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# Best Compromise Solutions for Stochastic Multi-Objective Environmental/Economic Dispatch of Power Systems using Evolutionary Chance-Constrained Nonlinear Programming and Latin Hypercube Sampling

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**Abstract**

In power systems optimization, the stochastic environmental/economic dispatch problem consists of simultaneously minimizing the fuel cost of generation and $NO_x$ emission to environment considering decision variables, power system loads and objective functions as stochastic. This stochastic approach represents a more realistic model since acquired data are subject to inaccuracies from measuring and forecasting of input data and changes of unit performance during the period between measuring and operation. The Non-Dominated Sorting Genetic Algorithm (NSGA-II) is used to generate the set of non-dominated solutions of the multi-objective problem formulated by a chance-constrained programming technique. Latin Hypercube Sampling is used to yield more precise estimates of these stochastic variables. The decision maker or power system operator may have imprecise or fuzzy goals for each objective function. In order to help the operator in selecting an operating point from the obtained set of Pareto-optimal solutions, fuzzy logic theory is applied to each objective function to obtain a fuzzy membership function. The best non-dominated solution can be found when the normalized sum of membership function values for all objectives is highest. This paper analyzes the changes in the best compromise solutions obtained from the evolutionary algorithm for coefficient of variation of 0.05, 0.1 and 0.2 under system reliability of 68.3% and 95.5% respectively. Simulation results are presented for the standard IEEE 30-bus test system.

**Keywords**: Power Systems, Environmental/Economic Dispatch, Multi-Objective Optimization, Evolutionary Algorithms, Fuzzy Logic.

**For correspondences and reprints*

## 1. INTRODUCTION

Economic dispatch aims at minimizing the fuel cost for power generation while environmental dispatch minimizes the emissions of fossil-fuel power plants. The multiobjective economic/environmental dispatch considers both problems simultaneously and attempts to find the set of compromise solutions since the two objectives, fuel cost and emission are conflicting. Classical techniques such as goal programming (Nanda 1988), epsilon-constraint method (Dhillon 1994) have been employed to solve the multiobjective optimisation problem. More recently, evolutionary algorithms such as NSGA (Abido 2003a), NPGA (Abido 2003b), SPEA (Abido 2003c) and NSGA-II (Ah King & Rughooputh 2003, Ah King et al. 2005) have been applied successfully to the deterministic problem. However, in real-world situations, data used in the optimization methods are subjected to inaccuracies and uncertainties due to inaccuracies in the process of measuring and forecasting of input data and changes of unit performance during the period between measuring and operation (Parti et al. 1983). Thus, it becomes imperative to consider these uncertainties for a more realistic solution. The problem therefore is known as the stochastic multiobjective environmental/economic dispatch problem. The stochastic problem has been solved using the weighted minimax technique (Dhillon et al. 1993), weighted sum technique (Dhillon et al. 1995), interactive fuzzy satisfying method (Bath et al. 2004) and NSGA-II (Ah King et al. 2006). More recently, the problem at hand was solved by a modified particle swarm optimization algorithm (Wang & Singh 2008). In this paper, the problem is formulated by a chance-constrained programming technique and solved by NSGA-II with Latin Hypercube Sampling which is used to yield more precise estimates of the stochastic variables. The decision maker or power system operator may have imprecise or fuzzy goals for each objective function. In order to help the operator in selecting an operating point from the obtained set of Pareto-optimal solutions, fuzzy logic theory is applied to each objective function to obtain a fuzzy membership function. Using the IEEE 30-bus system, this paper analyzes the changes in the best compromise solutions obtained from the evolutionary algorithm for coefficient of variation of 0.05, 0.1 and 0.2 under system reliability of 68.3% and 95.5% respectively.

## 2. DETERMINISTIC MULTIOBJECTIVE ENVIRONMENTAL/ECONOMIC DISPATCH

The environmental/economic dispatch involves the simultaneous optimization of fuel cost and emission objectives which are conflicting ones. The deterministic problem is formulated as described below.

Fuel Cost Objective. The classical economic dispatch problem of finding the optimal combination of power generation, which minimizes the total fuel cost while satisfying the total required demand can be mathematically stated as follows (Yokoyama et al. 1988):

$$C = \sum_{i=1}^{n} \left( a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) \text{ \$/hr} \tag{1}$$

where $C$ is total fuel cost ($/hr), $a_i$, $b_i$, $c_i$: are fuel cost coefficients of generator $i$, $P_{Gi}$: is power generated in per unit (p.u.) by generator $i$, and $n$ is number of generators.

$NO_x$ Emission Objective. The minimum emission dispatch optimizes the above classical economic dispatch including $NO_x$ emission objective, which can be modeled using second order polynomial functions (Yokoyama et al. 1988):

$$E = \sum_{i=1}^{n} ( a_{iN} + b_{iN} P_{Gi} + c_{iN} P_{Gi}^2 + d_{iN} \sin( e_{iN} P_{Gi} )) \text{ ton/hr} \tag{2}$$

The optimization problem is bounded by the following constraints:

Power balance constraint. The total power generated must supply the total load demand and the transmission losses.

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_L = 0 \tag{3}$$

where $P_D$ is total load demand (p.u.), and $P_L$ is transmission losses (p.u.).

The transmission losses is given by

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} \begin{bmatrix} (r_{ij} / V_i V_j) \cos(\delta_i - \delta_j)(P_i P_j + Q_i Q_j) + \\ (r_{ij} / V_i V_j) \sin(\delta_i - \delta_j)(Q_i P_j - P_i Q_j) \end{bmatrix} \tag{4}$$

where $N$ is number of buses, $r_{ij}$ is series resistance connecting buses $i$ and $j$, $V_i$ is voltage magnitude at bus $i$, $\delta_i$ is voltage angle at bus $i$, $P_i$ is real power injection at bus $i$, and $Q_i$ is reactive power injection at bus $i$.

Maximum and minimum limits of power generation. The power generated $P_{Gi}$ by each generator is constrained between its minimum and maximum limits, i.e.,

$$P_{Gimin} \le P_{Gi} \le P_{Gimax} \tag{5}$$

where $P_{Gimin}$ is minimum power generated, and $P_{Gimax}$ is maximum power generated.

