Relative Efficiency of Split-plot Design (SPD) to Randomized Complete Block Design (RCBD)

Oladugba, A. V+, Onuoha, Desmond O*, Opara Pius N.++

+Department of Statistics, University of Nigeria, Nsukka,
*Dept of Maths/Statistics, Fed. Polytechnic Nekede, Owerri, 08035442403,
++Datafield Logistics Services, Port Harcourt, Rivers State.

Abstract

The relative efficiency of split-plot design (SPD) to randomized complete block design (RCBD) was computed using their error variance, sensitivity analysis and design planning. The result of this work showed that conducting an experiment using split-plot (SPD) without replication is more efficient to randomized complete block design (RCBD) based on comparison of their error variances, sensitivity analysis and design planning consideration.

Key words: Split-plot Design, Randomized Complete Block Design, Error variance, Sensitivity Analysis and Design planning.

Introduction

In experimental design, the Relative Efficiency (RE)of design say A to another design say B denoted as RE(A:B) is defined in terms of the number of replicates of design B required to achieve the same result as one replicate of design A. In view of this, the relative efficiency of split-plot design (SPD) to randomized complete block design (RCBD) denoted as RE (SPD:RCBD) is the number of replicates of RCBD required to achieve the same result as one replicate of SPD. Relative efficiency can be expressed in terms of percentage by multiplying it by 100. If RE (SPD:RCBD) > 100%, SPD is said to be more efficient to RCBD and if RE(SPD:RCBD) ≤ 100% SPD is said to be less efficient to RCBD.

The relative efficiency of two designs is mostly measured in terms of comparing their error variances and the design with the smallest variance is said to be more efficient than the other. This measure of relative efficiency does not put into consideration the probability of obtaining significant difference or detecting significant difference if they exist between the treatments. RCBD is said to be more efficient to complete randomized design (CRD) based on the comparison of their error variance since the error variance of RCBD is always smaller than that of complete randomized design (CRD). There is a decrease in the error degree of freedom of RCBD compare to CRD and a decrease in the error degree of freedom leads to an increase in the tabulated value thereby reducing the probability of obtaining a significant result since the decision rule is always to reject the null hypothesis if F-calculated is greater than F-tabulated. Based on this assessment which is sensitivity analysis, CRD is said to be more efficient than RCBD; in other words, the sensitivity of RCBD is decreased. From above, it can be clearly seen that the relative efficiency of any two designs cannot be best judged by considering the ratio of their error variances only because it does not provide complete information about the designs. A better approach may be to consider the sensitivity of the two designs which is the probability of obtaining a significant result with respect to treatment comparisons and the design planning consideration to get a better comparison.

In design planning consideration, it is required to plan the sample sizes of the design in such a way that it enhances the sensitivity of the design. The sample sizes should be large enough to detect important differences with high probability. At the same time, the sample sizes should not be so large that the cost of the study becomes excessive and that unimportant difference becomes statistically significant with high probability [5]. The sample sizes of two designs can be used as a measure of relative efficiency of the two designs. The design that requires the smallest sample sizes to achieve a particular power is said to be more efficient than the other.

However, Gwowen and Show-Li [3] proposed three alternative criteria related to the sensitivity issue and design planning consideration. The proposed relative measures employ the p-value, scheffe confidence interval estimation and the power of both designs. The p-value approach evaluates the relative efficiency of RCDD to CRD in terms of the p-value for the expected value of the F-statistic, the estimation approach compares the expected squared half width of scheffe confidence Interval for both designs and lastly the power approach compares the power of both designs in terms of detecting treatment effects. They concluded that the proposed efficiency measures provide feasible solutions for the evaluation of efficiency regarding sensitivity in the context of RCBD relative to CRD, and also that the proposed three efficiency measures could be extended for use in more general block designs and models.

