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MODELLING THE NONLINEARITY OF PIEZOELECTRIC ACTUATORS IN ACTIVE VIBRATION CANCELLATION SYSTEMS USING NEURAL NETWORKS

by

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ABSTRACT

Piezoelectric actuators have great capabilities as elements of intelligent structures for active vibration cancellation. One problem with this type of actuator is its nonlinear behaviour. In active vibration control systems, it is important to have an accurate model of the control branch. This paper demonstrates the ability of neural networks to model the nonlinearity of a piezoelectric actuator in the control branch.

Keywords : Piezoelectric actuators, active vibration cancellation, system identification, neural networks.

INTRODUCTION

Vibration of mechanical surfaces is often a nuisance since it may cause damage to fixtures and produce undesirable audible noise. In active vibration control systems, sensors are used to measure the vibration in the structure at appropriate positions, and a counterwave is applied to the structure via an actuator so that the superposition of the disturbance and counterwave results in destructive interference (Fuller & Von Flotow, 1995).

There are many types of actuator-sensor systems currently used in active vibration control. Among these, piezoelectric actuators have great capabilities as elements of intelligent structures for active vibration cancellation (Crawley & Luis, 1987). Being light and compact, they may be glued to the mechanical surfaces to be controlled without significantly altering the structure.

In active vibration control systems, it is important to have an accurate model of the control branch, that is the transfer function between the signal applied on the actuator and the signal measured on the sensor. In most systems a linear model of the control branch is assumed. However, piezoelectric actuators have inherent nonlinearity (Chung Won & Sulla, 1994; Abe, 1995) and therefore cannot be represented by linear models. Some studies have been carried out to model hysteresis nonlinearity (Nobakht & Ardalan, 1992) which is present in piezoelectric actuators. However, the model considers only one type of nonlinearity (hysteresis) while the physical system may contain several types of nonlinearities acting altogether (hysteresis, nonlinear stiffness, type of glue used, etc.).
In active vibration control systems, it is common to perform system identification to obtain a model of the control branch. System identification consist basically of choosing a model structure (e.g. linear recursive and nonrecursive models, Volterra models, *etc*.) and then finding the model parameters from known input and output signals (Narendra & Parthasarathy, 1990; Schoukens & Pintelon, 1991; Haykin, 1996). Iterative algorithms such as LMS and RLS are commonly employed to find the model parameters (Schoukens & Pintelon, 1991; Haykin, 1996).

At the other end, neural networks have been greatly developed over the last two decades. Hornick *et al*. (1989) have shown that multilayer feedforward neural networks are universal approximators. Also, neural networks have been considered for the modelling and control of nonlinear and dynamical systems (Bozich & Mackay, 1991; Snyder & Tanaka, 1995; Narendra *et al*. 1990). It seems convenient then to model the control branch in a piezoelectric-based vibration control system using neural networks.

In this paper, the results of investigations on the modelling of the control branch in a piezoelectric-based vibration control system is presented. The mechanical structure was a cantilever beam fixed at one end. Since the system was very resonant (with a Q-factor of about 100), it was necessary to use recurrent models. Initially, a linear recursive model was assumed for the control branch, and the parameters were found using the equation-error approach (Shynk, 1989) and normalised LMS algorithm. Then, nonlinear recurrent models based on neural networks (proposed by Narendra & Parthasarathy, 1990) were assumed, and the model parameters found using a fast training algorithm proposed by Scalero & Tepedelenlioglu (1992). It was found that the neural network based models performed much better than the linear model.

ACTIVE CANCELLATION OF VIBRATION ON CANTILEVER BEAM

The arrangement used for this investigation is shown in Fig. 1 a and the dimensions are given in Fig. 1 b.