## 3. STOCHASTIC ENVIRONMENTAL/ECONOMIC DISPATCH

Previous stochastic approaches involved the inclusion of deviational (recourse) costs to account for mismatch between scheduled output and actual demand in the formulation of the objective function (Bunn & Paschentis 1986), and conversion of stochastic models into their deterministic equivalents by taking their expected values and formulating the problem as the minimization of cost and emission plus additional objective for the expected deviation between generator outputs and load demand (unsatisfied load demand) (Dhillon et al. 1993; 1995; Bath et al. 2004). The approach adopted in this paper is based on the reliability concept and simulations are performed to test the reliability of the stochastic system under different problem formulations. Decision variables $P_{Gi}$ ($i$ = 1,…,6) are assumed to be normally distributed with Mean $P_{Gi}$ and Standard Deviation (SD) $\sigma_i = CV\, P_{Gi}$. , where $CV$ is the coefficient of variation ($CV$ is chosen as 0.05, 0.1 or 0.2 of the mean). For each solution $P_{Gi}$ ($i$ = 2,…,6), 100 random instantiates having Mean $P_{Gi}$ and SD $\sigma_i$ are created within $k\sigma_i$ (where $k$ is chosen as 1 or 2). A good measure of system performance in the case of stochastic systems is its reliability (Deb & Chakroborty 1998). We define reliability $R$ as:

$$R = \frac{n_i}{m} \tag{6}$$

where $n_i$ is the number of instantiates satisfying a required criterion (here the number of instantiates falling in the range of power generated by generator $G_1$ or slack generator) and $m$ is the number of instantiates.
An additional constraint is thus included in the optimization problem:

$$R \geq \beta \tag{7}$$

where $\beta$ is the required reliability which is 68.3% (with $k$ = 1) or 95.5% (with $k$ = 2), i.e. P($P_{G1min} \leq P_{G1} \leq P_{G1max}$). Thus, Reliability $R$ is calculated according to the number of cases for which $P_{G1}$ is found to be within $P_{G1\min}$ and $P_{G1\max}$.
In the stochastic approach, the objective functions are now reformulated as follows using the chance-constrained programming technique (Rao 1996):

$$\text{Min.} \quad f_1 = \overline{C} + k\sigma_C \tag{8}$$
$$\text{Min.} \quad f_2 = \overline{E} + k\sigma_E$$

subject to the following constraints:

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_L = 0 \tag{9}$$
$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax}$$
$$\text{P}(P_{Gimin} \leq P_{Gi} \leq P_{Gimax}) \geq \beta_i, \qquad i = 1, 2, \ldots, \text{n}.$$

where $\overline{C}$, $\overline{E}$, $\sigma_C$ and $\sigma_E$ are the Expected Fuel Cost, Expected $NO_x$ emission, SD of Fuel Cost and SD of $NO_x$ emission respectively.
Note that $P_{G1}$ is calculated from the loadflow program and this satisfies implicitly the power balance constraint (Equation (3)). For $i$ = 2,…, $n$, we choose $P_{Gi}$ within $P_{Gimin}$ and $P_{Gimax}$, thereby making the P($P_{Gimin} \leq P_{Gi} \leq P_{Gimax}$) = 1. With the chosen $P_{Gi}$ ($i$ = 2,…, $n$) we then compute the equality constraint and calculate P($P_{G1min} \leq P_{G1} \leq P_{G1max}$) by using a Monte-Carlo method in which $m$ instantiates of $P_{G2}$ to $P_{Gn}$ are used to compute $P_{G1}$ and the number of events $n_i$ satisfying $P_{G1min} \leq P_{G1} \leq P_{G1max}$ is counted.

The probability is then computed by using P($P_{G1min} \leq P_{G1} \leq P_{G1max}$) = $\frac{n_i}{m}$.

The procedure used in this stochastic method is described as follows:

*For each feasible solution $\overline{P}_{Gj}(j = 2,\ldots,n)$ obtained by NSGA-II,*

*Create $m$ instantiates $P_{Gj}^{(i)}(i = 1,\ldots,100, j = 2,\ldots,n)$ by perturbing each $P_{Gj}$ as N($\overline{P}_j$, $\sigma_j$) and $\sigma_j$ = CV $\overline{P}_j$. (m is chosen as 100)*

*Count the number of instantiates $n_i$ for which $P_{G1}^{(i)} \in [P_{G1\min}, P_{G1\max}]$*

*Calculate* $R = \frac{n_i}{m}$

*Calculate Expected Cost $\overline{C}$, Expected $NO_x$ $\overline{E}$, SD of Cost $\sigma_C$ and SD of $NO_x$ $\sigma_E$*

In the above procedure, only the power generations $P_{Gi}$ ($i$=2,…,$n$) are perturbed as for case S1 considered below. The procedure is modified accordingly for the two other cases considered. Case S2 involves the perturbations of power generations $P_{Gi}$ ($i$=2,…,$n$) and system loads $P_{Loadi}$ (i=1,…,*21*) while case S3 considers the perturbations of power generations $P_{Gi}$ ($i$=2,…,$n$), system loads $P_{Loadi}$ ($i$=1,…,*21*) as well as the fuel cost ($a_i$, $b_i$ and $c_i$, $i$=1,…,6) and emission coefficients ($a_{Ni}$, $b_{Ni}$, $c_{Ni}$, $d_{Ni}$ and $e_{Ni}$, $i$=1,…,6).
A constrained Monte Carlo sampling scheme: Latin hypercube sampling (LHS) developed by McKay, Conover and Beckman (McKay et al. 1979) has been adopted to yield more precise estimates of variables for the $m$ instantiates as opposed to the work presented in (Ah King et al. 2005). This method selects n different values from each k variables $X_1$, $X_2$, …, $X_k$ by dividing the range of each variable into n nonoverlapping intervals on the basis of equal probability. One value from each interval is selected at random according to the probability density in the interval. For variable $X_1$, n values are thus obtained and are paired in a random manner (no correlation) or according to some correlation coefficient with the n values of $X_2$.

