NON-INTERACTIVE FEASIBLE REGION CONTRACTION INTERIOR-POINT ALGORITHM FOR SOLUTION OF MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

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ABSTRACT
This paper proposes an interior-point algorithm for solving multi-objective linear programming problems. The algorithm is non-interactive since it does not require an interaction with the decision maker during the process of solving multi-objective linear programming problems. It centered on locating a feasible interior-point each time the feasible region is contracted, thus generating a sequence of feasible interior-points that converge at the optimal compromise solution of a multi-objective linear programming problem. The convergence of the algorithm is proved to be on the boundary but not necessary at the extreme point of the constraint polytope. The algorithm was shown to perform with less number of iteration to reach the optimal compromise solution.

INTRODUCTION
Interior-point methods have become highly successful in solving linear programming problems especially large scale problems, while enjoying good theoretical convergence and complexity properties. (Ye (1997); Roos, Terlaky and Vial (1997); Rico-Ramirez and Westerberg (1999)]. An excellent complexity result of this paper, as well as the claim that the performance of the new method on real-world problem is significantly faster than the one with the simplex method. Since the introduction of Karmarkar's algorithm, interior-point algorithms have been developed with strong theoretical properties and excellent numerical performance. Although there have been a lot of theoretical advances and several interior-point methods on how to determine the optimal solutions for multi-objective optimization problems and improve the quality of this solution for optimal decisions, it is important to note that these advances require an accurate result that will give the objective function(s) the optimal compromise solution for a particular optimization problem at hand.

Benayoun*et al*, (1971) firstly presented an interactive algorithm for linear multi-objective programming. Its idea is the first in finding out a solution of an ideal value to every objective, obtaining better solution by improving unsatisfactory objective value. Geoffrion, Dyer and Feinberg (1972) gave an interactive approach to multi-criterion optimization where they defined a non-explicitly criterion function to show the decision maker's overall preference.

Anderson and Jibrin (2009) gave an interior-point method that uses weighted analytic centre. Anstreicher (2011) considered an extension of ordinary linear programming that adds weighted logarithmic barrier terms for some variables. The resulting problem generalized both linear programming and the problem of finding the weighted analytical centre of a polytope. They showed that the problem has a dual of the same form and give complexity results for several different interior-point algorithms. Li, Dong, Jia-wei and Qing-huai (2011) in their work transform multi-objective problem to unsmooth single-objective and construct new path-following. By path-tracking, they obtained minimal weak efficient solution of the multi-objective problem. Meng, Shen and Jiang (2011) presented an interactive algorithm to solve the inequality constrained multi-objective programming problems. They transformed multi-objective problem to single objective optimal problem with inequality constraints and prove that under some conditions, a best possible solution to LP problem is the same with multi-objective Pareto. They further designed the interaction algorithm to multi-objective problem accordingly.

After the acquisition of some fundamentals from interior-point multi-objective linear programming, and a broad review of several methods of solutions, we can draw conclusion that determination of accurate and quality optimal solution of a decision problem with multi-objectives still remains a persistent challenge.

A New interior-point technique known as Feasible Region Contraction Algorithm (FRCA) which focused on single objective linear programming problems with unique optimal solution was developed by Effanga (2011). New strategies of the interior-point methods of optimization has been generating enormous interest [Zhao, *et al*. (2012)]. Pandian and Jayalakshmi (2013) proposed a moving optimal method for solving multi-objective linear programming problems. Their method provides efficient solution with satisfactory level of percentage of each objective functions at each point on the line. Khanjerpanah and Pishvaee (2017) proposed a fuzzy robust programming approach to multi-objective portfolio optimization problem under uncertainty. Lachhwanni (2018), in his work presents an alternative method based on fuzzy programming for solving multi-objective linear bi-level multi-follower programming problem in which there is no sharing of information among followers. Effanga and Nsien (2019) presents a technique for generating Pareto Optimal solutions of Multi-objective Linear Programming Problems.

While a great variety of interior-point approaches exist to generate Pareto set, different authors wonder whether there would exist a fully based interior-point approach to generate the complete "Pareto-optimal set or a Pareto optimal point" of a Multi-Objective Linear Programming (MOLP) problem. However, determining a unique and quality optimal solution to multi-objective programming problem still remains difficult especially if the number of solutions and objectives are too large to allow the effective use of current existing decision making techniques. This work is devoted to the development of interior-point algorithm that gives an optimal compromise solution for multi-objective linear programming problem.