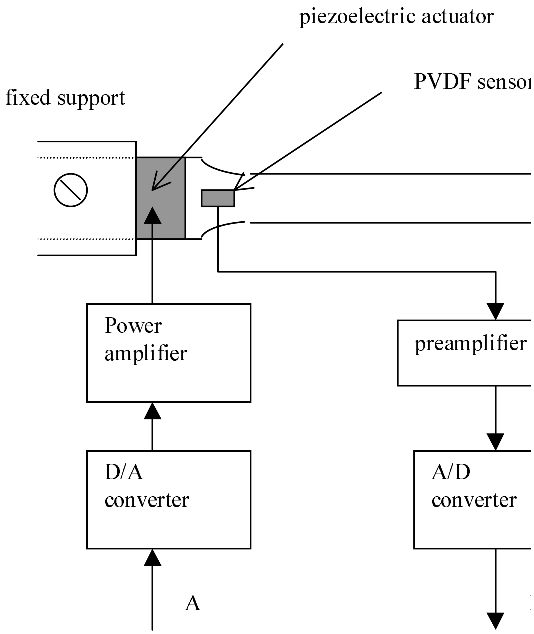

Fig. 1 a. Physical arrangement used in the investigations

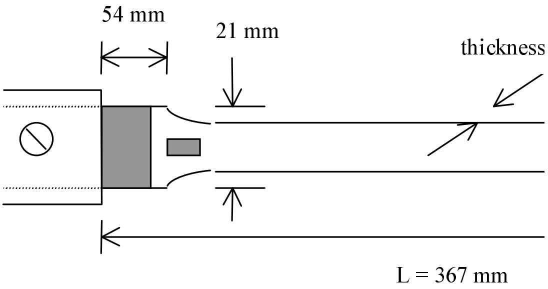

Fig. 1 b. Beam dimensions (side view shown)

The aluminium cantilever beam has several resonant frequencies depending on the vibration mode. The first resonant frequency is around 15Hz and the second resonant frequency is higher than 75Hz. The actuator consists of two piezoelectric wafers glued on both sides of the cantilever close to fixed end. By applying a sufficiently high voltage on the actuator, one of the wafers contracts while the other extends in the direction of the beam length. This results in a bending moment that causes the beam to deflect sideways.

The beam deflection produces charge on the PVDF sensor which is proportional to the deflection. This results in a voltage appearing at the input of the preamplifier which boosts the signal level. The analogue-to-digital converter transforms the output voltage of the preamplifier into a digital signal. The sampling frequency was set to 120.0Hz. Antialiasing filters are automatically configured to reject signal components above 60.0Hz. The high voltage applied on the actuator was reconstructed using a digital-to-analogue converter and amplified by about fifty times by the power amplifier.

The points labelled A and B correspond to specific points on a DSP board (DSP Starter Kit from Texas Instruments) for the input and output signals of the system. The DSP board was interfaced to a computer via serial port. The system between points A and B, from the signal point of view, is called the control branch and is labelled C in the following description.

In a vibration control problem, the cantilever is subjected to an external disturbance which may be due to mechanical coupling at the fixed end or some other force acting on the mechanical structure (e.g. wind gusts). In such case there will exist a vibrational wave on the structure. The principle of active vibration cancellation is to apply a counterwave on the structure (180 degrees out of phase with the initial wave) so that the superposition of the two waves result in destructive interference.

The counterwave is applied via the piezoelectric actuator and stems from a controller. The controller output signal is called the control signal. The effect of superposition of the vibrational wave and counterwave is measured by the PVDF sensor and a control error signal is obtained at point B.

Two control strategies are commonly employed: feedforward control and feedback control. The former is illustrated in Fig. 2 a. In this system, a reference signal, which comes from an upstream sensor at the disturbance, is fed to the controller. The transfer function between this upstream sensor and point B is represented by P. In the case when it is not possible to have a reference signal at the source of the disturbance, a feedback control strategy is more appropriate. This configuration is shown in Fig. 2 b. Notice that the controller input comes from the PVDF sensor.

