An Adaptive Fuzzy Control Scheme for Dyebath pH in Exhaust Dyeing

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Abstract

This paper presents the modeling, design and simulation of an adaptive fuzzy controlled pH tracking system for an exhaust dyeing process. The proposed system is shown to track both linear and exponential command pH profiles even when the measured pH signal is corrupted by random noise, and despite the inherent transport lag in the pH sensor. The controller gains can eventually be used as a starting point for quicker tuning of the actual control system under real dyeing conditions, thus reducing the number of experimental runs and associated costs in achieving adequate pH tracking performance.

Keywords: Adaptive Fuzzy Control, pH, Exhaust Dyeing.

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INTRODUCTION

Critical factors including temperature, dye concentration and pH largely affect the shade and colour uniformity of the dyed fabric in dyeing processes. The control of these key parameters is therefore essential for improving dye exhaustion and fixation, dyeing levelness, and shade reproducibility (Gore, 1995; Shamey and Nobbs, 2000). The availability of high performance temperature controllers is widespread and these are successfully applied in exhaust dyeing for achieving specific temperature profiles. During dyeing, however, the control of pH is quite complex since it has been shown that pH varies, to different extents, with dyeing parameters such as temperature, salt concentration, the quality of the dyed fabric, the water quality and the presence of dyes and auxiliaries (Huang and Yu, 1999). In batch dyeing processes using reactive dyes onto cotton, the pH is normally required to start from a value of about 7.0 (or slightly lower) to reach a final value of around 11.0. The pH profile against time may be either linear or non-linear. Moreover, with the intrinsic non-linearity between pH and concentration, together with the time lags associated with pH sensors, conventional Proportional Integral Derivative (PID) controllers cannot satisfactorily perform the control of pH over a desired profile. Such controllers produce oscillatory responses with large tracking errors, and are typically capable of tracking the pH profile only over a relatively small pH range (Shamey and Nobbs, 1999). Besides, accurate mathematical models of the dyeing process to include the non-linear effects of temperature, salt concentration, the quality of the cotton fabric, the water quality and the presence of dyes and auxiliaries on the dyebath pH, are quite complex (Huang and Yu, 1999). On the other hand, without any accurate model at hand, experienced human operators can potentially control the dyeing operation consistently by controlling the key dyeing parameters. Fuzzy controllers provide a means for implementing an operator's expert knowledge for controlling complex processes, and have been successfully implemented in various industrial processes. Fuzzy controllers possess non-linear features and may operate independent from an accurate system model. The fuzzy controller takes control decisions based on linguistic 'If-Then' rules (Driankov *et al.*, 1995), and can be designed for implementing non-linear, time varying control characteristics required for successful pH tracking. However, thorough heuristic knowledge of the dyebath pH dynamic behaviour is essential for proper controller design, and this may require several repeated process runs. Fine-tuning the fuzzy controller parameters may also require an extensive number of iterative experimental tests on the dyeing plant. A typical test involving preparation of dyes, and chemicals can be relatively time-consuming and expensive, thus significantly increasing the cost of implementing a working control system. This paper proposes the design of an adaptive fuzzy control system to track the desired pH profiles. The fuzzy controller parameters are tuned through computer simulations, with the aim of reducing the tracking error between the output and desired pH over the process duration. The simulated controller can then be used as a starting point for optimizing the implemented control system. Such an approach largely reduces the number of preliminary experimental tests required for obtaining a satisfactorily working fuzzy controller. The proposed fuzzy controller is shown to produce adequate pH tracking with both linear and non-linear reference profiles, despite the presence pH measurement noise. Its superior performance compared with a classical PID control scheme is also demonstrated.

MODELING OF SENSOR, ACTUATORS AND DYE BATH

In this paper, control of the dye bath pH is realized by a closed loop fuzzy control system, as shown in Figure 1. For a batch dyeing process, the desired pH profile, $pH_{ref}(kT)$, typically starts from pH 7 and reaches pH 11 at the end of the process. A pH sensor/ transmitter unit gives a steady-state voltage proportional to the actual dye bath pH.

The transmitter output is sampled at regular intervals by an Analog to Digital (A/D) converter, giving a discrete-time signal, $p\hat{H}(kT)$, where k represents the sample number, and T is the sampling interval.