The General Formulation Of Multi-Objective Linear Programming Problem

A standard multi-objective linear programming problem (MOLPP) can be modeled as follows:

$$
\begin{aligned}
&max \quad z_1(x) = \sum_{i=1}^{n} c_{1j}x_j \\
&max \quad z_2(x) = \sum_{j=1}^{n} c_{2j}x_j \\
&.max \quad z_p(x) = \sum_{j=1}^{n} c_{pj}x_j \\
&subjectto: \\
&\sum_{j=1}^{n} a_{ij}x_j \leq b_i \quad , i = 1,2\ .\ .\ .\ \mathrm{m} \\
&x_j \geq 0 \ , \quad j = 1,2.\dots n
\end{aligned}
$$

where a_{ij} is an mxn constraint matrix, $z(x)$ is a vector of p real -valued functions, x is an n-dimensional vector of solution (decision) vector and $b \in R^m$ is vector of available resources (decision space).

Let z denote the image of X in the objective function space: $z = \left\{ z \in \Re^n : \mathrm{z} = z(x) = (z_1(x), \ .\ .\ .\ z_p(x)), \ \ x \in X \right\}$. In multi-objective optimization there is no feasible solution that optimizes simultaneously all objective functions. Thus, the concept of optimal solution is replaced by Pareto optimal, efficient or non-dominated solution.

Given a set of p objective functions and m constraints, the multi-objective linear programming problem can be formulated in the decision space as:

$$z_i(x) \;=\; c^{(i)}x, \quad Ax \le b, \quad x \ge 0, \quad i = 1,2\ldots,p$$

Definition 2.1 The set defined by: $x \;=\; R = \{x \mid Ax \le b, \;\; x \ge 0\}$

$x \ge 0$ is called the set of feasible decision vectors or the feasible region in the decision space. For this study, X is assumed to be bounded and convex set. Furthermore, the convention of primal and dual problem is assumed for all objective functions.

Considering the formulation of MOLP problem in the decision space, X can be mapped into the set of feasible objective space vector F giving formulation in the objective space. Each decision variable $(x_j,\; \mathrm{j} = 1, 2, \;.\;.\;.\;, n)$ is solved algebraically in terms of objective function variables $(f_i\,,\;\; i = 1, 2, \;.\;.\;.\;, p)$. These expressions are then substituted for the decision variables in the defining equation of X. In set notation:

$$X \;\to\; F = \{f \mid g_r^t f \;\ge\; h_r\,,\;\; \forall\, r = 1,\; 2, \;.\;.\;.\;, m+n\,\}$$

where $g_r = p \times 1$ vector of objective space constraint coefficients for the r^{th} objective space constraint.

$f = p$ vector of objective function values for objectives f_i , $i = 1, 2, \;.\;.\;.\; p$

$h_r =$ right hand side element of the objective space constraint

Definition 2.2 The set $F = \{f \mid g_r^t f \;\ge\; h_r\,,\;\; \forall\, r = 1,\; 2, \;.\;.\;.\;\; m+n\}$ is called the feasible region in the objective space or the set of feasible objective vectors.

Definition 2.3 A solution $x^* \in X$ is said to be efficient solution for a given problem if no other $\hat{x} \in X$ exists such that: $z(\hat{x}) \;\le\; \mathrm{z}(x^*)$ for all $i = 1,2, \ldots, n$.

Efficient solution is also known as non-dominated solution or non-inferior solution.

In order words, A point $x^* \in X$ is called a Pareto optimal point, if there does not exist $x \in X$ such that $z(x) \le z(x^*)$. If x^* is a Pareto optimal, then $z(x^*)$ is called non-dominated or non-inferior point. Furthermore, a solution is called a weak Pareto optimal point if there does not exist $x \in X$ s.t. $z(x) < z(x^*)$. The set of all (weak) Pareto optimal points is called the (weak) Pareto set.

Definition 2.4 A feasible solution $\underline{x}^*$ to the multi-objective linear programming problem is said to be optimal if $c^k \underline{x} \;\le\; c^{\mathrm{k}} \underline{x}^*$ for all feasible $\underline{x}$ and all k.

