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MULTI-PHASE SHIFT KEYING MODULATION USING GENERALISED ARRAY CODES

by

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ABSTRACT

In this paper a Block Coded Modulation (BCM) scheme employing Generalised Array Codes (GAC) and M-ary Phase Shift Keying (MPSK) modulation technique is proposed. The GAC are encoded in the usual manner. The encoded data are then used to modulate the signals varying in phase. A maximum likelihood trellis decoding technique is used to demodulate/decode the received symbols. Simulation study results show that BCM employing GAC codes and QPSK modulation in AWGN channel yield 5dB or more coding gain over the corresponding uncoded case. Further as M increases the coding gain decreases over the corresponding uncoded case as expected.

Keywords: Coding and modulation, trellis decoding.

INTRODUCTION

Ungerboeck (1981) proposed a Combined Coding and Modulation (CCM) technique which improves the performance of synchronous data links without sacrificing the data rate or requiring more bandwidth. This scheme, as proposed by Ungerboeck, consists of encoding k information bits convolutionally into k+1 bits thus giving a code rate R = $\frac{k}{k+1}$. This rate limits the choice of signals to be transmitted. The k+1 bits in the code is used to select a point from the 2^n (n = k+1) multilevel (M-ary QASK) or multiphase MPSK signals according to a certain rule called mapping by set partitioning. The CCM technique is designed in order to achieve large free Euclidean distance d_{free}. Hence to improve the performance of the CCM codes it is important to maximise the Euclidean distance. The coding gain is a function of the amount of memory introduced by the encoder and the positioning of the signal space. Padovani & Wolf (1986) introduced a new scheme that combines FSK(frequency shift keying) and PSK (phase shift keying) modulation schemes and makes use of the trellis coding and Viterbi decoding (Viterbi, 1971) to improve the error performance over uncoded modulation. The transmitted signals consisted of Binary FSK and M-ary PSK resulting in a multidimensional signal space. Gercho & Lawrence (1986) have shown that multidimensional signal constellation (MPSK) can give further coding gain in CCM scheme over those employing 2-D MPSK. Until recently CCM have been associated with convolutional codes due to their trellis structure and their Maximum Likelihood Decoding (MLD) employing Viterbi algorithm. However, Huber (1989) and Kasami *et al.* (1991) have investigated the possibility of employing CCM with block codes. More recently Kaya & Honary (1993 a, 1993 b) and Kaya (1993) have investigated the application of CCM involving phase as well as frequency/phase modulation techniques with array codes. In this paper a combined coding and modulation technique employing Generalized Array Codes and MPSK modulation is proposed.

GENERALIZED ARRAY CODES

Array codes, such as product codes and concatenated codes are constructed by combining other codes (Kaya, 1993). Simple Row and Column (RAC) array codes may be square or rectangular and have the parameters (n,k,d_{min}=4), where n is the number of bits in the code, k is the number of information bits and d_{min} is the code's Hamming distance. These codes although flexible to design and simple to decode have a lower value of k than other linear block codes of the same size and Hamming distance. Honary *et al.* (1993, 1995) have proposed another block array code called Generalised Array Code (GAC) which is based on the augmentation of the RAC array code by superimposing repetition codes on parity row and/or columns.

GAC code construction

A generalised (n,k,d_{min}) array code is an array code in which the columns and rows subcodes may have different numbers of information and parity check symbols; the code length $n = n_1 n_2$ and the total length of the information digits

$$k = k_1 + k_2 + k_3 + \ldots\ldots\ldots + k_{n_2} \tag{1}$$

where n_1 and n_2 represent the number of rows and columns respectively and k_p is the number of information digits in the pth row. The procedures for designing a linear generalised array code (n_0,k_0,d_0) (Honary *et al.* 1993) which can also be described as an array representation of coset codes introduced by Forney (1988 a and 1988 b) are:

(i) Design a binary array code C_1 (n,k,d_{min}) as shown in Fig. 1a., with single parity check rows and columns and $R_1 = (n_2,k_2,d_2)$ row codes, where $d_2 = \left\lfloor \frac{d_0}{2} \right\rfloor$ where $\lfloor x \rfloor$ denotes the nearest greatest integer. If n_0 is a prime number we choose $n = n_0+1$,

(ii) Design a binary $n_1 n_2$ product code $C_2 = |PA|$, as shown in Fig. 1b. where P is a binary $k_2 n_1$, array with only parity check elements; A is a binary $(n_2\text{-}k_2)n_1$, matrix where the first row consists of only $k' = n_2\text{-}k_2$ information digits and all columns are repetition codes,

(iii) If there is information digits left over design a third binary product code C_3 as in Fig. 1c., Where $B=(n_1,1,d_0)$ is a repetition row code with k_0th information digit.