For each sample, the error between the reference pH and the dye bath pH is computed as

$$e(kT) = pH_{ref}(kT) - p\hat{H}(kT) \tag{1}$$
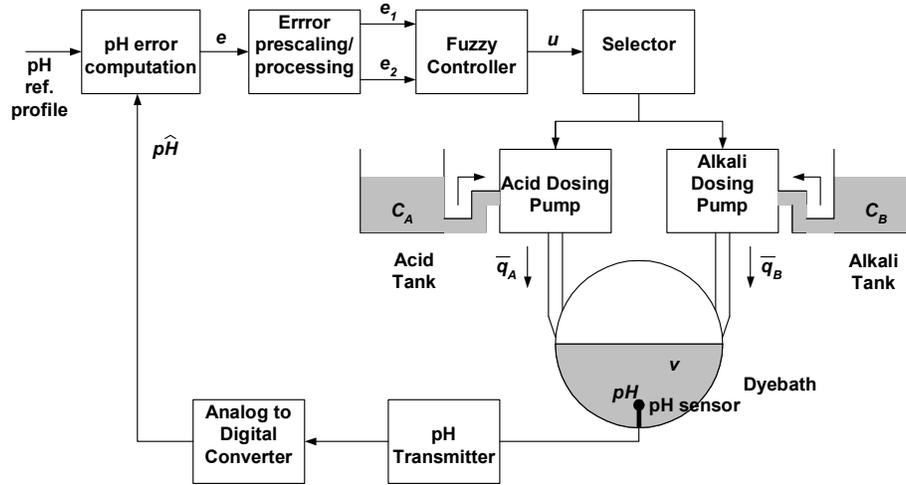
The error signal is further processed to generate two more signals, which are applied to the fuzzy controller, and computed respectively as:

$$e_1(k) = [G_e + G_i T]e(k) + e_1(k-1) \tag{2}$$

$$e_2(k) = G_{\delta e} \frac{e(k) - e(k-1)}{T} \tag{3}$$

where G_e, G_i and $G_{\delta e}$ are adjustable gains, which are used for tuning the fuzzy controller to give the desired pH tracking performance. It should be noted that $e_1(k)$

represents a weighted sum of the pH error and its time integral, while $e_2(k)$ gives an indication of the rate of change of the pH error. Depending on the sign and magnitude of $e_1(k)$ and $e_2(k)$, the fuzzy controller generates a control signal so as to dose either acid or alkali into the dyebath, with the aim of reducing the tracking error to zero.

pH reference profile
A linear profile for the reference pH is first considered and its time variation can be expressed as:

$$pH_{ref}(kT) = pH_{ref}(0)\left[1 + \left(\frac{pH_{ref}(T_f) - pH_{ref}(0)}{pH_{ref}(0)T_f}\right)kT\right] \qquad (4)$$

where T_f is the process terminal time.

pH sensor/ Transmitter and A/D Converter
pH sensors have an inherent transport lag typically in the range 10 to 50 s. To obtain the dynamic characteristics of the sensor, the dyes, and required auxiliaries were all added to the bath at an initial set pH of 7. The amount of alkali required to bring the pH to 11 was pre-calculated and added to the bath at once so as to simulate a step input, and the bath is stirred using the liquor circulating system. The experiment was performed using a *Roaches Jet M10 Soft Flow* dyeing machine, retrofitted with a *Liquisis*[TM] pH sensor / transmitter. The transmitter output was recorded and re-scaled to the range 0 to 11, to simulate zero initial conditions. The corresponding response is shown in Figure 2, from which the transfer function of the sensor in the Laplace domain can be approximated. The temperature of the bath was automatically maintained at 60^0C by an inbuilt closed loop temperature controller in the dyeing machine. The dynamic response is slightly under-damped and gives a characteristic time delay, T_d, of about 14.8s
Assuming that the response to be essentially due to the sensor/ transmitter unit, the *s*-domain transfer function is approximated by the following second order system:

$$\frac{p\hat{H}(s)}{pH(s)} = \frac{e^{-T_d s}\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (5),$$

where the natural angular frequency, $\omega_n = 0.337$ rad./s, and the damping ratio, $\xi = 0.83$.
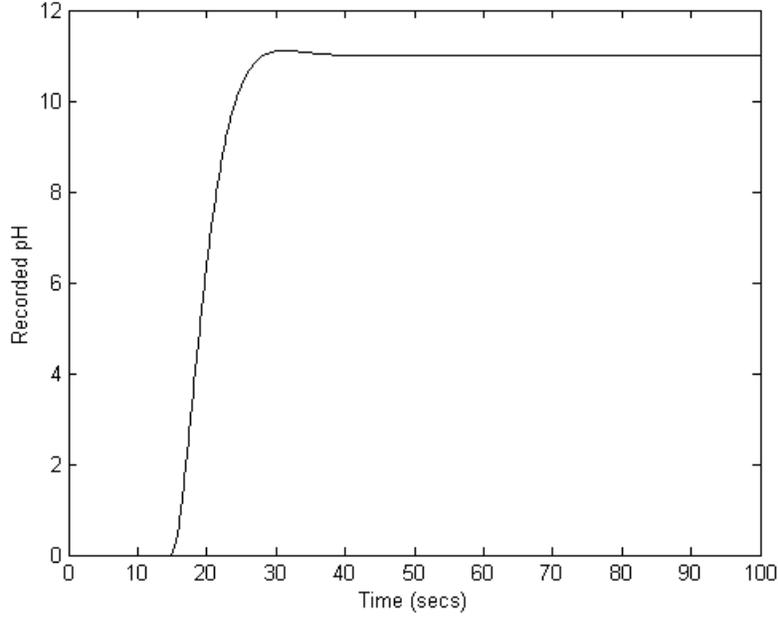
Figure 2 Step response of the pH sensor

