INTEGER PROGRAMMING APPROACH TO NURSE SCHEDULING PROBLEM (NSP) IN HOSPITAL MANAGEMENT

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ABSTRACT
The need for optimum allocation of available manpower for increase productivity and efficient management is a typical personnel scheduling problem faced by hospitals and other personnel management organizations. In this study, integer programming approach was used to solve a Nurse Scheduling Problem in the hospital using the days-off as decision variables. The result of implementation in St. Luke hospital, Anua, Uyo, Nigeria produces optimal days-off Nurses roster with ten Nurses each in Antenatal Ward, Casualty, Children, Male Surgical and Operating theatre Wards; nine Nurses each in Antenatal Clinic, Maternity Ward, Male Medical, Female Medical and Nursery Wards; three Nurses each in Anesthesia, Counseling and Immunization Units; and fourteen Nurses each in Labour Ward and Out-patients Department which ensures adequate continuous ward care services and minimizes 0.38% of the Hospital's total Nursing staff cost.

INTRODUCTION

Nursing staff scheduling is an essential task in manpower resource management in hospital and effective Nurse scheduling contributes to improved performance of Nurses. Consequently, it has a great impact on the quality of healthcare delivery, the recruitment of Nursing personnel, the development of Nursing budget, staff and patient safety, staff and patient satisfaction and administrative workload. Typically, personnel scheduling problems are highly constrained and complex optimization problem. The need to take into account individual preferences further complicates the process. Unlike many other organizations, healthcare institutions need to be staffed 24 hours a day over seven days a week. In preset shift, some Nurses are allowed to state their wishes on how they would love to be scheduled in the hospital, while other Nurses are scheduled around the preset shifts. Cheang *et al* (2003) added Nursing staff's preference into the factors to be considered when preparing work schedule, because according to Burke *et al* (2004) good quality solution can lead to a higher level of personnel satisfaction. In recent years, the emergence of larger and constrained problems such as Nurse Workforce, hospital work scheduling regulations, etc. has presented a real challenge. The Nurse scheduling problem becomes complex when additional factors such as patient admission, nurse qualifications or license to practice, type of disease as well as accidents and emergency are included. Operations/production management scheduling involves devising efficient methods to find the optimal order in which given tasks, events, or actions are carried out, usually subject to certain constraints, Fan (2020). Over the past six decades, a considerable amount of research has been expended on scheduling theory and practice. The literature contains many state-of-the-art studies on exact or approximation algorithms to solve a wide array of scheduling problems in various contexts, showing their performance analytically via establishing their computational complexity or empirically through numerical studies, Hsu and Liao (2020). Optimization problems where the variables are required to take on integer values are called integer programming (IP) problems. A linear integer programming problem is obtained when all the functions are linear, Jasbir (2004).

Efficient scheduling of nurses can greatly reduce the labour cost, which is generally a significant proportion of total cost for most organizations. The number of employees assigned to each days-off pattern must be determined in order to satisfy all daily labour demand with the minimum number or cost of employees. Smith and Wiggins (1997) proposed a Mathematical Programming Technique as the most suitable method for solving NSP. It is primarily used in days-off Nurse scheduling and is most suitable because it eliminates the problem of infeasible or less optimal solution by enforcing integer decision variables.

The relative importance of satisfying employee needs in staffing and scheduling decisions have grown, companies offer part-time contracts or flexible work hours and take into account employee preference when creating work schedules. Betchtold *et al* (1991) classify personnel scheduling solution methods in two categories: linear programming or construction based. Alfares (2002) proposed ten categories for tour scheduling approaches: manual solution, integer programming, implicit modeling, decomposition, goal programming, working set generation, linear programming–based solution, construction and improvement, Meta heuristics and other methods. Prior to the development of mathematical programming (optimization techniques), most Nurses schedules were based on the cyclical modeling, Rocha *et al* (2013). Cyclic models consist of regular patterns which can be rotated across multiple time periods. Howell (1966) provided the first cyclical scheduling approach which takes into consideration the behavior and preference of the individual Nurse. However, in a non-cyclical scheduling process, a new schedule is generated for each scheduling period. This process is more time-consuming but is more flexible to changes such as the variability of demand, Valouxis and Housos (2000). Subsequently, heuristic models which are able to consider all the requirements at the planning stage were adopted in Nurse scheduling. This method is time-consuming and less accessible, Guinet (1995).

