A Comparative Analysis of Fuzzy Inference Engines in Context of Profitability Control

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Abstract

Fuzzy Inference engine is an important part of reasoning systems capable of extracting correct conclusions from approximate data. Among the many different types of inference engine that are commonly used are, product inference engine, Mamdani minimum inference engine, Lukasiewicz, Zadeh, Dienes-Rescher inference engines and root sum square inference engine. Fuzzy inference engine has found successful applications in a wide variety of fields, such as automatic control, data classification, decision analysis, expert engines, time series prediction, robotics, pattern recognition, etc. This paper presents a comparative analysis of three fuzzy inference engines, max-product, max-min and root sum in fuzzy controllers using profitability control data. The presented results shows that RSS inference engine gives largest output membership function, while the product inference engine gives the smallest output membership function in this case; minimum inference engine is in between. This suggests that root sum square inference engine is one of the most promising strategies in profitability control.

Keywords: Max-Product Inference, Max-Min Inference, Root Sum Square Inference, Soft Computing, Membership Function, Profitability Control.

Introduction

Soft Computing is an emerging approach to computing which parallels the remarkable ability of the human mind to reason and learn in an environment of uncertainty and imprecision [1]. Fuzzy Inference engine is an important part of this reasoning system. The fuzzy inference engine (FIE) is a popular computing framework based on the concepts of fuzzy set theory, fuzzy if-then rules and fuzzy reasoning as shown in Figure 1. It has found successful applications in a wide variety of fields, such as automatic control, data classification, decision analysis, expert engines, time series prediction, robotics, and pattern recognition. Because of its multidisciplinary nature, the fuzzy inference engine is known by numerous other names, such as fuzzy-rule-based engine, fuzzy expert engine, fuzzy model [2], fuzzy associative memory [3], fuzzy logic controller [4], and simply (and ambiguously) fuzzy engine. The inference engine of the fuzzy system evaluates the different rules in the knowledge-base. The activation degree of each rule is calculated from the activation degree of its antecedents and according to the interpretation of the different connectives in use. From this point, the output of each rule is calculated applying the activation degree to the consequent by means of the implication function [5].

In general, both the inputs and outputs of a fuzzy inference engine are fuzzy variables; μA'(x) and μB'(y). Once membership functions have been defined for input and output variables, a control rule base can be developed to relate the output actions of the controller to the observed inputs. This phase is known as the inference, or rule definition portion, of fuzzy logic. Among the many different types of inference engine that are commonly used are, product inference engine, minimum inference engine and root sum square inference engine .

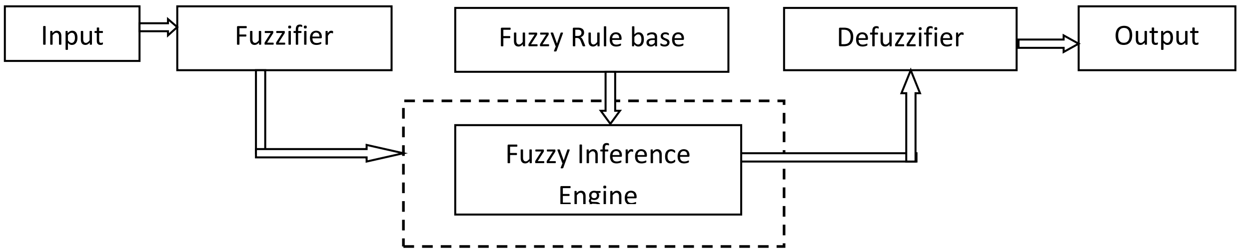

Fig. 1: Fuzzy Inference

In a fuzzy inference engine, fuzzy logic principles are used to combine the fuzzy IFTHEN rules in the fuzzy rule base into a mapping from a fuzzy set A' in *U* to a fuzzy set B' in V. Fuzzy IF-THEN rule is interpreted as a fuzzy relation in the input-output product space *U* x V, and a of implications are proposed by [6] that specify the fuzzy relation. If the fuzzy rule base consists of only a single rule, then the generalized modus ponens specifies the mapping from fuzzy set A' in *U* to fuzzy set B' in V. Because any practical fuzzy rule base constitutes more than one rule, there are two ways to infer with a set of rules: composition based inference and individual-rule based inference. In composition based inference, all rules in the fuzzy rule base are combined into a single fuzzy relation in *U* x V, which is then viewed as a single fuzzy IF-THEN rule. Composition based inference have two combinations: Mamdani and Gödel combinations. These models are shown in (1) and (2) respectively;

$$\mu Q_M(x, y) = \mu_{Ru}(1)(x, y) + \cdots + \mu_{Ru}(M)(x, y) \quad (1)$$

$$\mu Q_G(x, y) = \mu_{Ru}(1)(x, y) * \ldots * \mu_{Ru}(M)(x, y) \quad (2)$$

In individual-rule based inference, each rule in the fuzzy rule base determines an output fuzzy set and the output of the whole fuzzy inference engine is the combination of the M individual fuzzy sets. The combination can be taken either by union (3) or by intersection (4).

$$\mu B'(y) = \mu B'_1(y) + \cdots + \mu B'_M(y) \quad (3)$$

$$\mu B'(y) = \mu B'_1(y) * \ldots * \mu B'_M(y) \quad (4)$$

There are also fuzzy inference engines like; Dienes-Rescher implication, Lukasiewicz implication, Zadeh implication, Gödel implication, or Mamdani implications, and different operations for the t-norms and s-norms in their various formulas.