The A/D converter samples the measured pH and holds the output value constant until the next sampling instant, and as such, it is modeled as a zero order hold block (Dorf and Bishop, 1998) with a transfer function given by

$$C(s) = \left(\frac{1 - e^{-Ts}}{s} \right) \qquad (6)$$

Dosing pumps and dye bath
The control signal generated by the fuzzy controller, *u(kT)*, determines the amount by which the process pH needs to be increased or decreased, so as to match the reference profile. For a positive control signal, the alkali-dosing pump is turned on, while for negative control signal, the acid-dosing pump needs to be activated. It is assumed that there is homogeneous mixing in the tank.

The flow rate of the acid and alkali dosing pumps are preset to Q_A and Q_B litres/s, respectively. The control signal from the fuzzy controller represents the time duration for which the relevant dosing pump needs to be activated over one sampling period. A typical time profile of the flow rate from either pump is shown in Figure 3. With $T << T_f$, the local average value taken over one sampling interval of the acid and alkali flow rates, respectively, are given by

$$\overline{q}_A(t) = \frac{u(kT)}{T} Q_A \tag{7}$$

and $\quad \overline{q}_B(t) = \frac{u(kT)}{T} Q_B \tag{8}$
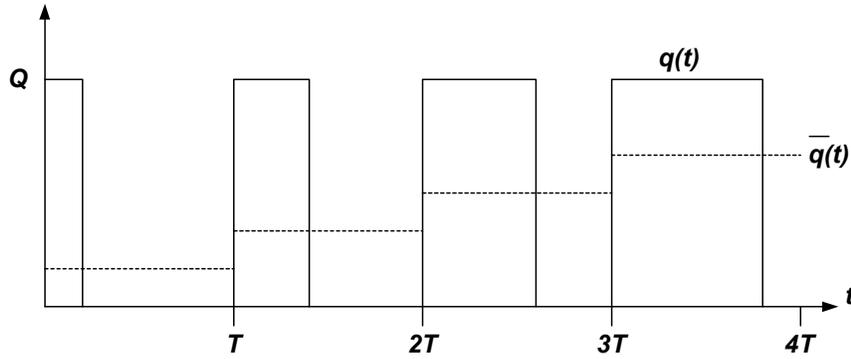

Figure 3 Instantaneous and local average flow rates from a dosing pump

To model the dyebath, the mixing dynamics are first considered as if there were no reactions, and then a non-linear titration model is incorporated to account for the pH change due to acid-base reaction (Astrom and Wittenmark, 1995). It is assumed that the dyebath temperature is kept constant for the process duration by an appropriate temperature regulation system. The rate of increase of concentration of alkali in the dye bath can be expressed as

$$\frac{dx_B}{dt} = \frac{\overline{q}_B(t) C_B}{v(t)} \tag{9}$$

where C_B is the alkali concentration at the inlet pipe of the dyeing machine, and $v(t)$ is the instantaneous volume of the dye bath, which is given by

$$v(t) = V_0 + \int \left[\overline{q}_A(t) + \overline{q}_B(t) \right] dt \tag{10}$$

V_0 is the initial volume of solution in the bath. From (9) and (10), the alkali concentration in the dye bath at time t is therefore

$$x_B = \int \frac{\bar{q}_B(t)C_B}{V_0 + \int \left[\bar{q}_A(t) + \bar{q}_B(t)\right]dt} dt \tag{11}$$

Likewise, considering the addition of acid at a flow rate $\bar{q}_A(t)$, the acid concentration in the dye bath at time t, is given by

$$x_A = \int \frac{\bar{q}_A(t)C_A}{V_0 + \int \left[\bar{q}_A(t) + \bar{q}_B(t)\right]dt} dt \tag{12}$$

Since the number of positive and negative ions in the dye bath should be equal, the concentration of hydroxyl ions, *[OH]*, and of hydrogen ions, *[H+]* are related by

$$x_A + [OH^-] = x_B + [H^+] \tag{13}$$

and $\quad [OH^-] \cdot [H^+] = 10^{-14} \tag{14}$

From Eq. (13) and (14), the dye bath pH can be found as

$$pH = -\log_{10}[H^+] = -\log_{10}\left[\sqrt{\frac{(x_B - x_A)^2}{4} + 10^{-14}} - \frac{(x_B - x_A)}{2}\right] \tag{15}$$