(iv) add the designed coded as follows:

$$C = C_1 \oplus C_2 \oplus C_3 \tag{2}$$

where $\oplus$ is a modulo-2 addition.

(v) If $n=n_0+1$, delete the symbol which is located in the n_1th row and n_2th column.

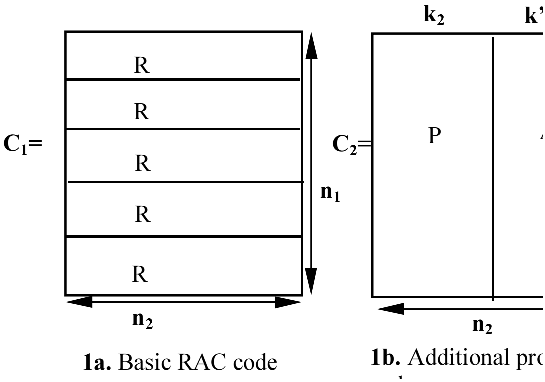


Fig. 1. Code stru

The procedures for the design of the (8,4,4) GAC code are:

(i) Design the basic (8,3,4) array code C_1 with single parity checks (4,3,2) columns and (2,1,2) repetition row codes.

$$C_1 = \begin{bmatrix} x_1 & p_1 \\ x_2 & p_2 \\ x_3 & p_3 \\ p_4 & p_4 \end{bmatrix}$$

(ii) Design an additional array code $C_2 = |PA|$ with the structure

$$C_2 = \begin{bmatrix} 0 & x_4 \\ 0 & x_4 \\ 0 & x_4 \\ 0 & x_4 \end{bmatrix}$$

where x_4 is an information digit and P is an all zero column.

(iii) Since all information digits have been used, there is no additional array code C_3.

(iv) Add the two codes C_1 and C_2 using modulo-2 addition to obtain C as

$$C = C_1 \oplus C_2 = \begin{bmatrix} x_1 & x_4 \oplus p_1 \\ x_2 & x_4 \oplus p_2 \\ x_3 & x_4 \oplus p_3 \\ p_4 & x_4 \oplus p_4 \end{bmatrix}$$

Since $n_0 = n$, there is no need to delete the parity check symbol on row 4 and column 2. This is a non systematic code and has a weight distribution of $w_0 = 1$, $w_4 = 14$ and $w_8 = 1.$, where w_i is a weight of i. This weight distribution is similar to that of the (8,4,4) RM code (Peterson & Weldon, 1975; Lin & Costello, 1993). Deleting the symbol at row 4 and column 2 gives the (7,4,3) Hamming code:

$$C = \begin{bmatrix} x_1 & x_4 \oplus p_1 \\ x_2 & x_4 \oplus p_2 \\ x_3 & x_4 \oplus p_3 \\ p_4 & \end{bmatrix}$$

Design of (15,7,5) code

(i) Design the basic (15,4,6) product code C_1:

$$C_1 = \begin{bmatrix} x_1 & x_2 & p_1 & p_2 & p_3 \\ x_3 & x_4 & p_4 & p_5 & p_6 \\ p_7 & p_8 & p_9 & p_{10} & p_{11} \end{bmatrix}$$

where x_i i=1,2,..,4 represents information digits and p_j j=4,5...,11 represents parity check symbols,

(ii) Design two additional product codes C_2 and C_3 with the structures,

$$C_2 = \begin{bmatrix} 0 & 0 & p_{12} & x_5 & x_6 \\ 0 & 0 & p_{12} & x_5 & x_6 \\ 0 & 0 & p_{12} & x_5 & x_6 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ x_7 & x_7 & x_7 & x_7 & x_7 \end{bmatrix}$$

where x_i i=5,6,7 are information bits and $p_{12} = x_5 \oplus x_6$.