In [6], product inference engine uses; (I) individual rule based inference with union combination, (ii) Mamdani's product implication, and (iii) algebraic product for all the t-norm operators and max for all the s-norm operators. If the fuzzy set A' is a fuzzy singleton, that is, if (5),

$$\mu A^{'}(x) = \begin{cases} 1 \; if \; x = x^{*} \\ 0 \; otherwise \end{cases} \tag{5}$$

where x* is some point in U, then the product inference engine is simplified and obtained as (6).

$$\mu B^{'}(y) = max_{l=1}^{M}[\prod_{i=1}^{n} \mu A_i^l(x_i^*) \mu B^{'}(y) \tag{6}$$

That is, given fuzzy set *A′* in *U,* the product inference engine gives the fuzzy set B' in V. The MAX-DOT or MAX-PRODUCT method scales each member function to fit under its respective peak value and takes the horizontal coordinate of the "fuzzy" centroid of the composite area under the function(s) as the output. Essentially, the member function(s) are shrunk so that their peak equals the magnitude of their respective function ("negative", "zero", and "positive"). This method combines the influence of all active rules and produces a smooth, continuous output.

Minimum Inference Engine uses: (i) individual-rule based inference with union combination, (ii) Mamdani's minimum implication, and (iii) min for all the t-norm operators and max for all the s-norm operators. The minimum inference engine is simplified and obtained as (7).

$$\mu B^{'}(y) = max_{l=1}^{M}[min(\mu A_1^l(x_1^*), \ldots, \mu A_n^l(x_n^*), \mu B^l(y))] \tag{7}$$

That is, given fuzzy set *A′* in U, the minimum inference engine gives the fuzzy set B' in V
The MAX-MIN inference technique tests the magnitudes of each rule and selects the highest one. The horizontal coordinate of the "fuzzy centroid" of the area under that function is taken as the output. This method does not combine the effects of all applicable rules but does produce a continuous output function and is easy to implement. The product inference engine and the minimum inference engine are the most commonly used fuzzy inference engines in fuzzy systems and fuzzy control. The main advantage of them is their computational simplicity; this is especially true for the product inference engine. Also, they are intuitively appealing for many practical problems, especially for fuzzy control. One disadvantage of the product and minimum inference engines is that if at some x∈U the $\mu A_i^1(x_i)'s$ are very small, then the $\mu B^{'}(y)$ will be very small and this may cause problems in implementation. Lukasiewicz, Zadeh and Dienes-Rescher fuzzy inference engines overcome the disadvantage associated with product and Mamdani’s fuzzy inference engines.

In Lukasiewicz inference engine, we use: (i) individual-rule based inference with intersection combination, (ii) Lukasiewicz implication, and (iii) min for all the t-norm operators. Wxe obtain as (8).

$$\mu B^{'}(y) = min_{l=1}^{M}\left\{sup_{X \in U} \min\left[\mu A^{'}(X), \mathbf{1} - min_{i=1}^{n}\left(\mu A_i^l(x_i)\right) + \mu B^l(y)\right]\right\} \tag{8}$$

That is, for given fuzzy set A' in U, the Lukasiewicz inference engine gives the fuzzy set B' in V.

In Zadeh inference engine uses: (i) individual rule based inference with intersection combination, (ii) Zadeh implication, and (iii) min for all the t-norm operators to obtain (9).

$$\mu B^{'}(y) = min_{l=1}^{M}\{sup_{X \in U} \min\left[\mu A^{'}(X), \max\left(\min\left(\mu A_1^l(x_1), \dots, \mu A^{'}(x_n), \mu B^l(y)\right), 1 - min_{i=1}^{n}\left(\mu A_i^l(x_i)\right)\right)\right]\} \quad (9)$$

In Dienes-Rescher inference engine, we use the same operations as in the Zadeh inference engine, except Zadeh implication is replaced with the Dienes-Rescher implication and presented in (10).

$$\mu B^{'}(y) = min_{l=1}^{M}\{sup_{X \in U} \min\left[\mu A^{'}(X), \max\left(1 - \min_{i=1}^{n}\left(\mu A_i^l(x_i), \mu B^l(y)\right)\right)\right] \quad (10)$$

In [6], if A' is a fuzzy singleton, then the Lukasiewicz, Zadeh and Dienes-Rescher inference engines are simplified to (11) – (13) respectively