A block diagram of the overall closed loop control system, based on Eqs. (1) to (15) is shown in Figure 4.
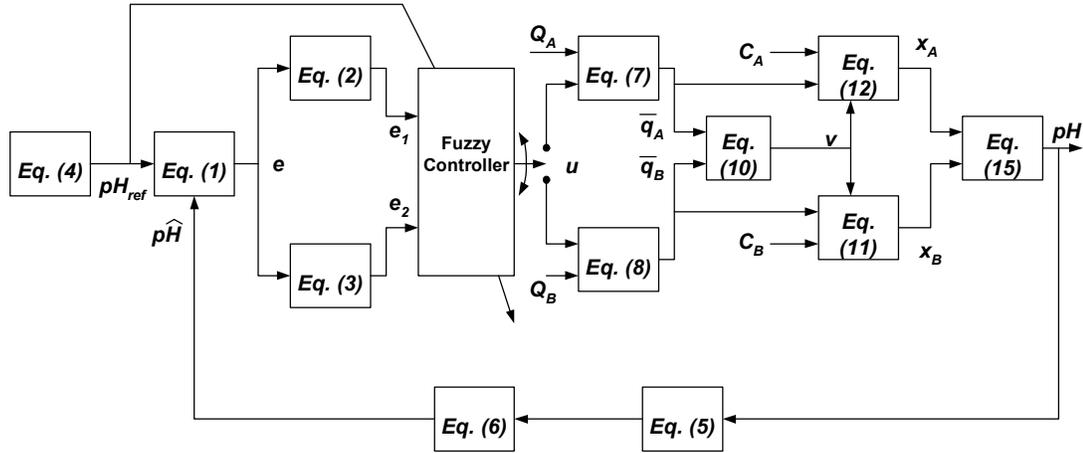
Figure 4 Block diagram of fuzzy controlled pH control model

ADAPTIVE FUZZY CONTROLLER FOR PH TRACKING

The pH control process is characterized by the highly non-linear dyebath model and the time delay associated with the dynamic response of the pH sensor. The proposed fuzzy controller is well suited for achieving tracking of the reference pH profile as it can implement non-linear and time-varying control actions. The inputs to the fuzzy controller are the pH error-related variables, e_1 and e_2, as described by Eqs. (2) and (3) respectively.

The control decisions are taken, based on linguistic rules as would be used by an experienced operator, for controlling the process. The output of the fuzzy controller, $u(kT)$ represents the time during which either the acid or the alkali dosing pump needs to be activated to make the output pH track the desired pH profile. The time during which the pump is on allows computation of the local average value of the flow rate of each dosing pump, according to Eqs. (7) and (8), respectively.

The inputs to the fuzzy controller, e_1 and e_2, are each characterized by membership functions, μ_{e1} and μ_{e2}, respectively, as shown in Figure 5.Linguistic-numeric values '–4', '-3', '-2', '-1', '0', '1', '2', '3' and '4' have been used to represent the linguistic terms for e_1 and e_2. Thus, from linguistic-numeric values '-4' to '4', e_1 (and e_2) are progressively changing from very large negative values to very large positive values, while the term '0' represents the set of real values which are very close to zero.
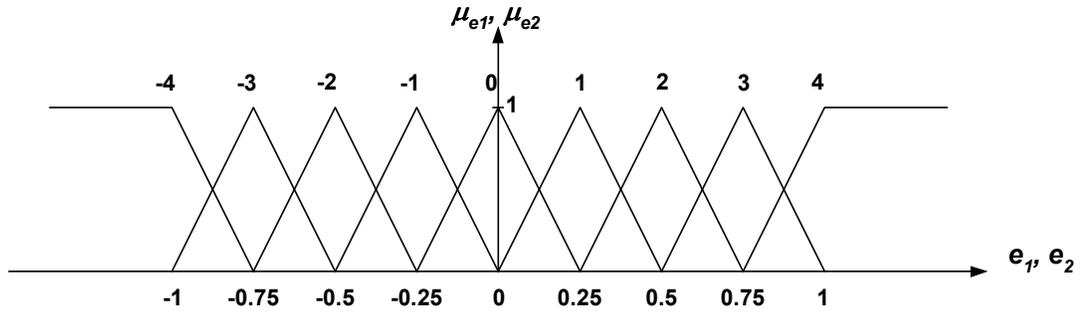
Figure 5 Membership functions for e_1 and e_2

The rule base for the fuzzy controller is shown in Table 1. In this paper, the *min* operator (Driankov *et al.* 1997) is used for evaluating the premise, $\mu_{p,q}$, for a given active rule, so that

$$\mu_{p,q}(e_1, e_2) = \min\{\mu_{e1,i}(e_1), \mu_{e2,j}(e_2)\} \tag{16},$$

where $i, j \in [-4, 4]$ are the linguistic-numeric values of e_1 and e_2, respectively, and the indices $p, q \in [1,9]$ locate the rule number by row- and column-wise in Table 1.