## 4. BEST COMPROMISE SOLUTION

The algorithm described in the previous section generates the non-dominated set of solutions known as the Pareto-optimal solutions. The decision maker (power system operator) may have imprecise or fuzzy goals for each objective function. To aid the operator in selecting an operating point from the obtained set of Pareto-optimal solutions, fuzzy logic theory is applied to each objective functions to obtain a fuzzy membership function $\mu_{f_i}$ as follows (Dhillon et al. 1993):

$$\mu_{f_i} = \begin{cases} 1 & f_i \le f_i^{min} \\ \dfrac{f_i^{max} - f_i}{f_i^{max} - f_i^{min}} & f_i^{min} < f_i < f_i^{max} \\ 0 & f_i \ge f_i^{max} \end{cases} \tag{10}$$

The best non-dominated solution can be found when eqn. (11) is a maximum where the normalized sum of membership function values for all objectives is highest.

$$\mu^k = \frac{\sum_{i=1}^{N} \mu_{f_i}^k}{\sum_{k=1}^{M} \sum_{i=1}^{N} \mu_{f_i}^k} \tag{11}$$

where $M$ is the number of non-dominated solutions.

## 5. SIMULATION RESULTS

### 5.1 Comparison with MOPSO

To assess the quality of the solutions obtained in the proposed method, the results are compared with those of a previous work on the same problem. A recent approach based on a stochastic formulation similar to Dhillon et al. 1993, 1995 but solved using a modified particle swarm optimization algorithm (Multi-Objective Particle Swarm Optimization MOPSO) has been proposed by Wang & Singh 2008. Simulations were performed on the standard IEEE 30-bus 6-generator test system (Fig. 1) with transmission losses calculated using loss coefficients as in (Wang & Singh 2008).

In this case, the transmission losses is given by

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j \tag{12}$$

where $N$ is number of buses, $B_{ij}$ .is the loss coefficients instead of Equation (4).

Note that $P_{G1}$ is still a slack variable and this satisfies implicitly the power balance constraint (Equation (3)). In the stochastic procedure, the perturbations of power generations $P_{Gi}$ ($i$=2,…,$n$) and the fuel cost ($a_i$, $b_i$ and $c_i$, $i$=1,…,6) and emission coefficients ($a_{Ni}$, $b_{Ni}$, $c_{Ni}$, $i$=1,…,6) are considered since emission coefficients $d_{Ni}$ and $e_{Ni}$ were not taken into account in (Wang & Singh 2008). Coefficients of variation $CV$ and correlation coefficients were taken as 0.1 and 1.0, respectively. The required reliability was taken as 68.3% (i.e. $k$ = 1).

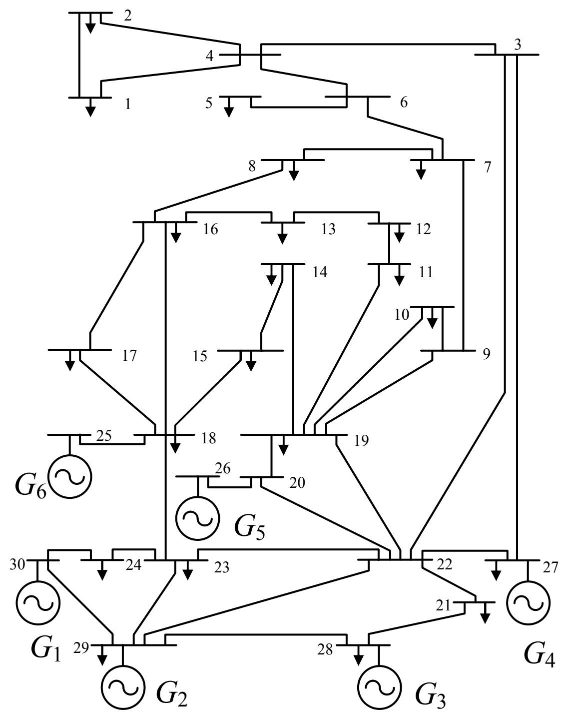



*Fig. 1: Single-line diagram of IEEE 30-bus test system (Abido, 2003c).*

The proposed algorithm was run with a population size of 100 for 500 generations with crossover probability of 0.9, mutation probability of 0.2. Fig. 2 shows the plot of $f_2$ against $f_1$. It should be noted that both objectives include the mean and standard deviation of fuel cost and emission respectively. The deterministic plot for the two objectives is also shown for comparison. As observed from both plots, the proposed algorithm achieves very good diversity. As far as convergence is concerned, the minimum values of objectives obtained (see Table 1) confirms the efficacy of the proposed method.

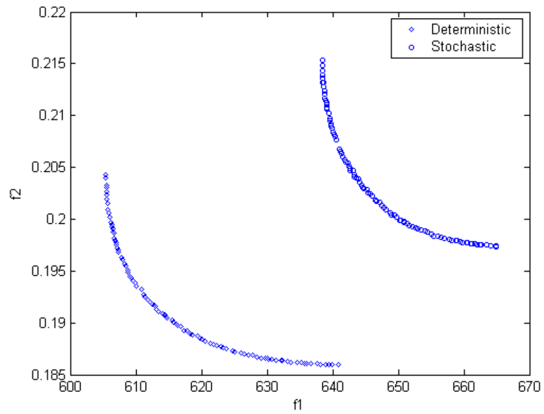
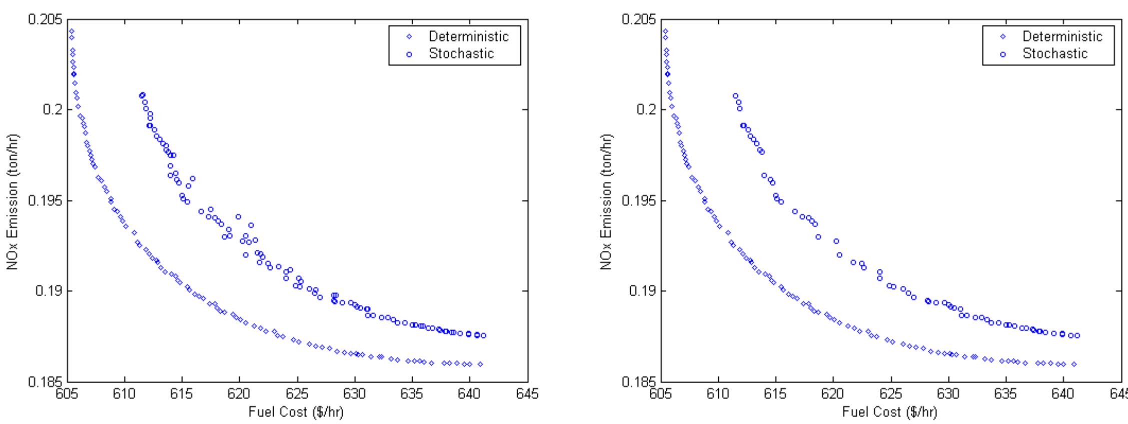



*Fig. 2. $f_2$ vs. $f_1$*

The expected (mean) values of the fuel cost and emission objectives are shown in Fig. 3. It can be observed that some solutions are dominated (see Fig. 3(a)) due the formulation of the proposed method (each objective being minimized is the sum of expected value and standard deviation of its expected value). The non-dominated solutions extracted from the obtained solutions of Fig. 3(a) are shown in Fig. 3(b). It is interesting to note that the distribution of solutions obtained by the proposed method shows nevertheless good diversity.