The focus of this work is on days-off scheduling methodology. A great deal of interest has been directed at the 5 working days, 7 days of the week (5, 7) days-off problem in which 2 consecutive days-off are given per week; El-Quliti and Al Darrab (2009), Alfares (1998) and Nanda and Browne (1992).However, Fitzsimmons and Fitzsimmons (2004) had proposed a formal complete formulation of Nurse scheduling problem as integer programming problem. We seek to adapt this approach to optimize days-off rosters for Nurses in the active wards using off-days as decision variables. This will reduce the hospital's labour cost and enhance efficient healthcare delivery.

St. Luke's hospital, Anua, Uyo, Nigeria has 20 wards and each ward has a minimum staffing requirement for each day. The hospital has a total of 181 full-time nurses (as at the time data were collected for this research) that work five days a week and take two consecutive days-off. Therefore, this work applies integer programming in days-off scheduling of Nurses for each ward in St. Luke's hospital, Anua, Uyo to ensure optimal scheduling of all Nurses in each ward/unit while meeting the ward's nursing-staff requirement for each day.

METHODOLOGY

The Branch and Bound Algorithm

1. **Initialization:** Consider the following integer programming problem (ILPP):

$$Maximze: Z = c_1x_1 + c_1x_2 + \ldots + c_1x_n$$

$$subjectto: \begin{Bmatrix} a_{21}x_1 & + & a_{22}x_2 & + & \ldots & a_{2n}x_n & = b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \ldots & a_{2n}x_n & = b_2 \\ \vdots & \vdots & \vdots & + & \ldots & \vdots & \vdots \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \ldots & a_{mn}x_n & = b_n \end{Bmatrix}$$

$$x_1, x_{2,} \ldots x_n \geq 0 \ andx_1, x_{2,} \ldots x_n areintegers$$

We obtain optimal solution of the problem ignoring the integer restriction on the variables.

2. **Branching Step**

i. Let X_k be one of the basic variables which does not have an integer value and also has the largest fractional value.

ii. Branch (that is, partition) the LP problem into 2 new LP sub-problems based on integer values of X_k that are immediately above and below its non-integer value (the fractional value). It is partitioned by adding 2 mutually exclusive constraints to the original LP problem. The constraints are;

$$X_k \leq [X_k] \text{ and } X_k \geq [X_k] + 1$$

Here $[X_k]$ is the integer portion of the current fractional value of the variable, X_k. This is done to exclude the non-integer value of X_k. The two new LP problems will be:

Maximize: $Z = \sum_{j=1}^{n} C_j X_j$
Subject to: $\sum_{j=1}^{n} a_{ij} X_j = b_i$
$X_k \leq [X_k]; X_j \geq 0$
And: Maximize $Z = \sum_{j=1}^{n} C_j X_j$
Subject to: $\sum_{j=1}^{n} a_{ij} X_j = b_i$
$X_k \geq [X_k] + 1 \,; X_j \geq 0$

3. **Bound Step:**
Obtain the optimal solution of the sub-problems. Let the optimal value of the objectives function of the sub-problems be Z_2 and Z_3 respectively. The best integer solution value becomes the lower bound on the integer LP problem. Let the lower bound de denoted by Z_L.

4. **Fathoming Step:**
   i. If a sub-problem yields an infeasible solution, terminate the branch.
   ii. If a sub-problem yields a feasible non-integer solution, return to step 2.
   iii. If a sub-problem yields a feasible integer solution, examine the value of the objective function. If the value is equal to the upper bound, an optimal upper solution is reached. But if it is not equal to the upper bound but exceeds the lower bound, this value is used as the new upper bound, then return to step 2. But, if this value is less than lower bound, terminate the branch.

5. **Termination:** The procedure of branching and bounding continues until no further sub-problem remains to be examined. At this stage, the integer solution corresponding to the current lower bound is the optimal all-integer programming problem solution.

The ILPP analysis of the Nursing personnel data was implemented with the aid of TORA software in order to reduce the complexities and errors associated with manual computing.

Data Requirement

St. Luke's hospital, Anua, Uyo is a missionary general hospital. It has 181Nurses, 74employed by government and 107employed by missionary. A full time Nurse works five days a week with two consecutive days-off. The hospital has about 20 wards with 17 operational wards/units. Each clinical ward/unit has a daily Nurse requirement (Table 1).