The result of premise quantification for each firing rule next determines how the output fuzzy sets of the fuzzy controller are modified to reflect the result recommended from the relevant cell in the rule-base given in Table 1.

Table 1 Rule base for proposed fuzzy controller

u_{out}		e_2								
		-4	-3	-2	-1	0	1	2	3	4
	-4	-4	-4	-4	-4	-4	-3	-2	-1	0
	-3	-4	-4	-4	-4	-3	-2	-1	0	1
	-2	-4	-4	-4	-3	-2	-1	0	1	2
	-1	-4	-4	-3	-2	-1	0	1	2	3

e_1	0	-2	-2	-2	-1	0	1	2	3	4
	1	-2	-2	-1	0	1	2	3	4	4
	2	-2	-1	0	1	2	3	4	4	4
	3	-1	0	1	2	3	4	4	4	3
	4	2	2	2	3	4	4	4	3	2

Figure 6 shows the proposed consequent membership functions for the fuzzy inference system. The index at the tip of each triangle represents the linguistic-numeric value for each output fuzzy set. Each membership function represents the consequent reached by each cell of the rule-base. In effect, the fuzzy inference stage generates one or more new implied output fuzzy sets, depending on the number of rules that fired at a particular sampling instant. The membership function formed by an implied fuzzy set arising from rule *(p, q)* in the rule-base is computed as

$$\mu_{uout,p,q}(u_{out}) = \min\{\mu_{p,q}(e_1,e_2), \mu_{uout,l}(u_{out})\} \qquad (17),$$

where $\mu_{p,q}(e_1,e_2)$ is found from Eq. (16), $\mu_{uout,l}(u_{out})$ is the consequent membership function of the firing rule in Figure 6, and l is the linguistic-numeric value of u_{out}, with $l \in [-4,4]$. This procedure is applied to all other rules that fire.
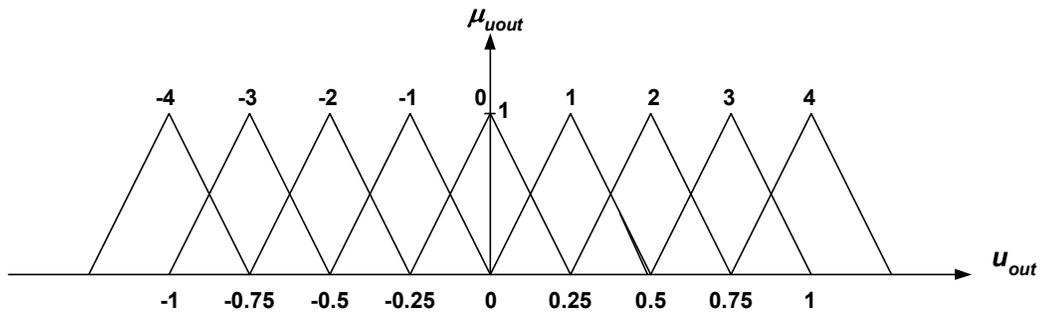

Figure 6 Consequent output membership functions and implied fuzzy sets

Once the implied fuzzy sets are formed, a real valued output needs to be computed so as to apply the required control action to the dosing pumps. In this paper, Center of Gravity defuzzification method is used to compute u_{out}, so that

$$u_{out} = \frac{\sum_{m=1}^{r} c_m \int \mu_{uout,p,q}(u_{out}) du_{out}}{\int \mu_{uout,p,q}(u_{out}) du_{out}} \tag{18},$$

where r is the total number of rules that fire at a given instant, c_m is the center of the consequent membership function for the mth rule, and the integral terms represent the area under each implied fuzzy set.

The final output from the fuzzy controller, $u(kT)$ represents the time interval during which the acid or alkali pump needs to be turned on so that the dyebath pH tracks the reference value during the kth sampling interval. The fuzzy controller output is computed as

$$u(kT) = G_0 u_{out}(kT) \tag{19},$$

where G_0 is the output gain of the fuzzy controller.
From Eq. (15), the change in pH is most sensitive to addition of acid or alkali at pH 7. As the pH progresses to its terminal value, this sensitivity is largely reduced, so that a much larger volume of alkali needs to be added to the dyebath to achieve a given change in pH. Hence the output gain of the fuzzy controller should be adapted to the current value of the pH reference, as shown in Figure 4. In this paper, G_0 is made to vary non-linearly with the pH so that

$$G_0 = 10^{[apH_{ref}(kT)-b]} \tag{20},$$

where the parameters a and b are tuned to provide the desired pH tracking performance. These constants are adjusted so as to set the minimum and maximum volumes of acid or alkali supplied by the dosing pumps. Hence, as the reference pH increases, larger volumes are dosed so as to achieve a given change in pH with minimal tacking error.