(iii) Add C_1, C_2 and C_3 using modulo 2 to obtain C:

$$C = C_1 \oplus C_2 \oplus C_3 = \begin{bmatrix} x_1 & x_2 & p_1 \oplus p_1 \\ x_3 & x_4 & p_4 \oplus p_1 \\ p_7 \oplus x_7 & p_8 \oplus x_8 & x_7 \oplus p_9 \oplus \end{bmatrix}$$

the designed code has the following parameters: (n_0=15, k_0=7, d_0=5)

TRELLIS DECODING OF GAC CODES

Trellis decoding of linear block codes has been under investigation since 1974 (Bahl *et al.* 1974; Wolf, 1978). Later, a number of soft decision maximum likelihood decoding algorithms for block codes were proposed (Conway & Stone, 1978; Kasami *et al.* 1993; Be'ery & Synders, 1986). Forney (1988 b) and Forney & Trott (1993) have stimulated interest in low complexity trellis decoding of block codes for both practical and theoretical reasons. Recently Honary *et al.* (1995) have proposed low complexity trellis design of a wide range of block codes. This trellis design procedure is described in the next section.

Trellis design procedure of Generalized Array codes

A Generalized Array Code can be represented as a set of row subcodes $(n_2,k_1,d_1),(n_2,k_2,d_2).......(n_2,k_{n1},d_{n1})$ where n_2 is the number of columns in the code and $k_1,k_2,...k_{n1}$ are the number of information bits in rows $1,2,3,...,n_1$. For each such code the generator matrices are $G_1,G_2,....,G_{n2}$ respectively.

The trellis design procedure (Honary *et al.* 1993; Honary *et al.* 1995) is as follows:

(i) Choose the trellis depth N_c and the number of states N_s as

$$N_c = n_1 + 1 \tag{3}$$

$$N_s = 2^{\max k_p} \tag{4}$$

(ii) Identify each state at depth p by a $\{k_p\}$-tuple binary vector $S_p(A)$ where $S_p(A)$ is given by

$S_p(A) = S_p\left(a_1, a_2, ..., a_j, ..., a_{\max\{k_p\}}\right)$ where $a_j = 0,1$

(iii) The trellis branches start at depth p=0 and ends at depth $p=n_2$; these are labelled as $S_0(00...0)$ and $S_{n2}(00...0)$ respectively

(iv) The trellis branches at depth p are labelled X_p/C_p where X_p represents the k_p-tuple binary vectors of information digits for the pth row and C_p corresponds to the encoded codewords in the pth row and is obtained from:

$$C_p = X_p^1 G_p + C_p^2 \tag{5}$$

where G_p is the generator matrix for the pth row code and X_p^1 and C_p^2 are codewords from the pth rows of the C_1 and C_2 codes respectively

(v) There are 2^{k_p} branches starting from each state $S_p(A)$ at depth p,(<Nc) each branch is connected with state $S_{p+1}(A)$ at depth p+1, which is defined as follows:

$$S_{p+1}(A_j) = S_p(A_i) + X_p \tag{6}$$

(vi) If a second additional code, C_3, is used for a code design, at the final depth all states must be connected to the final state S_n(00....0) with two parallel branches; the labels of these branches complement to each other.

There are

$$N_o = \prod_{p=1}^{n_2} 2^{k_p} \tag{7}$$

distinct paths through this trellis diagram and each path corresponds to a unique codeword from the code.

Trellis diagrams of the (8,4,4) GAC and (7,4,3) hamming codes

Following the technique outlined above, the trellis diagram of the (8,4,4) code will have N_c=4+1=5, and N_s=2^2=4.

We identify the states by a 2-tuple binary vectors and at depth p=0 and p=5 the trellis has only one state, namely S_0(00) and S_4(00), respectively. The trellis diagram of the (8,4,4) GAC code is given in Fig. 2. The trellis diagram of the (7,4,3) Hamming code (Fig. 3) is similar to the trellis of the (8,4,4) and differs only in the number of digits being used for labelling the branches at the final depth.