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$$\mu B^{'}(y) min_{l=1}^{M}[1, 1 - min_{i=1}^{n}\left(\mu A_i^l(x_i^*)\right) + \mu B^l(y)] \quad (11)$$

$$\mu B^{'}(y) = min_{l=1}^{M}\{\max\left[\min\left(\mu A_1^l(x_i^*), \dots, \mu A_n^l(x_n^*), \mu B^l(y)\right), 1 - \min_{i=1}^{n}\left(\mu A_i^l(x_i^*)\right)\right]\} \quad (12)$$

$$\mu B^{'}(y) = min_{l=1}^{M}\{\max\left[1 - min_{i=1}^{n}\left(\mu A_i^l(x_i^*)\right), \mu B^{'}(y)\right]\} \quad (13)$$

The ROOT-SUM-SQUARE (RSS) is often called RMS or Root Mean Squared. RMS is the incorrect term because we are adding the tolerances, not averaging the tolerances. In Fuzzy Inference Systems, RSS technique combines the effects of all applicable rules, scales the functions at their respective magnitudes, and computes the "fuzzy" centroid of the composite area. This method is more complicated mathematically than other methods. RSS is a common technique that is used mostly because it seems to give the best weighted influence to all firing rules. RSS analysis has the following form

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$$\text{RSS} = \sqrt{\sum R^2} = \sqrt{(R_1^2 + R_2^2 + R_3^2 +, \dots, + R_n^2)} \quad (14)$$

Where $R_1, R_2, R_3 \dots R_n$ are strength values of different rules which share the same. RSS calculation comes from the relationship for total assembly standard deviation as (15).

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$$\sigma_{Total} = (\sigma^2{}_1 + \sigma^2{}_2 + \dots + \sigma^2{}_n)^{1/2} \quad (15)$$

This paper performs comparative analysis based on Max Product (2), Max Min (3) and Root Sum Square (11) fuzzy inference methodologies in fuzzy controllers. We employ profitability control data based on [7], in the analysis of our work. Results indicate that RSS inference engine gives largest output membership function, while the product inference engine gives the smallest output membership function in this case; minimum inference engine is in between.

[8] study a complete framework of Modified Adaptive Fuzzy Inference Engine (MAFIE)

and its application. He designs a modified apriori algorithm technique is to reduce a minimal set of decision rules based on input-output dataset. In this paper a TSK type fuzzy inference system is constructed by the automatic generation of membership functions and fuzzy rules by the hybrid fuzzy clustering (Fuzzy C-Means and Subtractive Clustering) and apriori algorithms techniques, respectively. The generated adaptive fuzzy inference engine is adjusted by the least-square estimator and a conjugate gradient descent algorithm towards better performance with a minimal set of fuzzy rules. The proposed MAFIE is able to reduce the number of fuzzy rules which increases exponentially when large input dimensions are involved.

[9] design a TSK type fuzzy inference system by the automatic generation of membership functions and rules by the fuzzy c-means clustering and Apriori algorithm technique, respectively. The generated adaptive fuzzy inference engine is adjusted by the least-squares fit and a conjugate gradient descent algorithm towards better performance with a minimal set of rules. The proposed MAFIE is able to reduce the number of rules which increases exponentially when more input variables are involved. [10] propose models based on fuzzy inference systems (FISs) for calculating the resonant frequency of rectangular microstrip antennas (MSAs) with thin and thick substrates are presented. The study uses Mamdani FIS model and Sugeno FIS model to compute the resonant frequency. The parameters of FIS models are determined by using various optimization algorithms. The performances of FIS models are compared with each other and the best result is obtained from the Sugeno FIS model trained by the least squares algorithm. [11] describes the design and implementation of an inference engine for the execution of Fuzzy Inference Systems (FIS). The engine is implemented as a component to be referenced by other applications locally or remotely as a web service. The distinctive characteristic of this component is the ability to define fuzzy objects and attributes. [12] study a novel modified adaptive fuzzy inference system and its application to pattern classification. [13] studies application of fuzzy logic concept to profitability quantification in plastic recycling. [14] designs a neuro-fuzzy linguistic approach in optimizing the flow rate of a plastic extruder process. [15] presents fuzzy rule-base frame work for the management of tropical diseases. [16] proposes a fuzzy-neural network model for effective control of profitability in a paper recycling plant. [17] carryout performance evaluation of membership functions on fuzzy logic controlled AC voltage controller for speed control of induction motor drive. [18] study comparative analysis of fuzzy power system stabilizer using different membership functions. [19] present the design and implementation of three fuzzy inference systems (FIS) based on Mamdani's ethod for automatically assessing Dijkstra's algorithm learning by processing the interaction log provided by GRAPHs.

Research methodology

Figure 2 shows system architecture of fuzzy inference engine.