The optimal compromise solution (or optimal to the MOLP) is an efficient solution that optimizes the decision making's preference function. It is a solution $\bar{x} = (\bar{x}_i) \in X$ which is preferred by the decision maker to all other solutions, taking into consideration all criteria contained in the multi-objective function. One property required of the optimal compromise is that it is non-dominated.

A feasible solution $\bar{x} = \{\,\bar{x}_i\} \in X$ is said to be a non-dominated solution of the multi-objective linear programming problem if there is no other feasible solution $x = \{x_i\} \in X$.

The Pareto optimal solutions are ones within the search space whose corresponding objective vector component cannot be improved simultaneously. These solutions are also known as non-inferior or efficient solutions.

Lemma 1. Any point in a feasible solution set Ω is a convex combination of the extreme points of Ω, i.e. if $x^1, x^2, \ldots, x^k$, are extreme point of Ω, then $x = \sum_{j=1}^{k} w_j x_j$, $\forall x \in \Omega$, *where* $\sum_{j=1}^{k} w_j = 1$.

Proof: This is true by convexity of Ω.□

Theorem 1. (Anstreicher (2011)). If $\underline{x}^k$, $k = 1, 2, \ldots, p$ are feasible points, then

$\underline{x} = \sum_{k=1}^{p} w_k \underline{x}^k$, $\sum_{k=1}^{p} w_k = 1$ is also a feasible point.

Proof: By definition the set of feasible points of a primal problem is given by F_p : $\{ x \in R^n \mid Ax \leq b, \ x \geq 0 \}$

if $\underline{x}^k \in \Omega$, $k = 1, 2, \ldots, p$ and Ω , the solution space are feasible points, it implies that $\underline{x}^k$ are the interior points of a convex polytope.

By convexity the convex combination $w_1 \underline{x}^1 + w_2 \underline{x}^2 + \ldots + w_p \underline{x}^p \in \Omega$, $\sum_{k=i}^{p} w_k = 1$

is also a feasible solution. Hence $\underline{x} = \sum_{k=1}^{p} w_k \underline{x}^k$ is also a feasible point, this complete the proof.

Theorem 2. (Effanga and Nsien (2019)). If $\underline{x}^k$, $k = 1, 2, \ldots, p$ are Pareto optimal solutions,

then $\underline{x} = \sum_{k=1}^{p} w_k \underline{x}^k$ is either Pareto optimal or inferior .

$\underline{x}$ is Pareto optimum if it is a boundary point, $\underline{x}$ is inferior if it is interior point.

Proof: Let $x^* \in X$ be a solution of a given primal problem, then x^* is a Pareto optimal for the primal problem. Let $\underline{x}^{*k} \in \Omega$, $k = 1, 2, \ldots, p$ and Ω is the solution space, also be Pareto optimal set, then by convexity, the convex combination $w^1 x_1 + w^2 x_2 + \ldots + w^p x_p \in \Omega$ is also a Pareto optimal set. By definition, the Pareto optimal is on the boundary of the polytope then $\underline{x} = \sum_{k=1}^{p} w \, \underline{x}^k$ is either Pareto optimum or inferior. If $\underline{x}$ is a boundary point, then $\underline{x}$ is Pareto optimal otherwise it is interior point and it is inferior. This complete the proof.

Efficient starting point

In any interior point algorithm, the starting point determined how best the optimal solution would be. Here we present a simple method of finding initial starting point and hence subsequent points. We solve each linear programming problem in the vector programming problem to obtain optimal solutions.

Then take as the initial point

$$\underline{x}^0 = \sum_{k=1}^{p} w_k \underline{x}^k, \quad \text{where} \quad \sum_{k=1}^{p} w_k = 1$$

After determining the weights, we obtain an initial efficient starting point x_{eff} as shown in figure 1. The initial efficient point is in the interior of Ω.To locate this point, we compute $\underline{x} = \sum_{k=1}^{p} w_k x^k$ which is the convex combination of the optimum solution point of z_k of the given linear programming problem.

Theorem 3. (Ye (1997)).Let $\underline{x}^k$, $k = 1,2, \ldots, p$ be the optimal solution of a given MOLP problem. Let W_k be the weights attached to z_k. Then the convex combination $\underline{x} = \sum_{k=1}^{p} w_k x^k$ is a feasible solution to MOLP problem.