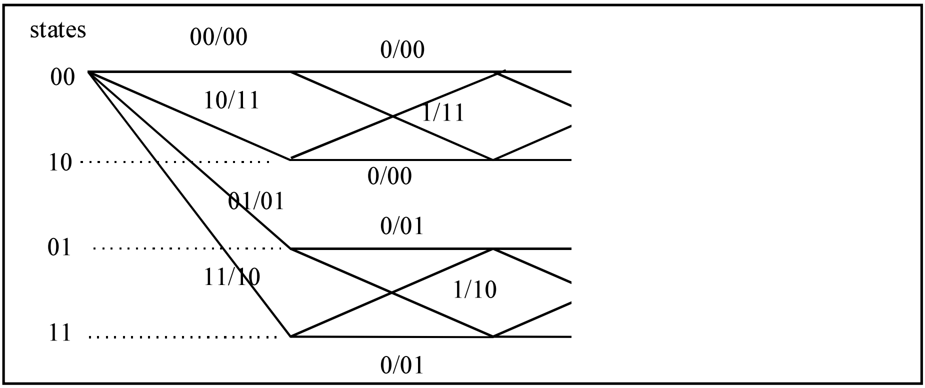


Fig. 2. Trellis structure of the (8,4,4) GAC code

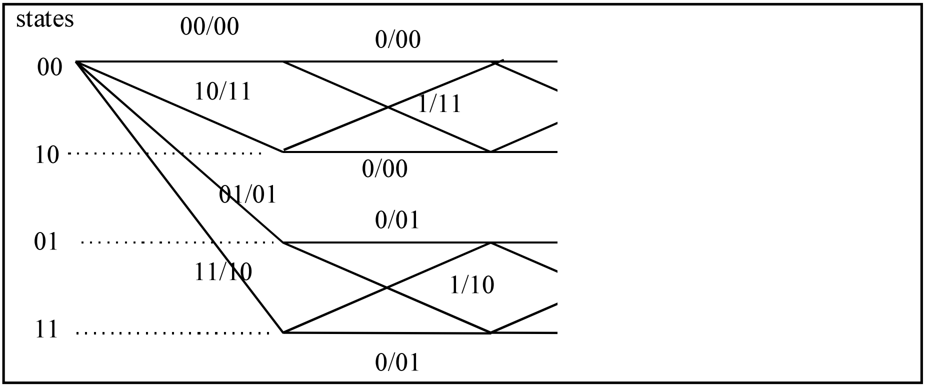


Fig. 3. Trellis structure of the (7,4,3) Hamming code

Trellis structure of the (9,6,3) GAC code

Following the same procedures we obtain the trellis structure of the (9,6,3) GAC code shown in Fig. 4.

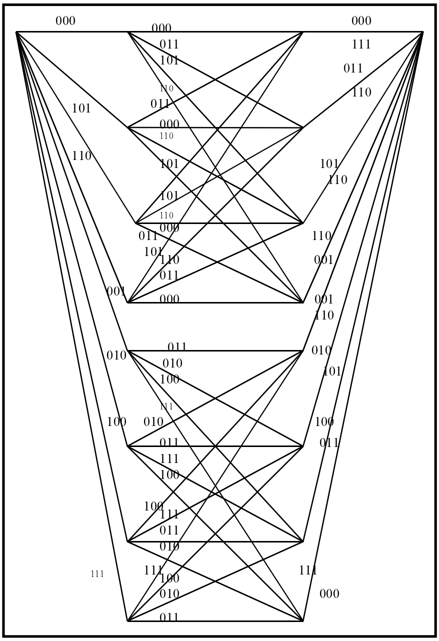


Fig. 4. Trellis diagram of the (9,6,3) GAC code

GAC IN BCM MPSK SCHEMES

In BCM employing MPSK modulation scheme and GAC codes, the encoded data sequences are used to modulate signals varying in phase. The information bits are encoded in the usual manner as with linear block codes encoder (Honary *et al*.1995). Each row of the block code is transmitted as a symbol. The total number of channel symbols per codeword is

$$N_{symbol} = n_1 \tag{8}$$

where n_1 is the number of rows in the code, and the required signal set M is given by

$$M = 2^{\max(k_p)} \tag{9}$$

For example for the (8,4,4) RM code $N_{symbol} = n_1 = 4$ and $M = 2^{\max(k_p)} = 2^{(2)} = 4$, that is, QPSK and similarly for the (16,5,8) RM code the $N_{symbol} = n_1 = 4$ and M = 8, that is, 8PSK is used. Thus the transmission of a complete codeword of the block code in a MPSK CCM scheme requires n_1 channel symbols. The row subcode determines the size of the signalling scheme to be used. Each row subcode is mapped to one of the MPSK signals, for example, if k_p=2, the CCM scheme employs a QPSK modulator and if k_p =3 an 8PSK modulator is employed.