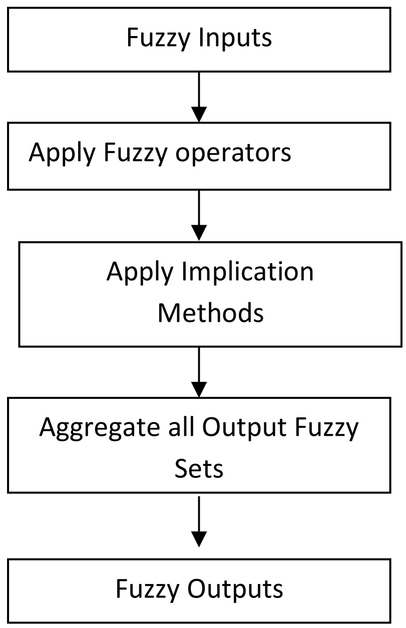

Fig. 2: System Architecture of Fuzzy inference Mechanisms for Profitability control.

Fuzzy Input takes fuzzy input labels (NB, NS, NE, ZE, PO PS, PB) and determines the degree to which they belong to each of the appropriate fuzzy sets via membership functions [20]. Once the inputs are fuzzified, we know the degree to which each part of the antecedent are satisfied for each rule. The fuzzy logical operators are applied to evaluate the composite firing strength of the rule if a given rule has more than one part. The implication method is defined as the shaping of the output membership functions on the basis of the firing strength of the rule. The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. Aggregation is a process whereby the outputs of each rule are unified. Aggregation occurs only once for each output variable. The input to the aggregation process is the truncated output fuzzy sets returned by the implication process for each rule. Fuzzy output of the aggregation process is the combined output fuzzy set. The aggregated output fuzzy set forms the input for the defuzzification process.

In this paper, fuzzy implication is modelled by Mamdani's minimum Operator, (MAX-MIN) method and the sentence connective *also* is interpreted as oring the propositions and defined by max operator. The firing levels of the 49 rules, denoted by α_i, i = 1, 2 are computed by

$$\alpha_1 = E_1(x_0) \wedge EC_1(y_0),\ \alpha_2 = E_2(x0) \wedge CE_2(y_0) \quad (16)$$

The individual rule outputs are obtained by

$$C'_1(w) = (\alpha_1 \wedge C_1(w)),\ C'_2(w) = (\alpha_2 \wedge C_2(w)) \quad (17)$$

Then the overall system output is computed by oring the individual rule outputs

$$C(w) = C'_1(w) \vee C'_2(w) = (\alpha_1 \wedge C_1(w)) \vee (\alpha_2 \wedge C_2(w)) \quad (18)$$

For example, if Rules 41, 42 48 and 49 fire from the rule base presented in [7](Umoh et al, 2010a)[19] (Umoh et al, 2010b), when error and change in error values are selected to be +95 and +95 and their corresponding degrees of membership are PS = 0.2 PB = 0.8 PO = 0.0 for error and PS = 0.2, PB=0.8 and PO = 0.0 for change in error. The MAX-MIN inference for Profit (P), Low Profit (LP) and High Profit (HP) membership function is calculated in (19).

For P, $\alpha_{48} = 0.2; C_{48}(w) = 0.2$
$C'_{48}(w) = (0.2 \wedge 0.2) = 0.2$
$C(w) = 0.2$
For LP, $\alpha_{41} = 0.2; C_{41}(w) = 0.2$
$C'_{41}(w) = (0.2 \wedge 0.2) = 0.2$ (19)
$C(w) = 0.2$
For HP, $\alpha_{42} = 0.2; C_{42}(w) = 0.2$
$A_{49} = 0.8; C_{49}(w) = 0.8$
$C_{42}(w) = (0.2 \wedge 0.2) = 0.2$
$C'_{49}(w) = (0.8 \wedge 0.8) = 0.8$
$C(w) = C_{42}(w) \vee C_{49}(w) = (0.2 \vee 0.8)$
$C(w) = 0.8$

The fuzzy implication is modelled using MAX- PRODUCT or MAX – DOT method called Larsen's product operator and the sentence connective *also* is interpreted as oring the propositions and defined by max operator. Let us denote αi the firing level of the i-th rule, i = 1, 2

$$\alpha_1 = E_1(x_0) * EC_1(y_0),\ \alpha_2 = E_2(x0) * CE_2(y_0) \quad (20)$$

The individual rule outputs are obtained by

$$C'_1(w) = (\alpha_1 * C_1(w)),\ C'_2(w) = (\alpha_2 * C_2(w)) \quad (21)$$

Then membership function of the inferred consequence *C* is pointwise given by

$$C(w) = (\alpha_1 C_1(w)) \vee (\alpha_2 C_2(w)) \quad (22)$$

For example, if Rules 41, 42, 48 and 49 fire from the rule base when error and change in error values are selected to be +95 and +95 and their corresponding degrees of membership are PS = 0.2 PB = 0.8 PO = 0.0 for error and PS = 0.2 PB = 0.8 PO = 0.0 for change in error. The MAX-PRODUCT inference for Profit (P), Low Profit (LP) and High Profit (P) membership function is calculated in (23).