(a) With dominated solutions (b) Without dominated solutions

*Fig. 3. Solutions obtained by proposed method*

Table 1 compares the minimum fuel cost obtained by the proposed method and that obtained by MOPSO (Wang & Singh 2008). It is observed that the proposed method achieves a lower minimum fuel cost compared to MOPSO both for the deterministic and stochastic models considered. An increase in fuel cost of 1.01% is found for the proposed model compared to 1.46% for MOPSO from deterministic to stochastic models. If the system is operated for a year, there is a significant increase in fuel cost of $53,550 for the proposed model if uncertainties are considered.

| | Deterministic model | | Stochastic model | |
|---|---|---|---|---|
| Generators/objectives | MOPSO (Wang & Singh 2008) | Proposed method | MOPSO (Wang & Singh 2008) | Proposed method |
| $P_{G1}(\overline{P}_{G1})$ | 0.0921 | 0.1135 | 0.1621 | 0.2612 |
| $P_{G2}(\overline{P}_{G2})$ | 0.2894 | 0.2930 | 0.2659 | 0.2809 |
| $P_{G3}(\overline{P}_{G3})$ | 0.5567 | 0.5784 | 0.6744 | 0.5363 |
| $P_{G4}(\overline{P}_{G4})$ | 1.0362 | 0.9924 | 1.0079 | 0.9468 |
| $P_{G5}(\overline{P}_{G5})$ | 0.5072 | 0.5250 | 0.4048 | 0.4951 |
| $P_{G6}(\overline{P}_{G6})$ | 0.3770 | 0.3549 | 0.3419 | 0.3400 |
| Minimum fuel cost ($/hr) | 605.735 | 605.427 | 614.571 | 611.540 |
| Emission (ton/hr) | 0.2064 | 0.2043 | 0.2065 | 0.2008 |

*Table 1: Minimum fuel cost for deterministic and stochastic models*

Table 2 compares the minimum emission obtained by the proposed method and that obtained by MOPSO (Wang & Singh 2008). It is observed that the proposed method compares equally in the minimum emission obtained in the deterministic model. However, a higher minimum emission (0.81% larger than the deterministic value) is obtained for the proposed stochastic model compared to MOPSO (0.32%). Based on the obtained result, there is a significant increase in emission of 13.14 tons if the system is operated for a year according to the proposed model under uncertainties.

| | Deterministic model | | Stochastic model | |
|---|---|---|---|---|
| Generators/objectives | MOPSO (Wang & Singh 2008) | Proposed method | MOPSO (Wang & Singh 2008) | Proposed method |
| $P_{G1}(\overline{P}_{G1})$ | 0.4125 | 0.3957 | 0.4142 | 0.4130 |
| $P_{G2}(\overline{P}_{G2})$ | 0.5104 | 0.4953 | 0.4707 | 0.4751 |
| $P_{G3}(\overline{P}_{G3})$ | 0.4964 | 0.5111 | 0.4983 | 0.5138 |
| $P_{G4}(\overline{P}_{G4})$ | 0.4543 | 0.4617 | 0.4671 | 0.4948 |
| $P_{G5}(\overline{P}_{G5})$ | 0.5023 | 0.5066 | 0.5081 | 0.4960 |
| $P_{G6}(\overline{P}_{G6})$ | 0.5018 | 0.4980 | 0.5154 | 0.4764 |
| Minimum emission (ton/hr) | 0.1860 | 0.1860 | 0.1866 | 0.1875 |
| Fuel cost ($/hr) | 645.300 | 640.898 | 649.658 | 641.160 |

*Table 2: Minimum emission for deterministic and stochastic models*

It is observed that the minimum values of the objectives obtained by the proposed method are better than MOPSO for the deterministic model while comparable results are achieved in the stochastic model and this confirms the validity of the method as well as the quality of solutions obtained.

### 5.2 Best Compromise Solutions

Simulations were performed on the standard IEEE 30-bus 6-generator test system (Abido, 2003a, 2003b, 2003c, Ah King 2005). LHS was used both without correlation and with correlation to test the system under 5%, 10% and 20% variations of the variables ($CV$ = 0.05, 0.1 and 0.2 respectively). The uncorrelated cases have zero correlation coefficient while a correlation coefficient of 0.9 has been assumed for all correlated cases. For each case, 11 independent runs for 2000 generations were performed using NSGA-II algorithm (Deb et al. 2002).

For the deterministic problem, the nondominated solutions out of the 11 runs is shown in Fig. 5 together with the best compromise solution calculated using eqn. (11). Table 3 gives the fuel cost, $NO_x$ emission and power generation of each unit for the best compromise solution.

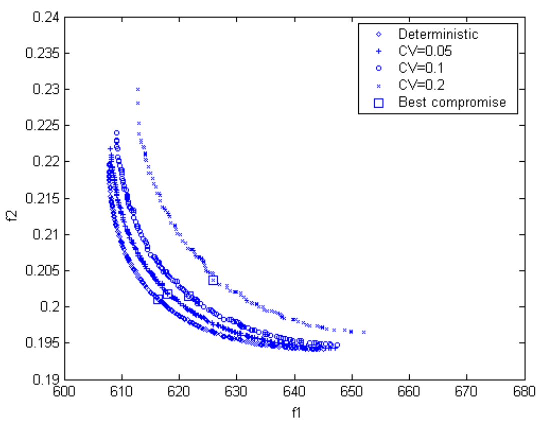
| Fuel Cost ($/hr) | $NO_x$ Emission (ton/hr) | $P_{G1}$ (p.u.) | $P_{G2}$ (p.u.) | $P_{G3}$ (p.u.) | $P_{G4}$ (p.u.) | $P_{G5}$ (p.u.) | $P_{G6}$ (p.u.) |
|---|---|---|---|---|---|---|---|
| 616.382202 | 0.200969 | 0.2621 | 0.3756 | 0.5432 | 0.6997 | 0.5610 | 0.4230 |