Management wants to minimize the hospital's labour cost while meeting the core coverage of each day and shift and also satisfying the schedule of 5 work days and 2 days-off for staff.

Table 1: Nursing staff availability and requirement at St. Luke's Hospital, Anua, Uyo

NURSES (CORE STAFF)

Wards/Units	Government	Mission	Total Availability	Daily Requirements
Antenatal clinic	7	4	11	6
Antenatal ward	7	5	12	7
Anaesthesia	3	1	4	2
Casualty (emergency)	6	6	12	7
Children's ward	5	8	13	7
Labour ward	10	10	20	10
Maternity ward II	2	10	12	6
Male medical ward	6	6	12	6
Male surgical ward	3	8	11	7
Female medical ward	5	9	14	6
Female surgical ward	4	8	12	7
Out-patients Dept.	7	10	17	10
Nursery	1	10	11	6
Operating theatre	2	11	13	7
Counselling unit	3	-	3	2
Immunization	3	1	3	2

Problem Formulation

The most critical element of the scheduling problem is deciding on the days. Therefore, the decision variables can be conceptualized as the two consecutive off-days that a Nurse is assigned in a scheduling cycle. There are seven possible pairs of consecutive off-days available: Saturday – Sunday; Sunday – Monday; Monday – Tuesday; Tuesday – Wednesday; Wednesday – Thursday; Thursday – Friday and Friday – Saturday. A satisfactory schedule will be produced if assignment can be made to guarantee these off-days to Nurses while meeting the unit requirement for each day.

Let X_1, X_2, X_3, X_4, X_5, X_6 and X_7 represents the number of Nurses to be off on Saturday-Sunday, Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday - Friday and Friday-Saturday respectively. The ILP formulated problem is:

Minimize $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$

Subject to: $X_1 + X_2 + X_3 + X_4 + X_5 \geq b_1$ (Saturday constraints)

$X_2 + X_3 + X_4 + X_5 + X_6 \geq b_2$ (Sunday constraints)

$X_3 + X_4 + X_5 + X_6 + X_7 \geq b_3$ (Monday constraints)

$X_1 + X_4 + X_5 + X_6 + X_7 \geq b_4$ (Tuesday constraint)

$X_1 + X_2 + X_5 + X_6 + X_7 \geq b_5$ (Wednesday constraints)

$X_1 + X_2 + X_3 + X_6 + X_7 \geq b_6$ (Thursday constraints)

$X_1 + X_2 + X_3 + X_4 + X_7 \geq b_7$ (Friday constraints)

$X_i \geq 0$; and integer

where Z is the objective function and b_j is the daily staff requirement for a day of the week. The above is represented in the following matrix form;

$$\text{Minimize } Z = [1\ 1\ 1\ 1\ 1\ 1\ 1] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix}$$

$$\text{Subject to: } \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix} \geq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix};$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \text{ and integer}$$

In Table 1, we note that some of the wards/units have the same number of daily required Nurses; therefore, their results will be similar and are therefore grouped into 4 on this basis. Let G_1 represent Antenatal ward, Casualty ward, Children's ward, Male surgical ward, Female surgical ward and Operating theatre with a daily requirement of 7 Nurses each. The integer linear problem for G_I is:

Minimize Z= $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$

Subject to: $X_1 + X_2 + X_3 + X_4 + X_5 \geq 7$

$X_2 + X_3 + X_4 + X_5 + X_6 \geq 7$

$X_3 + X_4 + X_5 + X_6 + X_7 \geq 7$

$X_1 + X_4 + X_5 + X_6 + X_7 \geq 7$

$X_1 + X_2 + X_5 + X_6 + X_7; \geq 7$

$X_1 + X_2 + X_3 + X_6 + X_7 \geq 7$

$X_1 + X_2 + X_3 + X_4 + X_7 \geq 7$

$X_i \geq 0$; and integer

The above is represent in the following matrix form:

$$\text{Minimize } Z = [1\ 1\ 1\ 1\ 1\ 1\ 1] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix}$$

$$\text{Subject to; } \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix} \geq \begin{bmatrix} 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \end{bmatrix}$$

Similarly, integer programming formulation for G_2 representing Antenatal clinic, Maternity ward II, Male medical ward, Female medical ward and Nursery with a daily requirement of 6 nurses, G_3 representing Anaesthesia unit, Counseling unit and Immunization with a daily requirement of 2 Nurses and G_4 representing Labour ward and Out-patient Department with daily requirement of 10 Nurses were formulated with values of b_j's on the right hand side of the constraints as 6, 2 and 10 respectively.