SIMULATION TESTS AND RESULTS

The model of fuzzy controlled dyebath pH system is implemented using Matlab™ and Simulink™ software (Mathworks, 2000) so as to evaluate the controller performance for tracking different reference pH profiles. Table 2 summarizes the values of the process and the controller parameters used for the tests. The input and output gain parameters of the fuzzy controller were tuned iteratively over a large number of simulation runs, until the desired pH tracking performance was obtained. Once tuned, these values were kept constant for all reference profiles. In practice the output signal from the pH transmitter is affected by electrical noise, which can be represented as a normally distributed random noise signal, adding to the incoming dyebath pH signal. Measurement noise with a variance equivalent to 0.001 pH is used in the simulations.

Linear reference profile
The reference pH profile defined by Eq. (4) is applied, along with the parameters defined in Table 2. Figure 7 shows the corresponding dynamic responses of both the reference and output pH. It is assumed that initially, the pH sensor output has stabilized to the dyebath pH, as would be the case in a typical setup. Figure 8 shows the error between the measured and actual pH, the tracking error and the time Integral of the Square of the tracking Error (ISE), used as a performance index. It is observed that the fuzzy controller can achieve quite robust tracking of the pH, even in the presence of noise. The maximum instantaneous tracking error magnitude is less than 0.1 pH and the ISE at the end of the process is less than 3.5 pH²s. The fuzzy controller was replaced by a PID controller, where the gains were adjusted for pH tracking at an operating point of 8.7 pH. The corresponding pH and ISE responses are shown in Figures 9 and 10, respectively. In this case the dyebath pH is characterized by high overshoots at the start of the process, and a steady-state error 0.7 pH at 2400 s, with a corresponding ISE exceeding 1400 pH²s.

Table 2 Process and Controller parameters

Parameter	Value
Process terminal time, T_f	2400 s
Initial dyebath pH: *pH(0)*	7
Desired terminal pH: *pH$_{ref}$(T$_f$)*	11
Sampling time period: *T*	20 s
Fuzzy Controller input gains: G_e, G_i, $G_{\delta e}$	1.7, 6e-4, 0.47
Dosing pumps preset peak flow rate: Q_A, Q_B	0.03 l /s

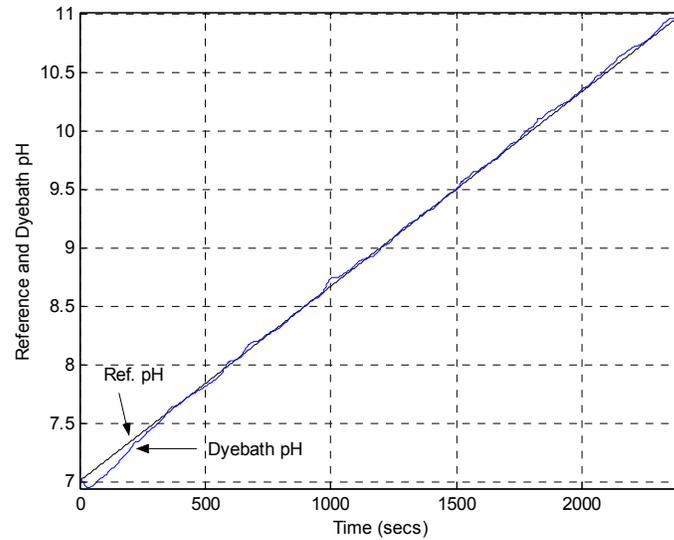| Acid concentration in inlet pipe: C_A	2 mol/l
Base concentration in inlet pipe: C_B	2 mol/l
Initial volume of solution in dyebath: V_0	20 l
Fuzzy controller output gain parameters: *a, b*	1.057, 15.113