For P, $\alpha_{48} = 0.2; C_{48}(w) = 0.2$
$C_{48}(w) = (0.2 \text{ x } 0.2) = 0.04$
$C(w) = 0.04$
For LP, $\alpha_{41} = 0.2; C_{41}(w) = 0.2$
$C_{41}(w) = (0.2 \text{ x } 0.2) = 0.04$ (23)
$C(w) = 0.04$
For HP, $\alpha_{42} = 0.2; C_{42}(w) = 0.2$
$A_{49} = 0.8; C_{49}(w) = 0.8$
$C_{42}(w) = (0.2 \text{ x } 0.2) = 0.04$

$$C'_{49}(w) = (0.8 \times 0.8) = 0.64$$
$$C(w) = C_{42}(w) \vee C_{49}(w) = (0.04 \vee 0.64)$$
$$C(w) = 0.64$$

Fuzzy implication modelled using Root Sum Square (RSS) method. In this procedure, the degrees of truths (R) of the rules are determined for each rule by evaluating the nonzero minimum values using AND operator. The RSS is evaluated as

$$RSS = \sqrt{\sum R^2} = \sqrt{(R1^2 + R2^2 + R3^2 + ,\ldots, + Rn^2)}$$

Where R1, R2, R3…. Rn are strength values of different rules which share the same conclusion.

For example, if rules 41, 42, 48 and 49 fire from the rule base when error and change in error are selected as +95 and +95 and their corresponding degrees of membership are PS = 0.2 PB = 0.8 PO = 0.0 for error and PS = 0.2 PB = 0.8 PO = 0.0 for change in error, the Root Sum Square inference for Profit (P), Low Profit (LP) and High profit (HP) membership function is calculated as follows

$$\text{Profit (P)} = \sqrt{R48^2} = \sqrt{(0.2)^2} = 0.2$$
$$\text{Low Profit (LP)} = \sqrt{R41^2} = \sqrt{(0.2)^2} = 0.2$$
$$\text{High Profit (HP)} = \sqrt{(R42^2 + R49^2)} = \sqrt{((0.2)^2 + (0.8)^2)}$$
$$= (0.04 + 0.64) = 0.82$$

Result and Discussion

Tables 1, 2 and 3 show the results of MAX-MIN, MAX-PRODUCT and RSS fuzzy inference methods based on profitability data. Where HL, LL, L, NPNL, P, LP, HP are the linguistic fuzzy subsets represent High Loss, Low Loss, Loss, No Profit No Loss, Profit, Low Profit and High Profit respectively (Umoh et al, 2010 b). The MAX-MIN method tests the magnitudes of each rule and selects the highest one. The horizontal coordinate of the "fuzzy centroid" of the area under that function is taken as the output. This method does not combine the effects of all applicable rules but does produce a continuous output function and is easy to implement. The MAX-DOT or MAX-PRODUCT method scales each member function to fit under its respective peak value and takes the horizontal coordinate of the "fuzzy" centroid of the composite area under the function(s) as the output. Essentially, the member function(s) are shrunk so that their peak equals the magnitude of their respective function ("negative", "zero", and "positive"). This method combines the influence of all active rules and produces a smooth, continuous output.

The ROOT-SUM-SQUARE (RSS) method combines the effects of all applicable rules, scales the functions at their respective magnitudes, and computes the "fuzzy" centroid of the composite area. This method is more complicated mathematically than other methods, but is selected for this work since it seemed to give the best weighted influence to all firing rules.

From research, it is revealed that product inference engine and minimum inference engines are the most commonly used fuzzy inference engines in fuzzy systems and control. The main advantage of them is their computational simplicity. And they are intuitively appealing for many practical problems, especially for fuzzy control. However, a disadvantage of the product and minimum inference engines is that if at some $x \in U$ the $\mu_{E^l_i}(x_i)$'s very small the $C'(w)$ obtained in these methods will be very small and this may cause problems in implementation. Root Sum Square inference engine overcome this disadvantage.

Tables 1: Result of MAX-MIN fuzzy inference method based on profitability data.