Research Methodology:

The following methods will be adopted in computing the relative efficiency of SPD to RCBD:

Comparison Of Error Variance*:*

The ratio of the error variances of SPD and RCBD will be use in obtaining the relative efficiency of SPD to RCBD. A correction factor will be use in a case where the error degrees of freedom for both designs are less than 20.

$$REV(SPD:RCBD)_{(S-P)} = \frac{\hat{\sigma}^2_{RCBD}}{\hat{\sigma}^2_{SPD(S-P)}} = \frac{MSE_{RCBD}}{MSE_{SPD(S-P)}}$$

$$REV(SPD:RCBD)_{(W-P)} = \frac{\hat{\sigma}^2_{RCBD}}{\hat{\sigma}^2_{SPD(W-P)}} = \frac{MSE_{RCBD}}{MSE_{SPD(W-P)}}$$

where,

$REV(SPD:RCBD)_{(S-P)}$ is the relative efficiency of SPD to RCBD with split-plot comparison based on comparison of their error variance.

$REV(SPD:RCBD)_{(W-P)}$ is the relative efficiency of SPD to RCBD with whole-plot comparison based on comparison of their error variance.

$\hat{\sigma}^2_{RCBD}$ is the error variance of RCBD;

$\hat{\sigma}^2_{SPD(S-P)}$ is the error variance of SPD with split-plot comparison;

$\hat{\sigma}^2_{SPD(W-P)}$ is the error variance of SPD with whole-plot comparison;

$MSE_{SPD(S-P)}$ is an estimate of the error variance of SPD with split-plot comparison;

$MSE_{SPD(W-P)}$ is an estimate of the error variance of SPD with whole-plot comparison;

(i) Split-plot comparison

$$MSE_{RCBD} = \frac{a(bn-1)MSE_{SPD(S-P)} + (a-1)MSE_{SPD(W-P)}}{abn-1}$$

(ii) Whole-plot comparison

$$MSE_{RCBD} = \frac{a(bn-1)MSE_{SPD(W-P)} + (a-1)MSE_{SPD(S-P)}}{abn-1}$$

The correction factor is given as:

$$m = \frac{(F_1+1)(F_2+3)}{(F_1+3)(F_2+1)}$$

where ,m is the correction factor; F_1 = d.f of mean square error for SPD; F_2 = d.f of mean square error for RCBD.

:

MSE_{RCBD} is an estimate of the error variance of RCBD.

Since RCBD was not performed, an estimate of error variance of RCBD will be obtained as:

Sensitivity Analysis:

The three criteria related to sensitivity analysis proposed by Gwowen and Show-Li [3] will be used in computing the relative efficiency of SPD to RCBD. The proposed methods are: estimation approach, p-value approach and power approach.

Estimation Approach:

The relative efficiency of SPD to RCBD denoted as RE (SPD: RCBD) using estimation approach is obtained in terms of the expected half width of Scheffe confidence interval which is defined as:

$$RE(SPD:RCBD)_{(S-P)} = \frac{S^2_{RCBD}}{S^2_{SPD(S-P)}} = \frac{(m-1)F[F_m, F_2, \alpha]\frac{\sigma^2_{RCBD}}{b}\sum_{i=1}^{m} C_i^2}{(b-1)F[F_b, F_1, \alpha]\frac{\sigma^2_{RCBD}}{b}\sum_{i=1}^{m} C_i^2}$$

$$= \left(\frac{(m-1)F[F_m, F_2, \alpha]}{(b-1)F[F_b, F_1, \alpha]}\right)\left(REV(SPD:RCBD)_{(S-P)}\right)$$

$i = 1, 2, \ldots, m$

$$RE(SPD:RCBD)_{(W-P)} = \frac{S^2_{RCBD}}{S^2_{SPD(W-P)}} = \frac{(m-1)F\left[F_m, F_2, \alpha\right]\frac{\sigma^2_{RCBD}}{b}\sum_{i=1}^{m} C_i^2}{(b-1)F\left[F_b, F_1, \alpha\right]\frac{\sigma^2_{RCBD}}{b}\sum_{i=1}^{m} C_i^2}$$

$$= \left(\frac{(m-1)F\left[F_m, F_2, \alpha\right]}{(b-1)F\left[F_b, F_1, \alpha\right]}\right)\left(REV(SPD:RCBD)_{(W-P)}\right)$$