(8,4,4) and (16,5,8) GACs in MPSK modulation schemes

The application of GAC codes in BCM scheme proposed is similar to that of RAC in similar scheme. Consider the (8,4,4) GAC code given by

$$C_1 = \begin{bmatrix} x_1 & x_4 \oplus p_1 \\ x_2 & x_4 \oplus p_2 \\ x_3 & x_4 \oplus p_3 \\ p_4 & x_4 \oplus p_4 \end{bmatrix}$$

In BCM MPSK each row is transmitted as a symbol. The uncoded bits, GAC coded bits, and mapping for the (8,4,4) GAC code are shown in Table 1.

Table 1. Uncoded bits, GAC coded bits and mappings for QPSK transmission for the (8,4,4) GAC code

Uncoded bits	**GAC coded**	**Mappings**
00	00	S_1
01	01	S_2
10	11	S_3
11	10	S_4

There are only four possible sequences for both coded and uncoded cases. Hence the GAC encoded subcode words are transmitted employing QPSK, which is also the signalling scheme used in the uncoded case. The mapping of the uncoded data bits and encoded sequences to the QPSK are shown in Fig. 5 on a two dimensional space.

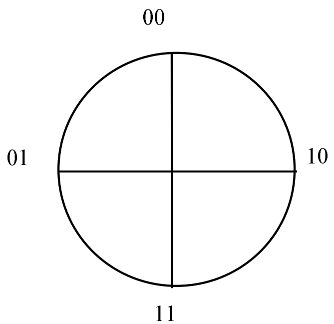


Fig. 5. Matching of codewords and signals points for the (8,4,4) GAC code (a) Uncoded signal set (b) Coded signal set and mapping

The trellis decoding of block codes in BCM MPSK schemes is similar to the process of demodulation and decoding separately. The trellis structure of the (8,4,4) GAC code employing QPSK scheme is given in Fig. 6.

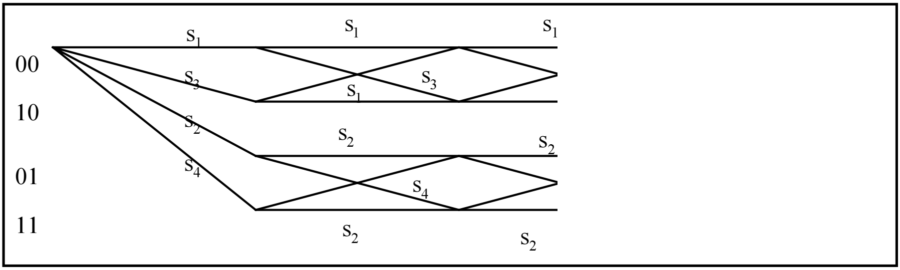


Fig. 6. Trellis structure of the (8,4,4) GAC. code in QPSK signalling scheme

In the case of the (16,5,8) GAC code in BCM MPSK scheme, all the possible data sequences and corresponding subcode words and mapping are as shown in Table 2.

Table 2. Uncoded bits, GAC coded bits and mappings for 8PSK transmission employing the (16,5,8) GAC code

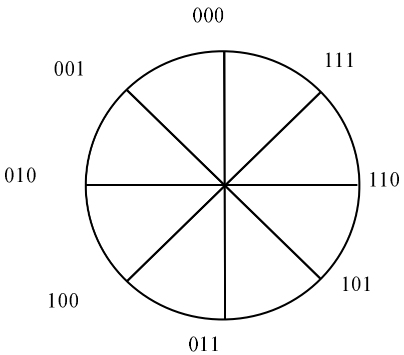
Uncoded data	GAC coded	Mapping
000	0000	S_1
001	0011	S_2
010	0101	S_3
011	0110	S_4
100	1111	S_5
101	1100	S_6
110	1010	S_7
111	1001	S_8

For each row of the GAC block code there are only 8 possible (i) 3-tuple uncoded data sequences and (ii) 4-tuple GAC coded subcode words. Therefore, both these can be transmitted employing the same signalling scheme, namely 8PSK, without expanding the size of the signalling scheme. The mapping for the uncoded and coded sequences to the 8PSK signals are illustrated in Fig. 7. The trellis structure for the decoding and demodulation of the (16,5,8) GAC code is given in Fig. 8.