Proof : $\underline{x}^k$, $k = 1,2, \ldots, p$ are extreme points of the feasible region of MOLP. Hence $\underline{x}$ is a feasible solution to the MOLP by lemma 1.

In any interior point algorithm, the starting point determined how best the optimal solution would be. Here we present a simple method of finding initial starting point and hence subsequent points. We solve each linear programming problem in the given multi-objective linear programming problem to obtain optimal solutions. After determining the weights w_i, we obtain an initial efficient starting point x_{eff} as shown in figure 1. The initial efficient point is in the interior of solution space. To locate this point, we compute $\underline{x} = \sum_{k=1}^{p} w_k \underline{x}^k$ which is the convex combination of the optimum solution point of z_k of the given linear programming problem. Then take as the initial point $\underline{x}^0 = \sum_{k=1}^{p} w_k \underline{x}^k$, where $\sum_{k=1}^{p} w_k = 1$

The optimal face where the optimal solution is located can be obtained by introducing the new constraints $c_j x_j \geq z_k^*$ to the given problem as shown in figure 1. The objective plane contract and demarcate the optimal face on the boundary of the constraints polytope as shown in Fig. 1

The reduced feasible region form a solution cone with the boundary of the constraint polytope. The new efficient point x_{neweff} is obtained by further reduction of the feasible region. This further reduces the optimal face and feasible region. The iteration is repeated until the current efficient point finally reach the boundary of the polytope as shown in fig. 1. When this happens, the termination criteria is met and the optimal compromise solution point is obtained.

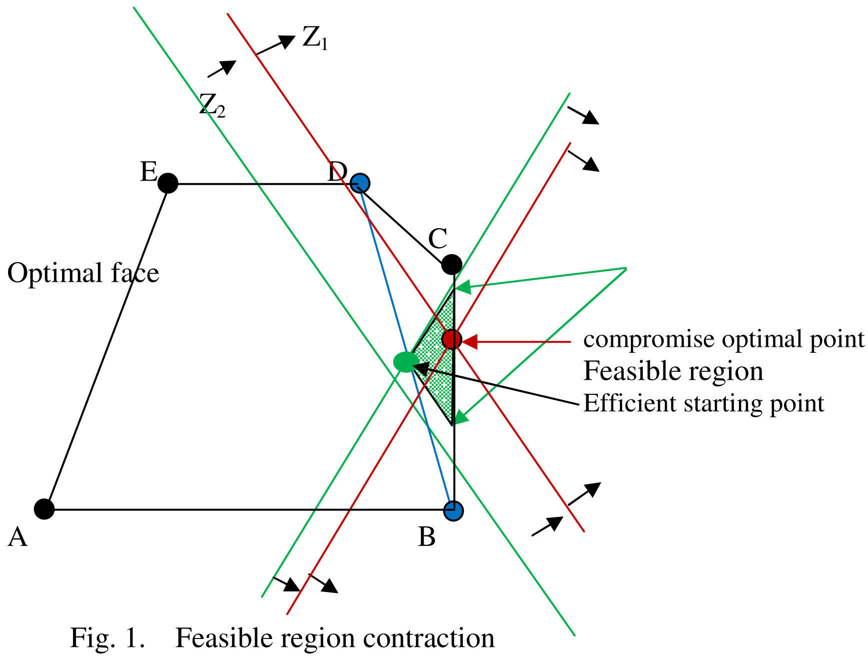

Fig. 1. Feasible region contraction

The Algorithm

Outline of "Feasible Region Contraction Interior-Point Algorithm (**FERCIPA)"** for obtaining optimal compromise solution of MOLP problem. Using the method for weights selection presented by Nsien (2015), and starting point in the technique for solving multi-objective linear programming problem, an interior-point algorithm for solving multi-objective linear programming problems was proposed and used to obtain the optimal compromise solution of a multi-objective linear programming problem as:

Step 0. Set r = 0. Solve each of the p linear programming problems to obtain p optimal solutions $\underline{x}^{k,r}$, $k = 1, 2, ..., p$ with objective function value $z_i^{k,r}$, $i, k = 1, 2, ..., p$, respectively.