ANALYSIS AND RESULTS

TORA (2.00)software was used to execute the branch and bound algorithm for solving the problem with the following results in Figures 2, 3, 4 and 5 which show the solutions to the Nurse Scheduling problem in St. Luke's Hospital, Anua, Uyo. The objective value (minimum) for $G_1$is 10 with$X_1 = 1$; $X_2 = 2$; $X_3 = 1$; $X_4 = 2$; $X_5 = 1$; $X_6 = 2$; X_7= 1.

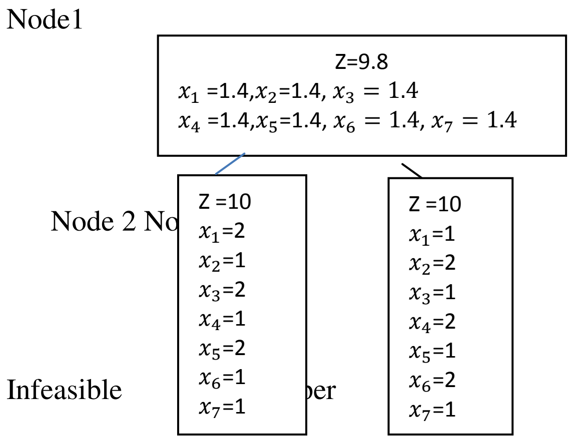

Figure 2: Branch-and-Bound Enumeration Tree for solution of G_1

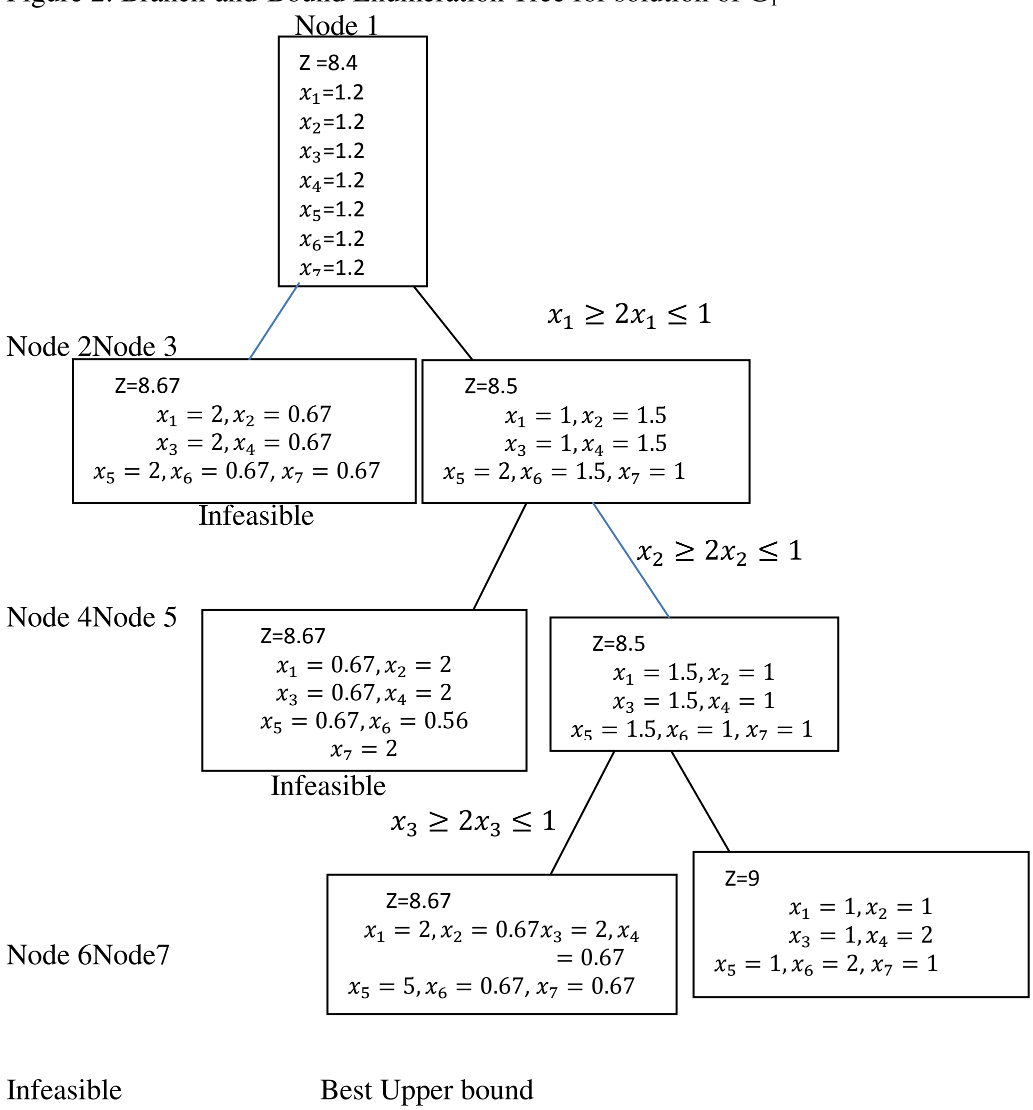

Figure 3: Branch-and-bound enumeration tree for solution of G_2

Figure 3 shows that the objective value (minimum) for $G_2$is 9 with$X_1 = 1;\ X_2 = 1;\ X_3 = 1;\ X_4 = 2;\ X_5 = 1;\ X_6 = 2;\ X_7 = 1.$

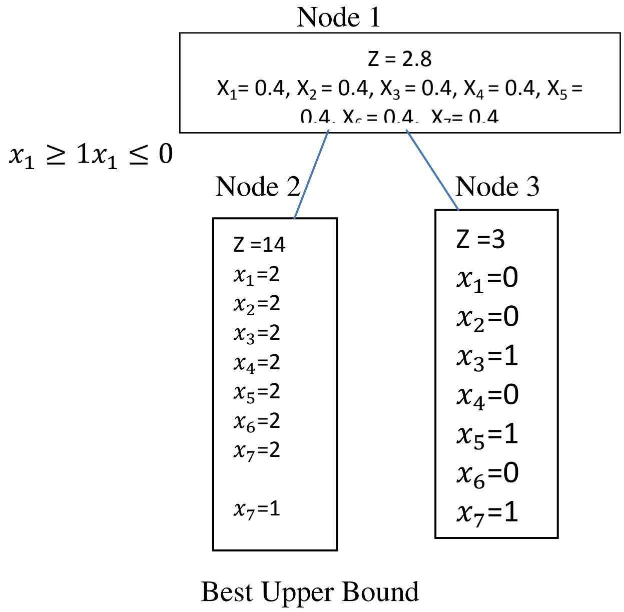

Figure 4: Branch –and –Bound Enumeration for G_3 solution

Similarly, the objective value (Minimum) for G_4consist of a single node with Z=14 with$X_1 = 2;\ X_2 = 2;\ X_3 = 2;\ X_4 = 2;\ X_5 = 2;\ X_6 = 2;\ X_7 = 2.$

Table 2: Summary of the optimal solution of the Nurse Scheduling problem

Group	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Objective value(Z)