*Table 3: Best compromise solution*

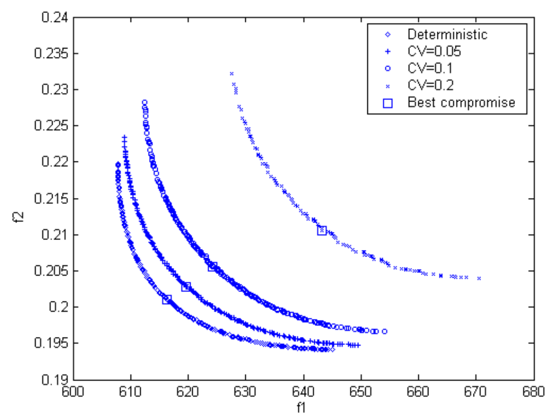
**5.2.1 Case S1: Stochastic power generation**

The multiobjective optimization problem is formulated as described above with fixed total system load $P_D$ = 2.834 p.u. Thus, power generated $P_{Gi}$ are random variables.

Figs. 4(a) and 2(b) shows the best nondominated solutions out of 11 runs together with the best compromise solution for CV=0.05, 0.1 and 0.2 with k=1 for LHS without correlation and LHS with correlation, respectively. Table 4 gives the percentage change in cost, emission and power generation for best compromise solution for the three coefficients of variation considered for the two cases respectively.




(a) LHS without correlation




(b) LHS with correlation

*Fig. 4: Best nondominated solutions and best compromise solutions for Case S1 with k=1.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.06 | -0.06 | -2.36 | 1.80 | 2.90 | -0.60 | -4.90 | 3.53 |
| 0.1 | 0.43 | -0.52 | 5.38 | -0.20 | 3.90 | -6.15 | 0.46 | 1.18 |
| 0.2 | 0.78 | -0.18 | -1.81 | 7.93 | 2.54 | -6.61 | -5.43 | 8.81 |

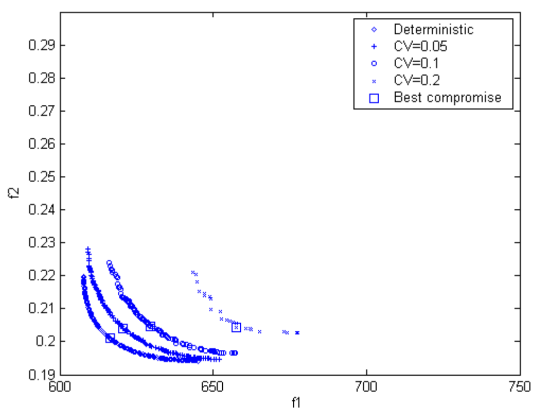
*Table 4(a): Percentage change in cost, emission and power generation for best compromise solution of Case S1 with k=1 for LHS without correlation.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.13 | 0.07 | -2.24 | -0.08 | 8.49 | -2.64 | -2.78 | -1.73 |
| 0.1 | 0.43 | 0.52 | 0.03 | 1.83 | 3.75 | -1.00 | -6.07 | 3.11 |
| 0.2 | 2.09 | 1.46 | 14.92 | -2.15 | 1.61 | -9.94 | 0.25 | 6.58 |

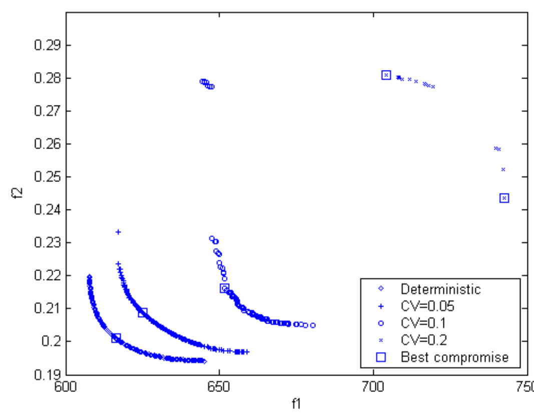
*Table 4(b): Percentage change in cost, emission and power generation for best compromise solution of Case S1 with k=1 for LHS with correlation.*

It can be observed from Fig. 4 that stochastic variables causes the nondominated front to shift up and right thus increasing both cost and emission values. The shift is more pronounced for correlated stochastic variables as can be verified from the graphs. This means that larger increase of cost and emission is expected with correlated stochastic variables.
From Table 4(a), it can be deduced that the cost is expected to increase and emission is expected to decrease for uncorrelated stochastic variables. For the largest variation (CV=0.2) in the stochastic variables considered, an increase of 0.78% (4.797 $/hr) in cost and a decrease of 0.18% (0.00035 ton/hr) in emission are expected for the uncorrelated case. However, from Table 4(b), both cost and emission are expected to increase with correlated stochastic variables. Increases of 2.09% (12.896 $/hr) and 1.46% (0.00293 ton/hr) are expected in cost and emission respectively for the correlated case. In terms of power generation, an increase of 8.81% (0.0373 p.u.) for unit 6 is observed for the uncorrelated case whereas the corresponding increase is 14.92% (0.0391 p.u.) for unit 1 for the correlated case.

Figs. 5(a) and 5(b) show the results obtained for Case S1 with k=2 and Table 3 gives the percentage change in cost, emission and power generation for best compromise solution.




(a) LHS without correlation




(b) LHS with correlation

*Fig. 5 Best nondominated solutions and best compromise solutions for Case S1 with k=2.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.09 | 0.07 | -1.46 | 0.63 | 2.50 | -1.41 | 0.85 | -1.77 |
| 0.1 | 0.79 | -0.52 | 1.05 | 4.04 | 1.25 | -8.69 | -0.01 | 8.42 |
| 0.2 | 3.07 | -0.93 | 33.87 | 15.64 | 1.57 | -23.73 | -7.22 | 11.81 |

*Table 5(a): Percentage change in cost, emission and power generation for best compromise solution of Case S1 with k=2 for LHS without correlation.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.14 | 0.67 | -7.47 | 3.27 | 4.70 | 0.43 | -3.69 | -0.30 |
| 0.1 | 1.23 | 3.23 | 43.34 | -17.22 | -8.97 | 13.07 | -18.86 | 4.07 |
| 0.2 | 5.93 | 29.84 | 16.52 | -86.42 | -30.22 | 114.36 | -42.07 | -22.30 |
| | 8.40 | 9.51 | 129.75 | -86.65 | -7.13 | -14.48 | 2.55 | 27.32 |