Step 1. If $\underline{x}^{k,r} = x^*$, $\forall k = 1, 2, ..., p,$ then stop x^* is the optimal compromise solution to the multi-objective linear programming problem, otherwise go to step 2

Step 2. Compute the weights w_k^r and $\underline{x}^{*r} = \sum_{k=1}^{p} w_k x^k$

Step 3. If x^{*r} is on the boundary of the feasible region, then stop,
x^{*r} is the optimal compromise solution to multi-objective linear programming problem, otherwise , go to step 4.

Step 4. Compute $z^{*kr} = c^k x^{*r}$, $k = 1,2,....p$

In a maximization problem, the feasible solution region is being contracted by introducing the constraints $c^k \underline{x} \geq z^{*kr}$, $k = 1,2,..., p$

or $c^k \underline{x} \leq z^{*kr}$, $k = 1,2,..., p$ in the minimization problem.

Set $r = r+1$, Return to step 0.

Convergence of FERCIPA

We shall delve a little into this algorithm by presenting the fundamental insight about a property of boundedness of a feasible region where the interior- point method could be used to obtain solutions for MOLP problems. This property of boundedness of a feasible solution region for a system of linear equations is summarized in the following theorem. The convergence of FERCIPA is summarized in the following theorem.

Theorem 4. (Roos, *et al.* (1997))

The sequence of points $\{\underline{x}^r\}_{r=0}^{\infty}$ generated FERCIPA converged to a Pareto optimum solution $\underline{x}^*$.

Proof : The sequence $\{\underline{x}^r\}_{r=0}^{\infty}$ is monotonically increasing interior point with respect to the objective function values $\{\mathrm{Z}^{\mathrm{k}}\}_{r=0}^{\infty}$, i.e. $z^{k,r} > z^{k,r-1}$, $\forall k, r = 1,2, ...$ But since the feasible region is compact and bounded, the sequence $\{\underline{x}^r\}_{r=0}^{\infty}$ is converged to $\underline{x}^*$ with objective function value z^*, i.e. $z^{k,r} \to z^{k*}$ *as* $\underline{x}^{\mathrm{r}} \to \underline{x}^*$.

Consider the following intervals $I_1^1, I_1^2, . . . , I_1^n;\ I_2^1, I_2^2, . . . , I_2^n;\ . \ . \ , I_k^1, I_k^2, . . . \ , I_k^n$

with the corresponding objective function values:

$z_1^1, z_1^2, . . . , z_1^n;\ z_2^1, z_2^2, . . . , z_2^n;\ . \ . \ , z_k^1, z_k^2, . . . \ , z_k^n$ and z^*

the optimal compromise objective function values.

Let $I_1^1 = [z_1^1, z^*],\ I_1^2 = [z_1^2, z^*],\ . . . ,I_1^n = [\ z_1^n, z^*]$

$I_2^1 = [z_2^1, z^*],\ I_2^2 = [z_2^2, z^*],\ . . . ,I_2^n = [\ z_2^n, z^*]$

$I_k^1 = [z_k^1, z^*],\ I_k^2 = [z_k^2, z^*],\ . . . ,I_k^n = [\ z_k^n, z^*]$

Then it is observed that for $k = 1,2,.....;\ n = 1,2,....;$ $I_k^n = [z_k^n, z^*]$ is a decreasing sequence of intervals as $k \to \infty$ and $I_k^n \to 0$.

Since $I_2 \subseteq I_1, I_3 \subseteq I_2,\ .\ .\ .\ I_k \subseteq I_{k-1}$. It implies that $I_{k+1} \subseteq I_k$. Hence the convergence of the algorithm. This completes the proof. ■

5. **Numerical Examples**

Let us consider the following example of multi-objective linear programming problem:

$$\begin{aligned} MOLP\ 1\quad & Maximize\ z_1 = x_1 + 2x_2 \\ & Maximize\ z_2 = x_1 + x_2 \\ & s.t. \\ & x_1 + 3x_2 \le 26 \\ & 2x_1 + x_2 \le 17 \\ & x_1 \le 7 \\ & x_2 \le 8 \\ & \underline{x} \ge 0 \end{aligned}$$

$$\begin{aligned} MOLP\ 2\quad & Maximize\quad z_1 = x_1 \\ & Maximize\quad z_2 = x_2 \\ & s.t.\quad x_1 + x_2 + x_3 \le 10 \\ & 2x_1 - x_2 + x_4 \le 14 \\ & 2x_1 + x_2 - x_5 \le 2 \\ & -2x_1 + x_2 + x_6 \le 7 \\ & \underline{x} \ge 0 \end{aligned}$$

$$\begin{aligned} \text{MOLP 3}\quad & \max z_1 = 2x_1 + 3x_2 \\ & \max z_2 = 3x_1 - x_2 \\ & \max z_3 = 4x_1 + 3x_2 \\ & s.t. \\ & x_1 + x_2 \le 20 \\ & 2x_1 + x_2 \le 30 \\ & x_2 \le 12 \\ & x_i \ge 0 \end{aligned}$$