G_1	1	2	1	2	1	2	1	10
G_2	1	1	1	2	1	2	1	9
G_3	0	0	1	0	1	0	1	3
G_4	2	2	2	2	2	2	2	14

Table 2 summarizes the optimal solution of the Nurse scheduling problem. The optimal solution for each ward is presented in Table 3.

DISCUSSION

From the result of the analysis, we found out the following:

1. In Antenatal Ward, Casualty Ward, Children's Ward, Male Surgical Ward, Female Surgical Ward and Operating Theatre (G_1) which has total availability of 12, 12, 13, 11, 12 and 13 employed Nurses respectively; (Table 1), an optimal total of 10 Nurses for each of the wards in this group is needed in order to satisfy their daily requirement of 7 Nurses; (Table 3).

Explicitly, $X_1 = 1, X_3 = 1,\ X_5 = 1\ and\ X_7 = 1$ indicates that one Nurse should be assigned to Saturday-Sunday off, Monday-Tuesday off, Wednesday-Thursday off and Friday-Saturday off respectively; $X_2 = 2,\ X_4 = 2\ and\ X_6 = 2$ indicates that two Nurses should be assigned to: Sunday-Monday, Tuesday-Wednesday and Thursday-Friday off respectively; see Table 2. This would minimizes the total Nursing staff cost by 0.3%.

Table 3: Optimal solution for each ward in St. Luke's hospital, Anua, Uyo.