$i = 1, 2, ..., m$

where,

$REV(SPD:RCBD)_{(S-P)}$ is the relative efficiency of SPD to RCBD based on comparison of their error variance with split-plot comparison;

$REV(SPD:RCBD)_{(W-P)}$ is the relative efficiency of SPD to RCBD based on comparison of their error variance with whole-plot comparison;

F_1= d.f for error of SPD; F_2 = d.f for error of RCBD;

C_i = possible contrasts among the whole-plot treatment effects;

F_m = d.f for the whole-plot treatment; F_b= d.f for the split-plot treatment

α= level of significance.

P-Value Approach

The relative efficiency of SPD to RCBD denoted as RE (SPD: RCBD) using p-value approach is obtained in terms of the ratio of the p-value of both SPD and RCBD and it is given as:

$$RE(SPD:RCBD)_{(S-P)} = \frac{P_{RCBD}}{P_{SPD(S-P)}}$$

$$RE(SPD:RCBD)_{(W-P)} = \frac{P_{RCBD}}{P_{SPD(W-P)}}$$

where ,

P_{RCBD} is the p-value associated with the F-statistic of RCBD;

P_{SPD} is the p-value associated with the F-statistic of SPD.

when the F-statistic of SPD (F_{SPD}) is available, an estimate is given as:

$$RE(SPD:RCBD) = \frac{\hat{P}_{RCBD}}{\hat{P}_{SPD}}$$

$$where\ \hat{P}_{SPD} = P\left[F_{F_1}^{F_b} > F_{SPD}\right]$$

$$\hat{P}_{RCBD} = P\left[\hat{P}_{F_2}^{F_m} > F_{RCBD}\right]$$

Since RCBD was not conducted, the F-statistic of RCBD is obtained as:

$$F_{RCBD} = \frac{F_{SPD} + REV(SPD:RCBD) - 1}{REV(SPD:RCBD)}$$

$$REV(SPD:RCBD) = \frac{MSE_{RCBD}}{MSE_{SPD}}$$

F_b = d.f for the split-plot treatment;
F_m = d.f for the whole-plot treatment;
F_1 = d.f for error of SPD;
F_2 =d.f for the error of RCBD;
F_{SPD} = F-statistic for SPD;
F_{RCBD} = estimated F-statistic for RCBD.

Power Approach

The relative efficiency of SPD to RCBD denoted as RE(SPD:RCBD) using power approach is obtained in terms of the power of both designs and it is given as:

$$RE(SPD:RCBD) = \frac{P_{SPD}(\delta_{SPD})}{P_{RCBD}(\delta_{RCBD})}$$

$$where\ P_{SPD}(\delta_{SPD}) = P\left\{F_{F_1}^{F_b}(\delta_{SPD}) > F[F_b, F_1, \alpha]\right\} is\ the\ associated\ power\ of\ F_{SPD}$$

$$P_{RCBD}(\delta_{RCBD}) = P\left\{F_{F_2}^{F_m}(\delta_{RCBD}) > F[F_m, F_2, \alpha]\right\} is\ the\ associated\ power\ of\ F_{RCBD}$$

F_b = d.f for the split-plot treatment;
F_m = d.f for the whole-plot treatment;
F_1 = d.f for error of SPD;
F_2 = d.f for error of RCBD;
F_{SPD} = F-statistic for SPD;
F_{RCBD} = F-statistic for RCBD;
δ_{SPD} *and* δ_{RCBD} *are the non − centrality parameter.*

An estimate of RE (SPD: RCBD) using power approach is given as:

$$RE(SPD:RCBD) = \frac{P_{SPD}(\hat{\delta}_{SPD})}{_{RCBD}(\hat{\delta}_{RCBD})}$$

$$where\ \hat{\delta}_{SPD} = \frac{F_b[(F_1-2)F_{SPD}]}{2(F_1-1)}$$

$$\hat{\delta}_{RCBD} = \frac{\hat{\delta}_{SPD}}{REV(SPD:RCBD)}$$

The power for both designs will be obtain using Table (10) by specifying the non-centrality parameter, the degree of freedom for treatment and error and the level of significance (α).