PERFORMANCE ANALYSIS

The performance of a combined coding and modulation scheme depends on the trellis structure of the code and the corresponding signal projection onto the signal space. The selection of points from the signal constellation and their mapping onto the branches of the trellis determines the free distance of the code and hence the coding gain. The asymptotic coding gain of a conventional CCM method employing convolutional codes is given by Pandovi & Wolf (1986).

$$G_a = 10\log_{10}\left[\frac{d_{free}^2}{d_0^2}\right] dB \tag{10}$$



(7a)

0

Fig. 7. Matching of codewords and signals points for (a) uncoded 8PSK transmission (b) (16,5,8) GAC code using 8PSK

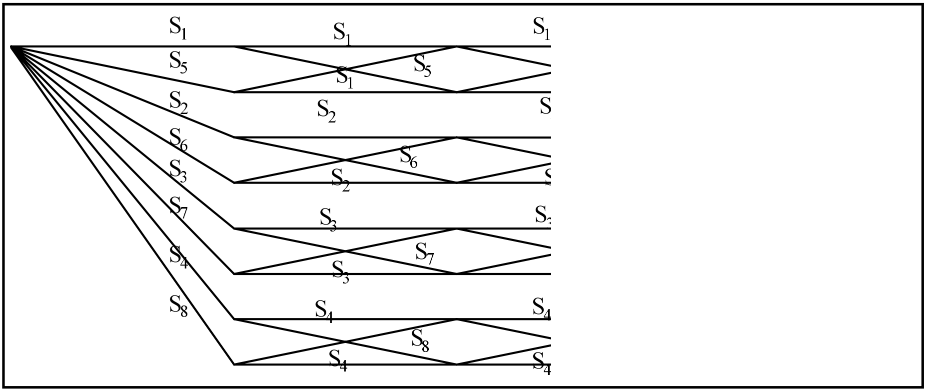


Fig. 8. Trellis structure of (16,5,8) GAC code employing 8PSK modulation.

where d_{free} is the free Euclidean distance of the code, and d_o is the minimum Euclidean distance of the MPSK signal constellations, which are taken as reference for comparison. d_{free} is determined by computing the Euclidean distances between all possible paths of the code's trellis. The expression for the gain G_a is used in determining the performance of block codes in BCM MPSK schemes. However, in the calculation of the overall coding gain of block codes the rate R_s defined as

$$R_s = \frac{\text{Number of uncoded symbols}}{\text{Number of block coded symbols}} \tag{11}$$

must be used. Hence the overall asymptotic coding gain of the GAC code in BCM MPSK scheme is given by

$$G_a = 10\log_{10}\left[\frac{d_{free}^2}{d_0^2}\right] R_s \mathrm{dB} \tag{12}$$

d_{free} can be calculated from the portion of the trellis given in Fig.9 and by using eqn. 6 as

$$d_{free}^2 = \min\mathrm{imum}(d_{ec}^2(s_i, s_j) + d_{ec}^2(s_i, s_j)) \tag{13}$$

where s_i and s_j can be any possible signal from the signal space as shown in Fig. 9 and d_{ec} is the minimum Euclidean distance.

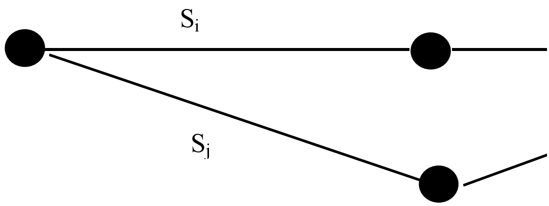


Fig. 9. The minimum path distance of GAC coded in BCM MPSK scheme.

For the (8,4,4) GAC code d^2_{free} is 4 and the minimum Euclidean distance d^2_0 is 1 while for the (16,5,8) RM d^2_{free} is 8 and d^2_0 is 2, both in BCM MPSK scheme. The asymptotic coding gain of the (8,4,4) GAC code and the (16,5,8) GAC code are respectively 6 dB and 4.8 dB.