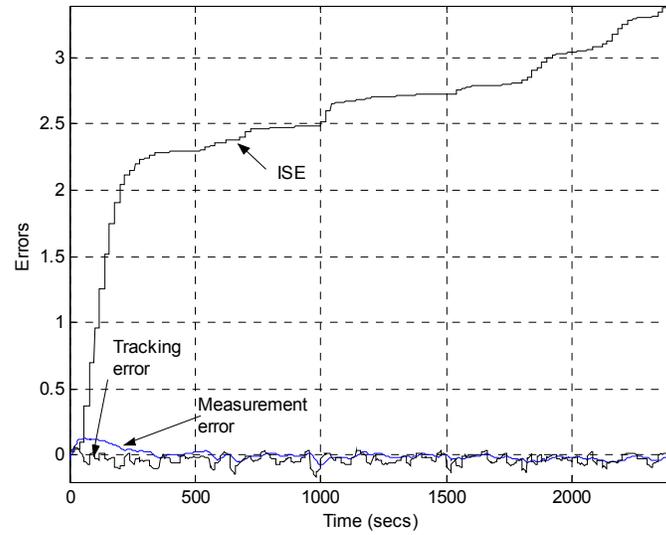*Figure 7 Dyebath pH response with a linear reference profile*

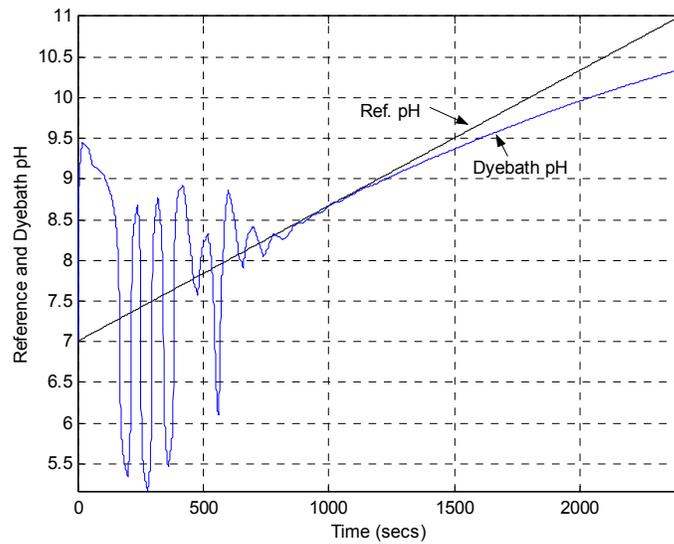*Figure 8 pH Error Performances For Linear Profile*

Figure 9 Dyebath pH response with a linear reference profile with a PID Controller
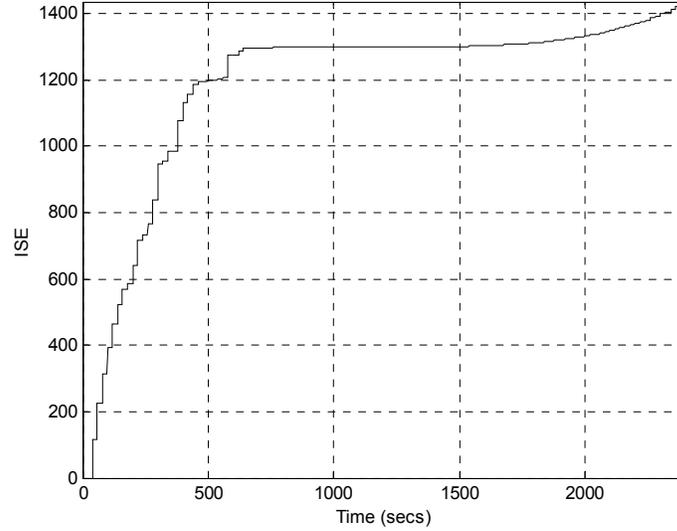
Figure 10 ISE Performance with a linear reference profile with a PID Controller

Exponential reference profile

To demonstrate the controller's ability to track reference profiles with varying slopes, the reference pH was changed to an exponential type of the form

$$pH_{ref}(kT) = \left[\exp\left(\frac{kT}{h} \right) - 1 \right] + pH_{ref}(0) \qquad (21),$$

where the parameter *h* is computed from the final and initial pH values as

$$h = \frac{T_f}{pH_{ref}(T_f) - pH_{ref}(0) + 1} \qquad (22)$$

The same fuzzy controller input and output parameters, as well as the pH measurement noise levels were maintained as in the linear pH profile. Figures 11 and 12 show the corresponding pH responses and error performances, respectively, for the exponential pH profile. The pH tracking error performances are of the same orders of magnitude as with the linear reference profile, with a maximum ISE and