Algorithm

Step 0 : solving MOLP 3 problem, the following extreme points and their corresponding objective function values were obtained:

A(0,0), $z_1 = 0$, $z_2 = 0$, $z_3 = 0$; B(15,0), $z_1 = 30$, $z_2 = 45$, $z_3 = 60$
C(10,10), $z_1 = 50$, $z_2 = 20$, $z_3 = 70$; D(8,12), $z_1 = 52$, $z_2 = 12$, $z_3 = 65$
E(0,12), $z_1 = 36$, $z_2 = -12$, $z_3 = 36$

$z_1,\ x^{*1} = (x_1^1, x_2^1) = (8,12),\quad z_1^1 = 52,\ z_2^1 = 12,\quad z_3^1 = 68$

$z_2,\ x^{*2} = (x_1^2, x_2^2) = (15,0),\quad z_1^2 = 30,\ z_2^2 = 45,\quad z_3^2 = 60$

$z_3,\ x^{*3} = (x_1^3, x_2^3) = (10,10),\quad z_1^3 = 50,\ z_2^3 = 20,\quad z_3^3 = 70$

Step 1: if $\underline{x}^{k,r} = x^*$, $\forall k = 1,2,3.$ then stop x^* is optimal compromise solution, otherwise go to step 2.

Since $\underline{x}^{k,r} \ne x^*$, i.e. $x^{*1} \ne x^{*2} \ne x^{*3}$, we go to step 2

Step 2: compute the weights w_k and $\underline{x}^{*r}$

$$w_k = \frac{|c_{1k}|}{|c_{11}| + |c_{12}| + |c_{13}|}, \qquad k = 1,2,3$$

$$\begin{aligned} c_{11} &= (z_2^2 - z_2^1)(z_3^3 - z_3^1) - (z_3^2 - z_3^1)(z_2^3 - z_2^1) \\ &= (45 - 12)(70 - 68) - (60 - 68)(20 - 12) \\ &= (33)(2) - (-8)(8) \\ &= 130 \end{aligned}$$

$$\begin{aligned} c_{12} &= (z_3^3 - z_3^1)(z_1^2 - z_1^1) - (z_1^3 - z_1^1)(z_3^2 - z_3^1) \\ &= (70 - 68)(30 - 52) - (50 - 52)(60 - 68) \\ &= (2)(-22) - (-2)(-8) \\ &= -60 \end{aligned}$$

$$\begin{aligned} c_{13} &= (z_1^2 - z_1^1)(z_2^3 - z_2^1) - (z_2^2 - z_2^1)(z_1^3 - z_1^1) \\ &= (30 - 52)(20 - 12) - (45 - 12)(50 - 52) \\ &= (-22)(8) - (33)(-2) \\ &= -110 \end{aligned}$$

$$|c_{11}| + |c_{12}| + |c_{13}| = 130 + 60 + 110 = 300$$

$$w_1 = \frac{|c_{11}|}{|c_{11}| + |c_{12}| + |c_{13}|} = \frac{130}{300} = 0.43$$

$$w_2 = \frac{|c_{12}|}{|c_{11}| + |c_{12}| + |c_{13}|} = \frac{60}{300} = 0.20$$

$$w_3 = \frac{|c_{13}|}{|c_{11}| + |c_{12}| + |c_{13}|} = \frac{110}{300} = 0.37$$

$$w_1 + w_2 + w_3 = 0.43 + 0.20 + 0.37 = 1$$

Compute $\underline{x}^{*r}$, i.e.

$$\begin{aligned} w_1 x_1^1 + w_2 x_1^2 + w_3 x_1^3 &= 0.43(8) + 0.20(15) + 0.37(10) \\ &= 3.44 + 3 + 3.7 \\ &= 10.14 \end{aligned}$$

$$\begin{aligned} w_1 x_2^1 + w_2 x_2^2 + w_3 x_2^3 &= 0.43(12) + 0.20(0) + 0.37(10) \\ &= 5.16 + 0 + 3.7 \\ &= 8.86 \end{aligned}$$

$x^* = (10.14,\ 8.86)$ which is the efficient starting point.