Model and Definition of Variables*:*

$$Y_{ijkl} = \mu + R_i + A_j + (RA)_{ij} + B_k + (AB)_{jk} + e_{ijkl}; i=1,2,3(r), j=1,2,3(a), k=1,2,3,4(b),$$
$$l=1,2,3,4(n)$$

where,
Y_{ijkl} is the observed response from the l^{th} replication of the i^{th} level of the land preparation method , j^{th} level of the phosphorus and the k^{th} level of the poultry manure;
μ is the universal constant or overall mean;
R_i is the effect of the i^{th} level of the land preparation method (block);
A_j is the effect of the j^{th} level of the phosphorus (whole-plot treatments);
$(RA)_{ij}$ is the whole-plot error;

B_k is the effect of the k^{th} level of the poultry manure (split-plot treatments);

$(AB)_{jk}$ is the interaction effect of the j^{th} level of the phosphorus (whole-plot treatments) and k^{th} level of the poultry manure (split-plot treatments);

e_{ijkl} is the random error component associated with the $(ijkl)^{th}$ observation (split-plot error);

Table B: ANOVA TABLE with Replication				
Source	$D.F$	SS	MS	F-ratio
Block (A_i)	2	3.98	1.99	0.2392
Whole-plot (B_j)	2	3.2429	1.62145	0.1949
Block/whole-plot Interaction $(AB)_{ij}$	4	33.2821	8.3205	_
Split-plot (C_k)	3	24.0792	8.0264	0.4894
Whole-plot /split-plot Interaction $(BC)_{jk}$	6	26.3854	4.3976	0.2682
Error (e_{ijkl})	126	2066.2979	16.3992	_
Total	143	2157.2675	-	_

Table C - Summary		
Criteria	Split-plot comparison	Whole-plot comparison
Error variance	72%	79%
Sensitivity(estimation approach)	60%	65%
Sensitivity(p-value approach)	108%	85%
Sensitivity(power approach)	116%	116%

Analysis of Variance Table

Summary

In computing the relative efficiency of SPD to RCBD, we are interested in the number of replicates required by RCBD to achieve the same result as one replicate

(100%) of SPD. Relative efficiency can be expressed in terms of percentage by multiplying it by 100, if RE (SPD:RCBD) is greater than 100, it implies that SPD is more efficient or better than RCBD and there is a gain in using SPD instead of RCBD. If RE (SPD:RCBD) is less than or equal to 100, it implies that SPD is less efficient to or not better than RCBD and the experiment could have been performed using RCBD. The summary of the statistical analysis of this experiment is given in the table below:

Conclusion

From the summary table above, the conclusion of this study is given as follows:

1. SPD (with split-plot comparison) is less efficient to RCBD based on the comparison of their error variances, sensitivity analysis using estimation approach but more efficient to RCBD based on sensitivity analysis using p-value and power approach.
2. SPD (with whole-plot comparison) is less efficient to RCBD based on comparison of their error variances and sensitivity analysis based on estimation and p-value approach but more efficient to RCBD based on sensitivity analysis using power approach.

Recommendation

From the summary of analysis above, it can be seen that SPD (with split-plot comparison) is less efficient to RCBD based on comparison of their error variances, sensitivity analysis using estimation approach but SPD is more efficient to RCBD based on sensitivity analysis using p-value and power and power approach. It can therefore be recommended that for a maximum plant height of African yam bean, the experiment should be conducted using SPD by combining the levels of the poultry manure (split-plot treatments) and phosphorus application (whole-plot treatments) instead of RCBD which is one at a time experiment.

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