*Table 5(b): Percentage change in cost, emission and power generation for best compromise solution of Case S1 with k=2 for LHS with correlation.*

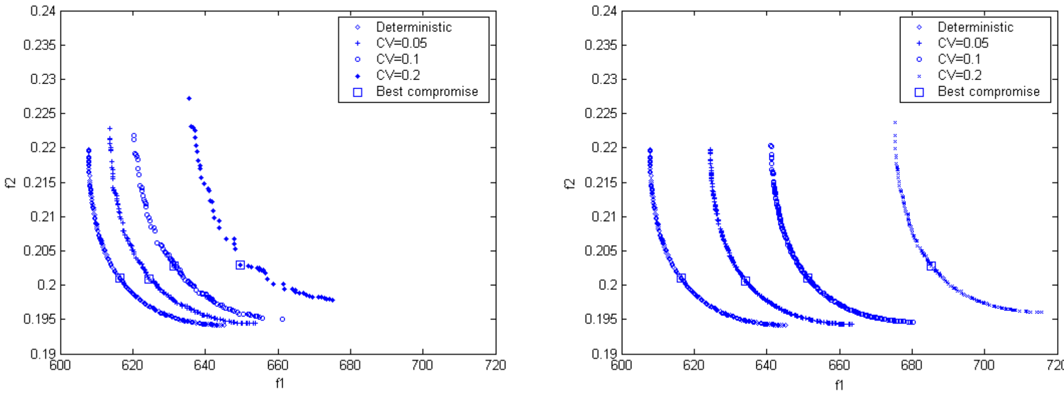
The same trend as in Fig. 4 is observed in Fig. 5with an even larger shift of the nondominated fronts. It should be pointed out that it is harder for the NSGA-II algorithm to obtain the nondominated front as CV is increased since a smaller number of solutions is found. Again, cost is expected to increase and emission to decrease (except for CV=0.05) for the uncorrelated case. For the largest variation (CV=0.2) in the stochastic variables considered, an increase of 3.07% (18.896 $/hr) in cost and a decrease of 0.93% (0.00186 ton/hr) in emission are expected for this case. Similarly, both cost and emission are expected to increase when the stochastic variables are correlated. From Fig. 5(b), it is found that two best compromise solutions (the minimum cost and minimum emission solutions) are identified for CV=0.2 since the nondominated front obtained is concave. If the minimum cost solution is chosen as the best compromise solution, increases of 5.93% (36.522 $/hr) and 29.84% (0.05996 $/hr) are expected in cost and emission respectively. If the minimum emission solution is chosen, increases of 8.40% (51.777 $/hr) and 9.51% (0.01912 ton/hr) in cost and emission are expected. In terms of power generation, an increase of 33.87% (0.0887 p.u.) for unit 1 is observed for the uncorrelated case whereas the corresponding increase is 114.36% (0.8002 p.u.) for unit 4 and 129.75% (0.3400 p.u.) for unit 1 for the two best compromise solutions of the correlated case.

In general, the changes in cost and emission is expected to be larger for the correlated case.

**5.2.2 Case S2: Stochastic power generation and system loads**

The multiobjective optimization problem is formulated as above but the individual loads on the system are treated as stochastic variables. Thus, power generation and system loads are random variables. Each of 21 loads is normally distributed with mean $P_{Li}$ and $\sigma_i = CV\, P_{Li}$ . Power factor for each load is maintained as at the base load, i.e. ratio $P_{Li}$ to $Q_{Li}$ is constant.

Figs. 6(a) and 6(b) shows the best nondominated solutions out of 11 runs together with the best compromise solution for CV=0.05, 0.1 and 0.2 with k=1 for LHS without correlation and LHS with correlation, respectively. Table 6 gives the percentage change in cost, emission and power generation for best compromise solution for the three coefficients of variation considered for the two cases respectively.




(a) LHS without correlation (b) LHS with correlation

*Fig. 6: Best nondominated solutions and best compromise solutions for Case S2 with k=1.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.32 | -0.47 | 10.72 | -3.80 | 6.11 | -4.53 | -2.25 | -0.90 |
| 0.1 | 0.30 | -0.06 | 5.82 | -4.83 | 3.88 | -2.14 | -4.71 | 5.36 |
| 0.2 | 1.38 | -0.57 | 11.39 | 1.44 | 9.63 | -14.40 | -1.86 | 5.13 |

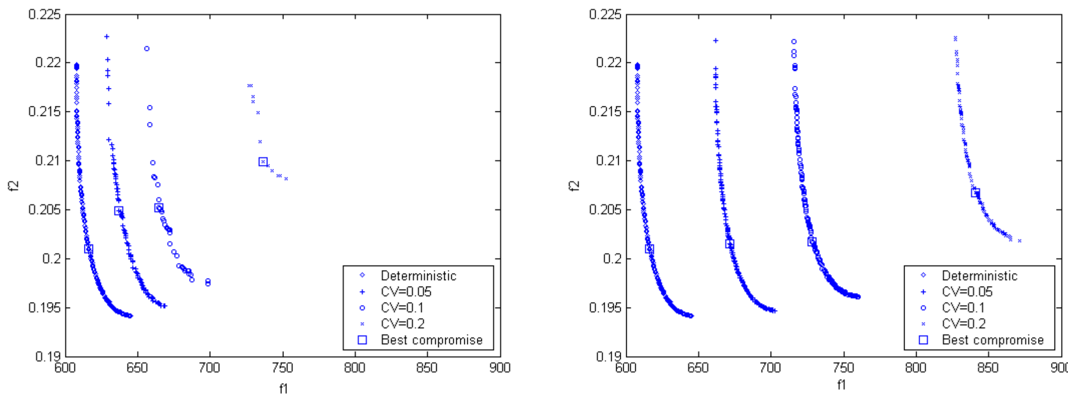
*Table 6(a): Percentage change in cost, emission and power generation for best compromise solution of Case S2 with k=1 for LHS without correlation.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.18 | -0.35 | 11.39 | 1.44 | 9.63 | -14.40 | -1.86 | 5.13 |
| 0.1 | 0.27 | -0.34 | 3.06 | -0.43 | 4.04 | -3.41 | -4.22 | 4.37 |
| 0.2 | 0.46 | 0.04 | -0.71 | 3.21 | 5.81 | -4.84 | -2.53 | 1.23 |