SIMULATION RESULTS AND DISCUSSION

The proposed combined coding and modulation scheme employing block codes has been simulated for the MPSK modulation scheme with a correlator detection method. The simulation tests were carried out under AWGN channel conditions with a mean of zero. For all cases perfect symbol and block synchronisation are assumed. Fig. 10 shows the decoding results of a number of block codes employing QPSK modulation scheme. The performance of these codes increases with increasing coderate, Hamming distance and trellis depth. Further, it can be seen that (8,4,4) GAC code has about 5 dB gain over uncoded QPSK at a bit error rate of 10^{-3}. Fig. 11 gives the performance of block codes employing 8PSK modulation scheme. The results of the block codes improve slightly with coderate, Hamming distance and trellis depth as in the case of QPSK modulation and are about 5 dB better than the uncoded 8PSK at a bit error rate of 10^{-3}. Fig. 12 gives the performance of block codes employing 16PSK modulation scheme. The results show that the block codes give 2 dB or more coding gain over the uncoded 16PSK case at a bit error rate of 10^{-3}. As expected the coding gain decreases with increasing M.

Fig. 10 Performance of GAC codes emplo

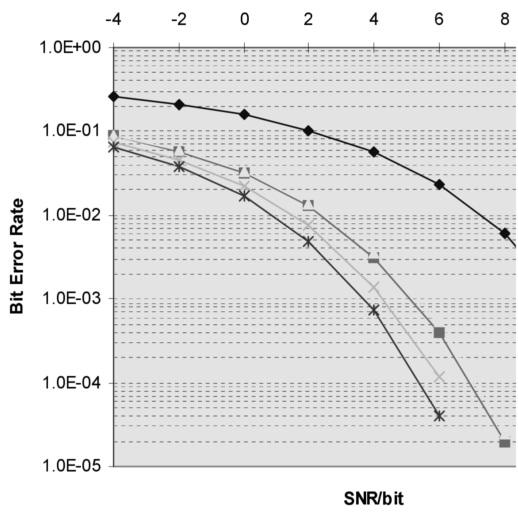
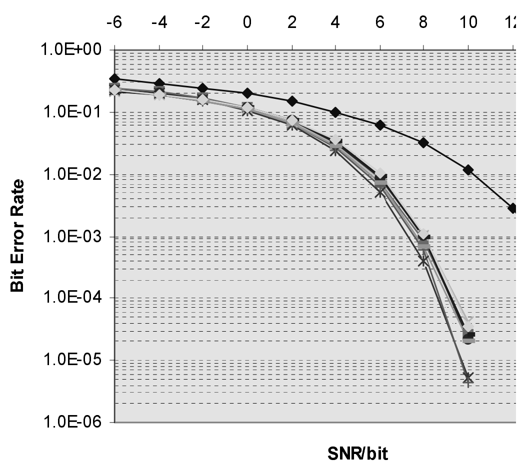
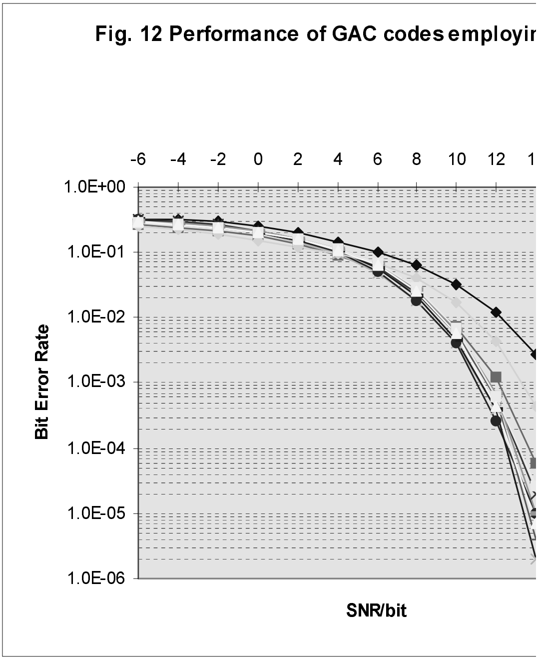


Fig. 11 Performance of GAC codes employi





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