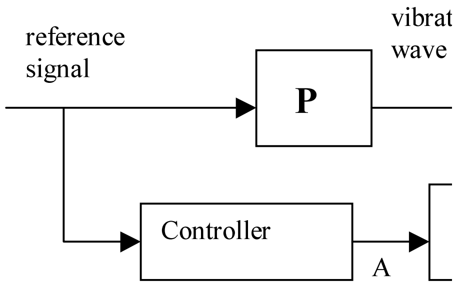

Fig. 2 a. Feedforward vibration control system

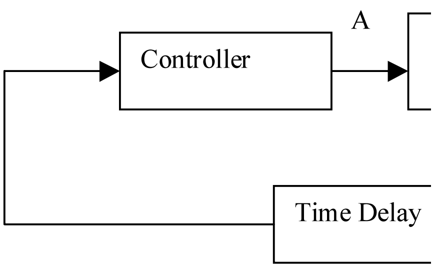

Fig. 2 b. Feedback vibration control system

In both of the above control systems, the problem is to find the optimal controller. In this end, it is important to have an accurate model of the control branch C. The process of obtaining a model for the control branch is called system identification. Since the control branch is in general not linear, nonlinear models are required for system identification.

IDENTIFICATION OF THE CONTROL BRANCH USING A LINEAR RECURSIVE MODEL

In this section, identification of the control branch using a linear recursive model is described. Since there is only one resonant frequency (around 15Hz) for the cantilever beam on the range from zero to 60.0Hz, a recursive model with only two coefficients for the autoregressive part would be sufficient. If x(n) is the input signal and y(n) the output signal of the model representing the control branch, then an adequate linear representation of the system is:

$$y(n) = \sum_{i=0}^{N-1} b_i x(n-i) + \sum_{j=1}^{2} a_j y(n-j) \tag{1}$$

The system identification problem reduces to choosing a suitable N, and finding the a- and b-coefficients using an appropriate algorithm. The physical system was excited with a broadband signal (white noise this case) and the error e(n) between its output d(n) and the model output y(n) evaluated. The model coefficients were found using the equation-error formulation (Shynk, 1989) and iteratively updating the coefficients with the normalised LMS algorithm based on error signal e(n). The algorithm is very well known and details may be found in the book by Haykin (1996). After a sufficiently large number of iterations, the parameter set converges to the optimal set, giving the model closest to the physical system. The normalised LMS algorithm was chosen for simplicity and also because the rate of convergence was not the issue here: only the solution at convergence was important. A measure of the accuracy of the model at convergence is the error-to-signal ratio η in decibels:

$$\eta = 10\text{xlg}\left(\frac{energy\ of\ e(n)}{energy\ of\ d(n)}\right) \tag{2}$$

In the identification process, N was set to 256. After 4000 iterations the error-to-signal ratio was –34 ± 3 dB. Subsequent iteration does not improve this value significantly. The ± 3 dB was due to variations of the amplitude of target signal d(n) and system nonlinearity which is not accounted for by the model.

The b-coefficients at convergence are shown in Fig. 3. The coefficients a_1 and a_2 at convergence were –1.3992 and 0.9916 respectively, which correspond to a resonant frequency of 15.12 Hz and Q-factor of 98.8. The signal spectra as well as the identified transfer function are shown in Fig. 4. Examination of Fig. 3 suggests that 128 b-coefficients would have been sufficient. This result was used later in nonlinear system identifications.