E	CE	MAX-MIN						
		HL	LL	L	NPNL	P	LP	HP
-100	-86	0.5	-	-	-	-	-	-
-86	-55	0.5	0.5	0.5				
-60	-30	-	-	0.8	0.5	-	-	-
-50	-18	-	-	0.5	0.5	-	-	-
-30	0	-	-	-	0.9	-	-	-
-18	+18	-	-	-	0.5	0.5	0.5	-
0	+18	-	-	-	0.5	0.5	0.5	-
+10	+10	-	-	-	0.5	0.25	0.5	-
+18	+18	-	-	-	0.5	0.5	0.5	-
+25	+34	-	-	-	-	0.7	0.28	-
+40	+50	-	-	-	-	0.5	0.5	-
+50	+55	-	-	-	-	0.35	0.5	-
+55	+67	-	-	-	-	-	0.65	-
+55	+55	-	-	-	-	0.35	0.65	-
+67	+75	-	-	-	-	-	0.75	0.25
+75	+75	-	-	-	-	0.75	0.75	0.25
+84	+75	-	-	-	-	0.25	0.5	0.25
+95	+75	-	-	-	-	0.25	0.25	0.75
+90	+95	-	-	-	-	0.35	0.25	0.65
+90	+100	-	-	-	-	0.35	0.25	0.65
+95	+95	-	-	-	-	0.2	0.2	0.8
+95	+100	-	-	-	-	-	-	0.75
+100	+95	-	-	-	-	0.2	-	0.8
+100	+85	-	-	-	-	0.5	-	0.5
+100	+100	-	-	-	-	-	-	1.0

Tables 2: Results of MAX-Product fuzzy inference method change in error inputs for error and based on profitability data.

E	CE	MAX-PRODUCT						
		HL	LL	L	NPNL	P	LP	HP
-100	-86	0.25	-	-	-	-	-	-
-86	-55	0.25	0.25	0.09	-	-	-	-
-60	-30	-	-	0.65	0.225	-	-	-
-50	-18	-	-	0.25	0.25	-	-	-
-30	0	-	-	-	0.8	-	-	-
-18	+18	-	-	-	0.25	0.25	0.25	-
0	+18	-	-	-	0.25	0.25	0.25	-
+10	+10	-	-	-	0.25	0.063	0.25	-
+18	+18	-	-	-	0.25	0.25	0.25	-
+25	+34	-	-	-	-	0.49	0.56	-
+40	+50	-	-	-	-	0.25	0.25	-
+50	+55	-	-	-	-	0.123	0.25	-
+55	+67	-	-	-	-	-	0.423	-
+55	+55	-	-	-	-	0.122	0.423	-
+67	+75	-	-	-	-	-	0.563	0.063
+75	+75	-	-	-	-	0.653	0.563	0.063
+84	+75	-	-	-	-	0.25	0.25	0.063
+95	+75	-	-	-	-	0.063	0.063	0.563
+90	+95	-	-	-	-	0.123	0.063	0.423

+90	+100	-	-	-	-	0.123	0.123	0.423
+95	+95	-	-	-	-	0.04	0.04	0.64
+95	+100	-	-	-	-	-	-	0.563
+100	+95	-	-	-	-	0.04	-	0.64
+100	+85	-	-	-	-	0.25	-	0.25
+100	+100	-	-	-	-	-	-	1.0

Tables 3: Results of Root Sum Square (RSS) fuzzy inference method based on profitability data.

E	CE	ROOT SUM SQUARE						
		HL	LL	L	NPNL	P	LP	HP
-100	-86	0.701	-	-	-	-	-	-
-86	-55	0.6	0.5	0.3	-	-	-	-
-60	-30	-	-	0.943	0.447	-	-	-
-50	-18	-	-	0.79	0.707	-	-	-
-30	0	-	-	-	1.0295	-	-	-
-18	+18	-	-	-	0.707	0.707	0.707	-
0	+18	-	-	-	0.5	0.707	0.5	-
+10	+10	-	-	-	0.5	0.353	0.5	-
+18	+18	-	-	-	0.5	0.707	0.5	-
+25	+34	-	-	-	-	0.7	0.28	-
+40	+50	-	-	-	-	0.5	0.574	-
+50	+55	-	-	-	-	0.35	0.789	-
+55	+67	-	-	-	-	-	0.738	-
+55	+55	-	-	-	-	0.35	0.805	-
+67	+75	-	-	-	-	-	0.75	0.25
+75	+75	-	-	-	-	0.75	0.75	0.35
+84	+75	-	-	-	-	0.5	0.5	0.35
+95	+75	-	-	-	-	0.25	0.25	1.09
+90	+95	-	-	-	-	0.35	0.25	0.739
+90	+100	-	-	-	-	0.35	0.35	0.739
+95	+95	-	-	-	-	0.2`	0.2	0.82
+95	+100	-	-	-	-	-	-	1.09
+100	+95	-	-	-	-	0.2	-	0.8
+100	+85	-	-	-	-	0.5	-	0.5
+100	+100	-	-	-	-	-	-	1.0

Table 4: Comparison of product, minimum and root sum square inference engines based on 2 inputs, +95 and + 95 for error and change in error in profitability control.

	Low Profit (P)	Profit (P)	High Profit (HP)
Max-Product Inference	0.04	0.04	0.64
Max-Min Inference	0.2	0.2	0.8
Root Sum Square Inference	0.2	0.2	0.82

Table 5: Comparison of product, minimum and root sum square inference engines based on 2 inputs, -86 and -55 for error and change in error in profitability control.