*Table 6(b): Percentage change in cost, emission and power generation for best compromise solution of Case S2 with k=1 for LHS with correlation.*

As in Case S1, considering stochastic system loads in addition to stochastic power generation causes the nondominated front to shift slightly up and more to the right. For uncorrelated stochastic variables, it is found from Table 6(a) that a 1.38% (8.494 \$/hr) increase in cost and 0.57% (0.00114 ton/hr) decrease in emission are expected for CV=0.2 (20% variation in both power generation and system loads). For correlated stochastic variables from Table 6(b), for the same coefficient of variation, the expected increase in cost and emission are 0.46% (2.842 \$/hr) and 0.04% (0.00008 ton/hr) respectively. In terms of power generation, a decrease of 14.40% (0.1008 p.u.) for unit 4 is observed for the uncorrelated case whereas the corresponding increase is 5.81% (0.0315 p.u.) for unit 3 for the correlated case.

Figs. 7(a) and 7(b) shows the results obtained for case S2 with k=2 and Table 7 gives the percentage change in cost, emission and power generation for best compromise solution.




(a) LHS without correlation (b) LHS with correlation

*Fig. 7: Best nondominated solutions and best compromise solutions for Case S2 with k=2.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.02 | 0.32 | -3.42 | 2.39 | 2.81 | 0.30 | -0.82 | -3.15 |
| 0.1 | 0.94 | -0.18 | 21.32 | -9.91 | 2.77 | -6.26 | -3.24 | 6.59 |
| 0.2 | 4.96 | -0.06 | 47.97 | 18.22 | 21.90 | -37.35 | -17.99 | 11.01 |

*Table 7(a): Percentage change in cost, emission and power generation for best compromise solution of Case S2 with k=2 for LHS without correlation.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.23 | -0.33 | 2.60 | 2.41 | 7.56 | -3.91 | -4.48 | -1.38 |
| 0.1 | 0.73 | -0.67 | 16.63 | -2.61 | 8.35 | -8.38 | -4.54 | 0.79 |
| 0.2 | 1.55 | 0.04 | 22.51 | -4.30 | 5.91 | -12.69 | 2.69 | -0.63 |

*Table 7(b): Percentage change in cost, emission and power generation for best compromise solution of Case S2 with k=2 for LHS with correlation.*

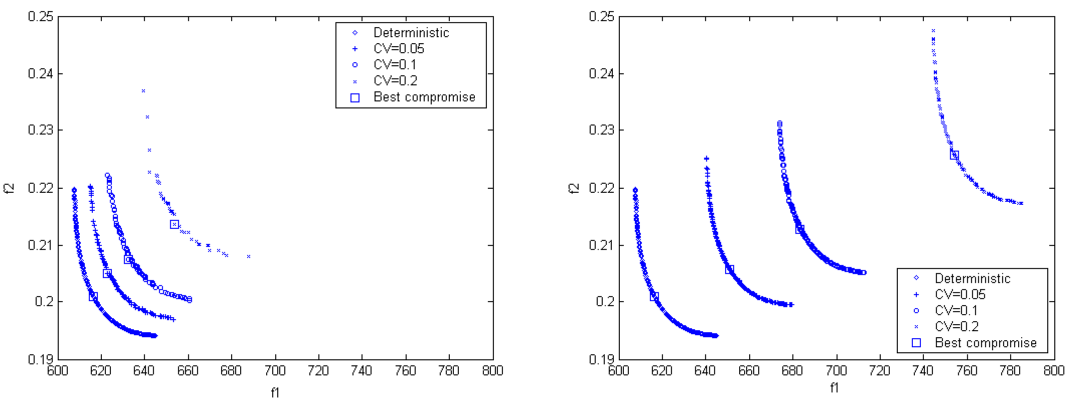
A larger shift to the right of the nondominated solutions is observed from Fig. 7 compared to Fig. 6. Relatively larger changes are expected for the uncorrelated case with 4.96% (30.592 \$/hr) increase in cost and 0.06% (0.00011 ton/hr) decrease in emission compared to increases of 1.55% (9.564 \$/hr) and 0.04% (0.00009 ton/hr) in both cost and emission respectively for the correlated case for the largest variation (CV=0.2) in the stochastic variables considered. In terms of power generation, an increase of 47.97% (0.1257 p.u.) for unit 1 is observed for the uncorrelated case whereas the corresponding increase is 22.51% (0.0590 p.u.) for unit 1 for the correlated case.

In general, the expected increase in cost is larger for the uncorrelated case as opposed to case S1.

### 5.2.3 Case S3: Stochastic power generation, system loads, fuel cost and emission coefficients

This case is similar to Case S2 but in addition the fuel cost and $NO_x$ emission coefficients are considered as stochastic variables with mean as deterministic values and standard deviation as $CV = 0.05$, 0.1 and 0.2 of their respective means.

Figs. 8(a) and 8(b) shows the best nondominated solutions out of 11 runs together with the best compromise solution for CV=0.05, 0.1 and 0.2 with k=2 for LHS without correlation and LHS with correlation, respectively. Table 8 gives the percentage change in cost, emission and power generation for best compromise solution for the three coefficients of variation considered for the two cases respectively.




(a) LHS without correlation (b) LHS with correlation

*Fig. 8: Best nondominated solutions and best compromise solutions for Case S3 with k=1.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | -0.08 | 0.40 | -14.91 | 4.84 | 8.17 | -0.62 | -3.81 | 0.21 |
| 0.1 | 0.31 | 0.07 | -7.48 | -3.70 | 2.63 | -4.33 | 1.25 | 9.94 |
| 0.2 | 1.30 | 0.06 | -5.60 | 15.73 | 4.60 | -11.68 | -1.06 | 4.01 |