Wards/units	Optimal allocation of Nurses for days –off							Optimal number of Nurses required
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	
Antenatal clinic	1	1	1	2	1	2	1	9
Antenatal ward	1	2	1	2	1	2	1	10
Anaesthesia	0	0	1	0	1	0	1	3
Casualty (emergency)	0	2	1	2	1	2	1	10
Children' ward	1	2	1	2	1	2	1	10
Labour ward	2	2	2	2	2	2	2	14
Maternity ward II	1	1	1	2	1	2	1	9
Male medical ward	1	1	1	2	1	2	1	9
Male surgical ward	1	2	1	2	1	2	1	10
Female medical ward	1	1	1	2	1	1	1	9
Female surgical ward	1	2	1	2	1	2	1	10
Out-Patient Department	2	2	2	2	2	2	2	14
Nursery	1	1	1	2	1	2	1	9
Operating theatre	1	2	1	2	1	2	1	10
Counselling unit	0	0	1	0	1	0	1	3
Immunization	0	0	1	0	1	0	1	3

2. In Antenatal Clinic, Maternity Ward II, Male Medical Ward, Female Medical Ward and Nursery (G_2) with total availability of 11,12,12,14 and 11 Nurses respectively and a daily requirement of 6 Nurses each in Table 1, an optimal total of 9 Nurses is required in each ward. Also, $X_1 = 1;\ X_2 = 1;\ X_3 = 1;\ X_5 = 1;\ and\ X_7 = 1$ indicate that one Nurse should be assigned to Saturday-Sunday off, Sunday-Monday off, Monday-Tuesday off, Wednesday-Thursday off and Friday-Saturday off respectively; $X_4 = 2$ and $X_6 = 2$ implies that two Nurses should be assigned to Tuesday-Wednesday off and Thursday-Friday off respectively; see Table 2. This would minimize the total Nursing staff cost by 0.15%.

3. In Anesthesia, Counseling and Immunization Units (G_3) with a total of 4,3 and 3 employed Nurses respectively and with a daily requirement of 2 Nurses each in Table 1, an optimal total of 3 Nurses is required; (see Table 3).$X_1 = 0;\ X_2 = 0;\ X_4 = 0$ and $X_6 = 0$ implies that no Nurse should be assigned to Saturday-Sunday off, Sunday-Monday off, Monday-Tuesday off and Thursday-Friday off respectively;$X_3 = 1,\ X_5 = 1$ and $X_7 = 1$ implies that one Nurse should be assigned to Tuesday-Wednesday off, Thursday-Friday and Friday-Saturday off respectively; see Table 2. This would minimize the total labour cost by 0.01%.

4. In Labour Ward and Out-Patients Department (G_4), which needs to be staffed with more Nurses than others due to their peculiar duties, the hospital employed a total of 17 Nurses and a daily requirement of 10 Nurses (Table 1), an optimal total of 14Nurses is required; see Table 3. $X_1 = 2;\ X_2 = 2;\ X_3 = 2;\ X_4 = 2;\ X_5 = 2;\ X_6 = 2;\ X_7 = 2$ implies that two Nurses should be scheduled on Saturday-Sunday off, Sunday-Monday off, Monday-Tuesday off, Tuesday-Wednesday off, Wednesday-Thursday off, Thursday-Friday off and Friday-Saturday off respectively; see Table 2. This would minimize the total Nursing staff cost by 0.09%.
In general, the hospital would minimizes the total Nursing staff cost by 0.38% if the optimal solutions of G_1=10 Nurses, G_2=9 Nurses, G_3=3 Nurses and G_4-14 Nurses is implemented.

Table 4: Sample one week Roster for Nurses in G_1 Wards

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	ON	ON	ON	ON	ON	OFF	OFF
2	OFF	ON	ON	ON	ON	ON	OFF
3	OFF	OFF	ON	ON	ON	ON	ON
4	OFF	OFF	ON	ON	ON	ON	ON
5	ON	OFF	OFF	ON	ON	ON	ON
6	ON	ON	OFF	OFF	ON	ON	ON
7	ON	ON	OFF	OFF	ON	ON	ON
8	ON	ON	ON	OFF	OFF	ON	ON
9	ON	ON	ON	ON	OFF	OFF	ON
10	ON	ON	ON	ON	OFF	OFF	ON
Required	7	7	7	7	7	7	7
Assigned	8	7	7	7	7	7	7
Excess	1	0	0	0	0	0	0