	High Loss (HL)	Low Loss (LL)	Loss (L)
Max-Product Inference	0.25	0.25	0.09
Max-Min Inference	0.5	0.5	0.5
Root Sum Square Inference	0.6	0.5	0.3

From the Tables 1, 2, and 3, we have the following observations: (i) If the membership value of the IF part at points E(x) and CE(x) is small (say, (μ_{MF} < 0.5), then the product inference engine gives the values of 0.123 and 0.25 for profit and low profit respectively, indicating a very small membership function values. Max- Min inference engines gives the values of 0.2 and 0.2 for profit and low profit respectively, indicating a small membership values. Root sum square gives the values of 0.2 and 0.2 for profit and low profit respectively, also showing a small membership values. (ii) If the membership value of the IF parts at points E(x) and CE(x) is large (say, (μ_{MF} >0.5), then the product inference engine gives the values of 0.64 high profit, ax-Min inference engines gives the values of 0.8 for high profit and Root sum square gives the values of 0.82 high profit. From (ii) it is observed that, RSS inference engine gives largest output membership function, while the product inference engine gives the smallest output membership function in this case; minimum inference engine is in between.

Table 4 shows the comparison of the three inference engine methods, product, minimum and RSS based on inputs, error and change in error selected at +95 and +95 derive from profitability control data. The result indicates that, product inference engine gives the values of 0.04 and 0.04 for profit and low profit respectively and a high profit with 0.64 values. This indicates a very small output membership function values for profit and low profit and a large output membership value for high profit. Max- Min inference engine gives the values 0.2, 0.2 and 0.8 for profit, low profit and high profit respectively. The result indicates a small output membership values for profit and low profit and a larger output membership function for high profit. Whereas root sum square gives the values of 0.2, 0.2 and 0.82 for profit low profit and high profit respectively. This result presents small values output membership for profit and low profit and the largest membership value for high profit. Considering the high profit (HP) situation in the three fuzzy inference techniques, RSS inference engine gives largest output membership function of 0.82; minimum inference engine gives 8.0 value, while the product inference engine gives the smallest output membership function of 0.64. Table 5 shows the comparison of the three inference engine methods, product, minimum and RSS based on inputs, error and change in error selected at -86 and -55 derive from profitability control data. Result indicates that RSS fuzzy inference engine method gives the largest output membership value of 0.6 for high loss and very small membership function value of 0.25 is indicated for high loss in the case of product inference engine. While, max-min method has the value of 0.5 for high loss. The overall results present the fact that root sum square has the largest output membership function values; max-product inference engine has the smallest output membership function values, while Max-min is in between.

The plot of output membership functions ($\mu_{outputMF}$(CW)) using the Product, Max Min and Root Sum Square inference engines for error and change in error selected in Table 4 for high profit for (μ_{MF} >= 0.5) case is shown in Figure 3. While Figure 4 shows the plot of output membership functions ($\mu_{outputMF}$(CW)) using the Product, Max Min and Root Sum Square inference engines for error and change in error selected in Table 4 for profit for (μ_{MF} <= 0.5) case. The plot of output membership functions ($\mu_{outputMF}$(CW)) using the Product, Max Min and Root Sum Square inference engines for error and change in error selected in Table 5 for high loss and loss for (μ_{MF} >= 0.5) and (μ_{MF} <= 0.5) cases are shown in Figures 5 and 6 respectively.

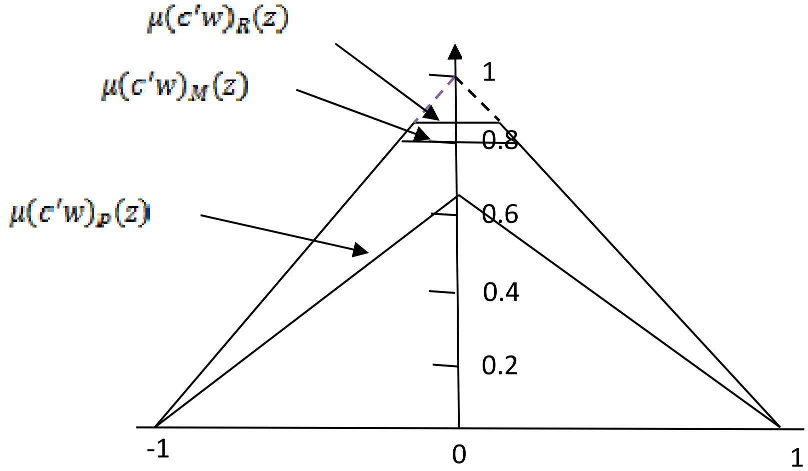

Fig. 3: Output membership functions using the product, minimum and root sum square inference engines for the $\mu(x,y) \geq 0.5$ case in Table 4.