tracking error of 1.5 and 0.16, respectively. The time variation of the fuzzy controller output gain for the linear profile (Case I) and exponential profile (Case II) are shown in Figure 13. For both cases, the initially low controller output gain produces very little dosing so as to avoid large overshoots in the pH response. The gains increase to the same final value at a pH of 11, so that a larger dosing volume is provided to achieve a given pH change.
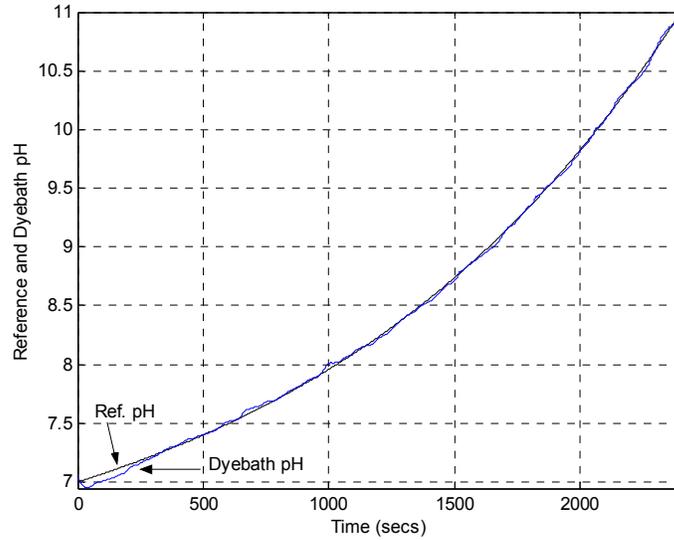

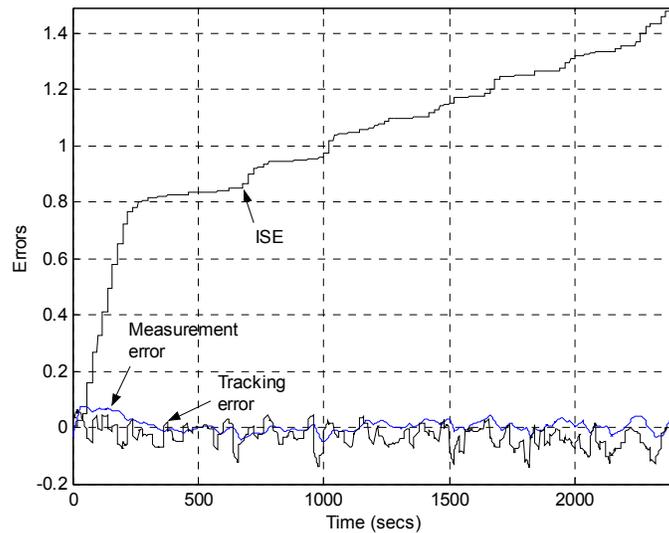*Figure 11 Dyebath pH response with an exponential reference profile*
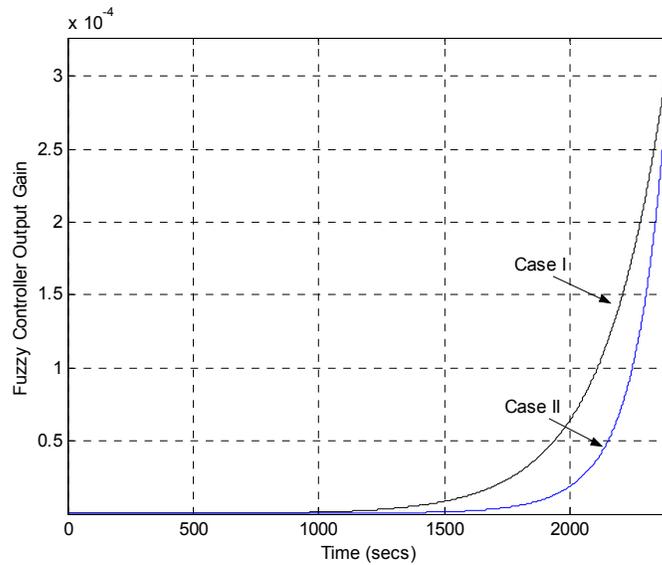

Figure 12 Error Performances For Exponential Profile

Figure 13 Fuzzy Controller Output Gain For Linear And Exponential Profiles

CONCLUSION

The design and simulation of an adaptive fuzzy pH control for exhaust dyeing has been presented. Mathematical models were derived for the various stages forming the control system, including the pH sensor, analog to digital converter, fuzzy controller, dosing pumps and dyebath. The proposed adaptive fuzzy controller has a variable output gain, which is a non-linear function of the reference pH. The performance of the controller has been evaluated by applying both linear and exponential pH reference profiles, and with the pH transmitter output affected by random noise signals. The proposed controller is shown to closely track command pH profiles even with measurement errors due to the sensor's characteristic delay and electrical noise in the system. The fuzzy controller is also shown to perform significantly better than a conventional PID controller. The simulated controller gains and process parameters can be used as a good starting point for further fine-tuning of the real control system. Such an approach will considerably reduce the number of experiments and required to obtain a satisfactorily working pH controller, with consequent savings on time and chemicals.

ACKNOWLEDGEMENT

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