Table 5: Sample one week Roster for Nurses in G_2 Wards

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	ON	ON	ON	ON	ON	OFF	OFF
2	ON	OFF	OFF	ON	ON	ON	ON
3	ON	ON	ON	OFF	OFF	ON	ON
Required	2	2	2	2	2	2	2
Assigned	2	3	2	2	2	2	2
Excess	0	0	0	0	0	0	0

Table 6: Sample one week Roster for Nurses in G_3 Units

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	ON	ON	ON	ON	ON	OFF	OFF
2	OFF	ON	ON	ON	ON	ON	OFF
3	OFF	OFF	ON	ON	OFF	ON	ON
4	ON	OFF	OFF	ON	ON	ON	ON
5	ON	ON	OFF	OFF	ON	ON	ON
6	ON	ON	OFF	OFF	ON	ON	ON
7	ON	ON	ON	OFF	OFF	ON	ON
8	ON	ON	ON	ON	OFF	OFF	ON
9	ON	ON	ON	ON	OFF	OFF	ON
Required	6	6	6	6	6	6	6
Assigned	7	7	7	6	6	6	6
Excess	1	1	1	0	0	0	0

Table 7: Sample one week Roster for Nurses in G_4 Wards

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	ON	ON	ON	ON	ON	OFF	OFF
2	ON	ON	ON	ON	ON	OFF	OFF
3	OFF	ON	ON	ON	ON	ON	OFF
4	OFF	ON	ON	ON	ON	ON	OFF
5	OFF	OFF	ON	ON	ON	ON	ON
6	OFF	OFF	ON	ON	ON	ON	ON
7	ON	OFF	OFF	ON	ON	ON	ON
8	ON	OFF	OFF	ON	ON	ON	ON
9	ON	ON	OFF	OFF	OFF	OFF	ON
10	ON	ON	OFF	OFF	OFF	OFF	ON
11	ON	ON	ON	OFF	OFF	ON	ON
12	ON	ON	ON	OFF	OFF	ON	ON
13	ON	ON	ON	ON	ON	OFF	ON
14	ON	ON	ON	ON	ON	OFF	ON
Required	10	10	10	10	10	10	10
Assigned	10	10	10	10	10	10	10
Excess	0	0	0	0	0	0	0

Tables 4, 5, 6 and 7 show the resulting sample scheduling for the wards with ON representing assignments and OFF representing days-off. The last three rows of each table show the requirements, total number of assignments and any excess in a given day.

CONCLUSION

Nurse scheduling analysis was carried out in this work using integer linear programming with the aid of TORA (2.00) optimization software. The results produce the following optimal days-off schedule for Nurses with 0.38% reduction in staffing cost in St. Luke's hospital, Anua, Uyo:

i. G_1 wards-Antenatal Ward, Casualty, Children, Male Surgical, Female Surgical and Operating Theatre Wards when assigned an optimal total of 10 Nurses each would minimize 0.13% of the total nursing staff cost with individual ward contributions of 0.02%, 0.02%, 0.03%.0.01%,0.02%,0.03% respectively.

ii. G_2 wards-Antenatal Clinic, Maternity Ward II, Male Medical, Female Medical and Nursery Wards when assigned an optimal total of 9 Nurses each would minimize 0.15% of Nursing staff cost with individual ward contributions of 0.02%, 0.03%, 0.03%,0.05%,0.02% respectively.

iii. G_3 wards-Anaesthesia, Counselling and Immunization Units when assigned an optimum of 3 Nurses each would minimize 0.01% of total Nursing staff cost with individual ward contributions of 0.01%, 0.00%, 0.00% respectively.

iv. G_4 wards-Labour Ward and Out-Patients Department when assigned an optimum of 14 Nurses would minimize 0.090% of total Nursing staff cost with individual contributions of 0.06% and 0.03% respectively.

This study employs an integer programming model approach for solving the 5 working days a week and 2 consecutive days-off Nurse scheduling problem. The case of St. Luke's Hospital, Anua, Uyo shows that the total Nursing staff cost of the hospital would be minimized by 0.38% if the optimal days-off schedule proposed in this work is utilized. Therefore, the use of integer programming model is essential in Nurse scheduling problem in hospital management and personnel scheduling in general in organizations with similar structure. This would help reduce labour cost and provide optimum service delivery.

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