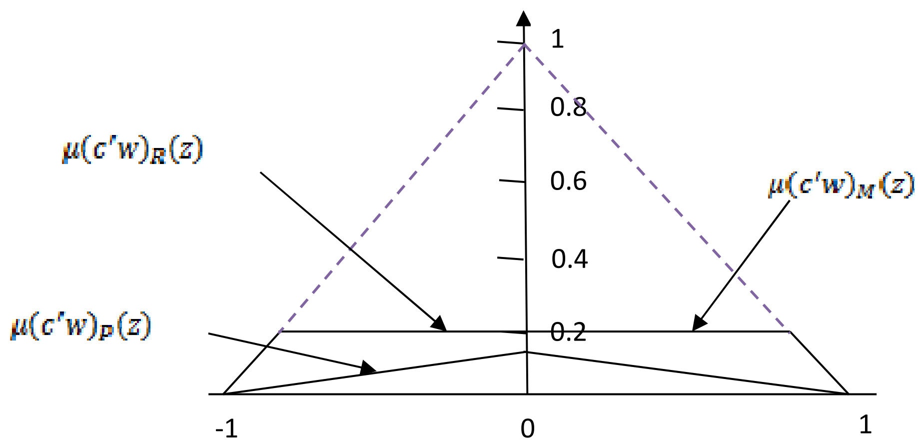

Fig. 4: Output membership functions using the product, minimum and root sum square inference engines for the $\mu(x,y) \leq 0.5$ case in Table 4.

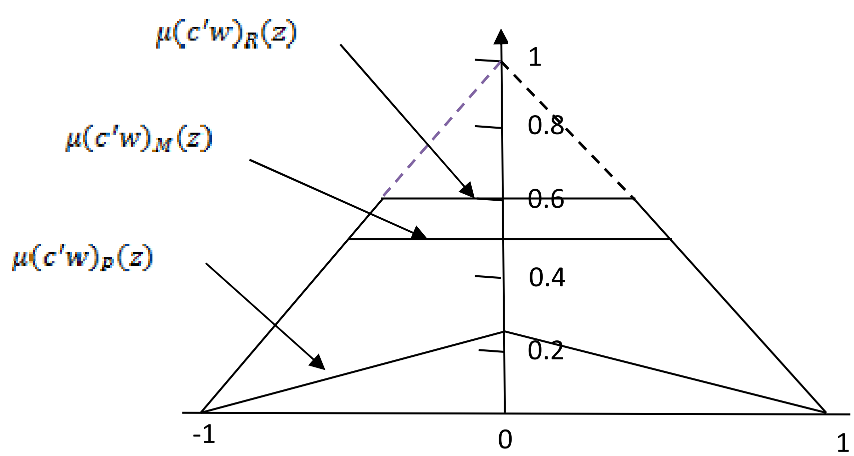

Fig. 5: Output membership functions using the product, minimum and root sum square inference engines for the $\mu(x,y) \geq 0.5$ case in Table 5.

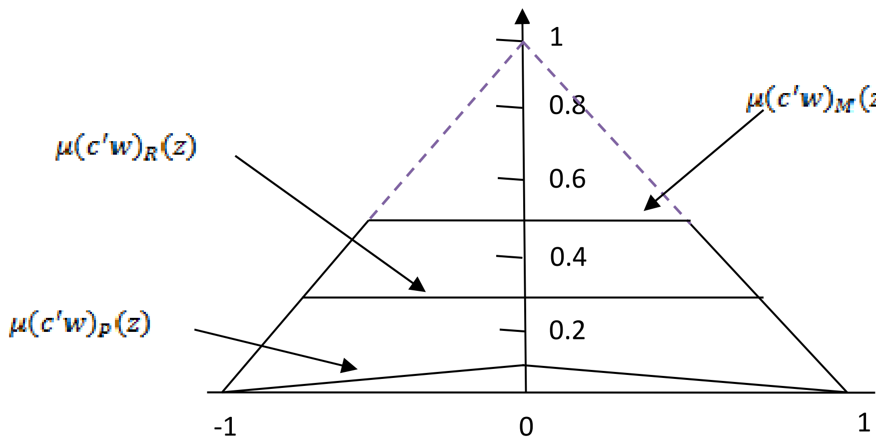

Fig. 6: Output membership functions using the product, minimum and root sum square inference engines for the $\mu(x,y) \leq 0.5$ case in Table 5.

Conclusion

This paper presents a comparative analysis of three fuzzy inference engines, max-product, max-min and root sum in fuzzy controllers which can be used to implement FIS systems in profitability control. The presented results shows that RSS inference engine gives largest output membership function, while the product inference engine gives the smallest output membership function in this case; minimum inference engine is in between. This suggests that root sum square inference engine is one of the most promising strategies in profitability control. Our future work will focus on the comparison and implementation of more inference mechanisms such as Lukasiewicz, Zadeh and Dienes-Rescher using profitability data. Also the definition of other types of Membership Functions is also needed

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