Clearly, the channel is better-off with discount than without discount, however, we need to answer the question of which of the discount options (game scenarios) should be adopted. Obviously, a comparison of Tables 1, 2 and 4 shows that total discount (that is, provision of discount by all the players) should be off the table. From Tables 1 and 2 we observe that the channel is better-off with a single player's involvement in discount.

However, that player should not be the retailer because his provision of discount in the absence of any assistance from neither the distributor nor the manufacturer places a strain on his payoff which eventually affects the channel payoff. Thus, the channel performs best with the distributor and the manufacturer's provision of discount. However, it is pertinent to note that the resulting large receivable payoff by the non-discount-providing players should be shared. The players will need to agree on a sharing formula that will ensure that no player, especially the discount-providing player is short-changed.

CONCLUSION

This work considered a three-level game-theoretic model on sequential provision of discount from the manufacturer to the consumer through the distributor and the retailer in a supply channel. The manufacturer is the Stackelberg leader, while the distributor is the first follower, and the retailer is the last follower.

The work examined four scenarios, and observed that provision of discount reduces a player's payoff, and that the channel should avoid both total provision and non-provision of discount. The channel performs best with the provision of discount by only one player. However, the channel members should agree on a sharing formula that ensures that the player who engages in the provision of discount is not short-changed.

This work has some limitations. A non-linear demand function can be used instead of a linear demand function. Further, the work can be extended by incorporating multiple manufacturers, distributors and retailers. These can provide more insight into sequential discount transfer.

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