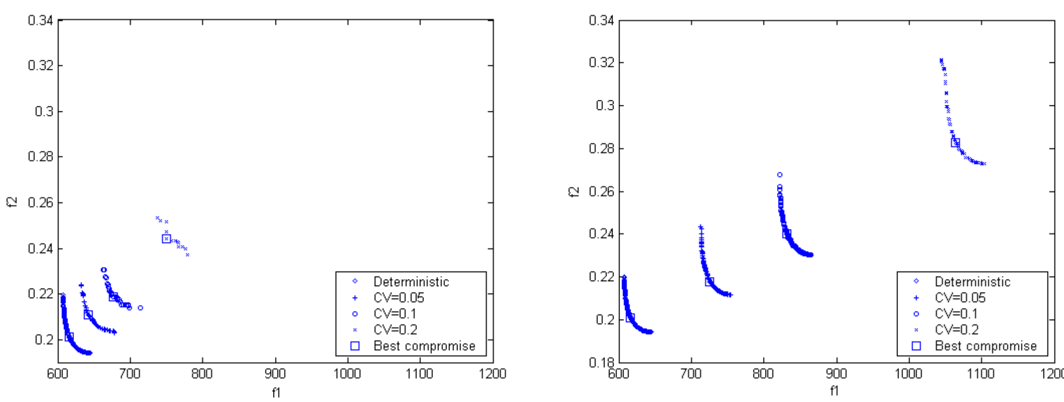
*Table 8(a): Percentage change in cost, emission and power generation for best compromise solution of Case S3 with k=1 for LHS without correlation.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.35 | -0.53 | -0.61 | 3.02 | 7.48 | -5.64 | -3.90 | 2.26 |
| 0.1 | 0.41 | 0.08 | -5.11 | -2.77 | 8.77 | -4.14 | -0.19 | 1.11 |
| 0.2 | 1.45 | 0.63 | -4.49 | -0.90 | 8.58 | -4.78 | -0.76 | 1.10 |

*Table 8(a): Percentage change in cost, emission and power generation for best compromise solution of Case S3 with k=1 for LHS with correlation.*

With three set of parameters (power generation, system loads and cost/emission coefficients) considered as stochastic variables, a much larger shift of the nondominated solutions is observed compared with previous cases with k=1. Slightly larger increase in expected cost and emission for the correlated is observed from Table 8. With 20% variation of the stochastic variables, increase of 1.30% (8.023 \$/hr) and 0.06% (0.00013 ton/hr) for cost and emission are expected for the uncorrelated case while the corresponding values for the correlated case are 1.45% (8.963 \$/hr) and 0.63% (0.00126 ton/hr) respectively. In terms of power generation, an increase of 15.73% (0.0591 p.u.) for unit 2 is observed for the uncorrelated case whereas the corresponding increase is 8.58% (0.0466 p.u.) for unit 3 for the correlated case.

Fig. 9 shows the results obtained for case S3 with k=2 and Table 9 gives the percentage changes in cost, emission and power generation for the best compromise solution.




(a) LHS without correlation (b) LHS with correlation

*Fig. 9: Best nondominated solutions and best compromise solutions for Case S3 with k=2.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.38 | -0.35 | -0.49 | 9.57 | 0.13 | -3.54 | -3.71 | 2.36 |
| 0.1 | 1.83 | -1.23 | 17.64 | 5.64 | 9.54 | -19.14 | -7.07 | 12.42 |
| 0.2 | 4.45 | 0.33 | 82.29 | 4.40 | -9.15 | -26.87 | -3.95 | 6.99 |

*Table 9(a): Percentage change in cost, emission and power generation for best compromise solution of Case S3 with k=2 for LHS without correlation.*

| CV | Cost | Emission | $P_{G1}$ | $P_{G2}$ | $P_{G3}$ | $P_{G4}$ | $P_{G5}$ | $P_{G6}$ |
|---|---|---|---|---|---|---|---|---|
| 0.05 | 0.64 | -0.69 | 8.96 | -0.04 | 6.25 | -6.95 | -2.23 | 0.60 |
| 0.1 | 0.96 | 0.50 | 7.21 | -8.97 | 4.42 | -1.37 | -2.19 | 2.86 |
| 0.2 | 4.08 | 1.55 | 7.31 | -3.91 | 9.33 | -10.60 | -0.32 | 4.49 |

*Table 9(b): Percentage change in cost, emission and power generation for best compromise solution of Case S3 with k=2 for LHS with correlation.*

An even larger shift of the nondominated solutions is observed from Fig. 9 for both cases compared to the previous plots with k=2. The same observation about the larger shift up and right for the correlated can be made again. However, cost and emission are expected to be larger in the uncorrelated case than the correlated one. The increase in cost and emission as observed from Table 9 shows that with the largest variation of CV=0.2, there are increases of 4.45% (27.447 $/hr) and 0.33% (0.00067 ton/hr) in cost and emission respectively and 4.08% (25.158 $/hr) and 1.55% (0.00312 ton/hr) in cost and emission respectively for the two cases. In terms of power generation, an increase of 82.29% (0.2157 p.u.) for unit 1 is observed for the uncorrelated case whereas the corresponding decrease is 10.60% (0.0741 p.u.) for unit 4 for the correlated case.

In general, the expected increase in cost is larger in the uncorrelated case as for Case S2. In fact, case S3 is closer to the real-world problem where most of the parameters would be expected to be uncertain.

It should be pointed out that all the changes in cost and emission are quoted on a per hour basis and consequently the overall impact on the power system operation will not be negligible. Consider for instance a 1% increase in both cost and emission, this will correspond to an increase of $ 53,995 and 17.605 ton in annual cost and annual emission respectively.

## 6. CONCLUSIONS

The stochastic multiobjective environmental/economic dispatch problem has been formulated as a chance-constrained nonlinear programming problem and solved by NSGA-II. A comparison with previous work using a modified particle swarm algorithm revealed the efficacy of the proposed method. Three cases with the following stochastic variables: (i) power generation (Case S1), (ii) power generation and system loads (Case S2), and (iii) power generation, system loads and cost and emission coefficients (Case S3) have been analyzed. Latin Hypercube Sampling with both uncorrelated and correlated variables with correlation coefficient of 0.9 has been considered. To aid the operator in selecting an operating point from the obtained set of Pareto-optimal solutions, fuzzy logic theory is applied to each objective functions to obtain a fuzzy membership function. The best non-dominated solution is the compromise solution chosen for which the normalized sum of membership function values for all objectives is highest. The best compromise solution for different coefficient of variation of the stochastic variables for the three cases considered were identified. It is found that in general, the expected increase in cost of the best compromise solution is larger if the stochastic variables are correlated for Case S1. On the other hand, the expected increase in cost is larger if the stochastic variables are uncorrelated for Case S2 and Case S3. In general, the expected increase in fuel cost is less than 5% in most scenarios. As far as the emission changes are concerned, the expected increase or decrease in emission is less than 1% in most of the scenarios with the exception of the concave front obtained.

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