Step 3: The point $x^* = (10.14, 8.86)$ is not on the boundary of the polytope, we go to step 4.

Step 4: Compute $z^{kr} = c^k x^{*r}$, $k = 1,2,3$.

$$\begin{aligned} z^{1r} &= 2(10.14) + 3(8.86) \\ &= 20.28 + 26.56 \\ &= 46.86 \end{aligned}$$

$$
\begin{aligned}
z^{2r} &= 3(10.14) - (8.86) \\
&= 30.42 - 26.56 \\
&= 21.56 \\
z^{3r} &= 4(10.14) + 3(8.86) \\
&= 40.56 + 26.58 \\
&= 67.14
\end{aligned}
$$

Contract the feasible region by introducing the constraints $c^k \underline{x} \geq z^{*kr}$ into the given problem, we obtained:

$$
\begin{aligned}
&\max z_1 = 2x_1 + 3x_2 \\
&\max z_2 = 3x_1 - x_2 \\
&\max z_3 = 4x_1 + 3x_2 \\
&s.t. \\
&\qquad x_1 + x_2 \leq 20 \\
&\qquad 2x_1 + x_2 \leq 30 \\
&\qquad x_2 \leq 12 \\
&\qquad 2x_1 + 3x_2 \geq 46.86 \\
&\qquad 3x_1 - x_2 \geq 21.56 \\
&\qquad 4x_1 + 3x_2 \geq 67.14 \\
&\qquad x_i \geq 0
\end{aligned}
$$

Introducing the new constraints, we obtained the new extreme points of the contracted feasible region.

F(10.14, 8.86), $z_1 = 46.86$, $z_2 = 21.56$, $z_3 = 67.14$
G(10.79, 8.43), $z_1 = 46.85$, $z_2 = 23.94$, $z_3 = 68.45$
H(10.31, 9.38), $z_1 = 48.76$, $z_2 = 21,55$, $z_3 = 69.38$

Step 3: The point $x^* = (10.31,\ 9.38)$ is on the boundary of the constraints polytope. Thus the point x^* is the optimal compromise solution for the given MOLP 3 problem.

$x^* = (10.31,\ 9.38)$, $z_1 = 48.76$, $z_2 = 21.55$, $z_3 = 69.38$

The results obtained by Feasible Region Contraction Interior-Point Algorithm (FERCIPA) will be compared with that obtained by Affine-Scaling Primal Algorithm (ASPA).

Table 1. Summary of results using FERCIPA and ASPA on MOLP problems

Method	Problem	Decision Variable	Objective function values			Number of iteration	Time in (sec)
			Z1	Z2	Z3		
ASPA	MOLP 1	(5.75, 5.5)	16	11.25		9	
FERCPA	MOLP 1	(5, 7)	19	121	0.125		
ASPA	MOLP 2	(4.49, 5.51, 0, 0, 12.5, 0)	4.49	5.51		11	-
FERCIPA	MOLP 2	(4.5, 5.5,0,0, 12.5, 0)	4.5	5.5 1	0.125		
FERCIPA	MOLP 3	10.31, 9.39	48.77	21.55	69.38	2	0.125

CONCLUSION

In this paper we have developed a feasible region contraction interior-point algorithm (FERCIPA) that is a viable multi-objective linear optimization technology for both single and multi-objectives linear programming problems. The interior-point algorithm will give an optimal compromise solution with accuracy to the multi-objective linear programming problems provided the issues related to computational complexity are properly addressed through problem formulation and through efficient implementation techniques. The comparison of the results in table 1 shows that FERCIPA is more efficient than ASPA because of the systematic method of determining weights which is used to determine the efficient starting point within the constraints polytope. This starting point is closer to the optimal face than the one obtained by approximation centrality used by ASPA. This is seen in the objective function values and the number of iterations needed to reach the optimal compromise solution of a particular problem. Also, this shows the fast convergence of our algorithm to the optimum.

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