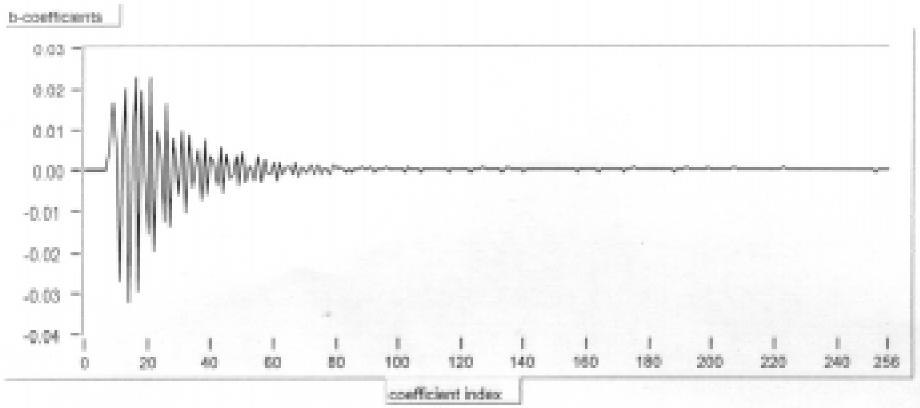

Fig. 3. The b-coefficients identified for the linear recursive model

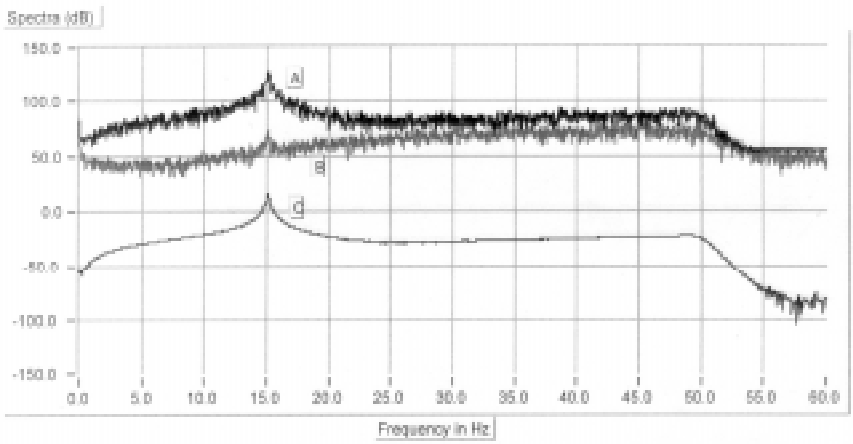

Fig. 4. A = spectrum of target signal d(n); B = spectrum of error signal e(n); C = transfer function evaluated from the identified a- and b-coefficients

IDENTIFICATION OF THE CONTROL BRANCH WITH NEURAL NETWORK BASED MODELS

In this section, identification of the control branch based on the recurrent nonlinear models, proposed by Narendra *et al.* (1990), is considered. The four models are:

Model I : $y(n) = \sum_{i=1}^{M} a_i y(n - i) + g[x(n), x(n - 1), ..., x(n - N)]$ (3)

Model II : $y(n) = f[y(n - 1), y(n - 2), ..., y(n - M)] + \sum_{i=0}^{N} b_i x(n - i)$ (4)

Model III : $y(n) = f[y(n - 1), y(y - 2), ..., y(n - M)] + g[x(n), x(n - 1), ..., x(n - N)]$ (5)

Model IV : $y(n) = f[y(n - 1), y(y - 2), ..., y(n - M), x(n), x(n - 1), ..., x(n - N)]$ (6)

In each case, x(n) is the model input, y(n) the model output, f(.) and g(.) are nonlinear functions to be identified, and the orders M and N to be appropriately chosen. Note that model III is more general than the first two and model IV is most general.

Being universal approximators, feedforward neural networks can be used to model the nonlinear functions f(.) and g(.) as suggested by Narendra & Parthasarathy (1990). In the investigations presented here, two-layer feedforward neural networks were used. The hidden layer consisted of a weight matrix followed by sigmoidal functions. The output layer simply consisted of a weighted sum of the hidden layer outputs. For example, to model function f[x(n),x(n-1),…,x(n-N)], the following construct was used. The signal x(n) was fed into a tapped delay line with N delay elements to give the vector $\mathbf{X}_0$:

$$\mathbf{X}_0 = [x(n),x(n-1),\ldots,x(n-N)]^T \quad (7)$$

The vector $\mathbf{X}_0$ is the input to the hidden layer:

$$\mathbf{X}_0^* = [1,x(n),x(n-1),\ldots,x(n-N)]^T \quad (8)$$
$$\mathbf{Y}_1 = \mathbf{W}_1.\mathbf{X}_0^* \quad (9)$$

where $\mathbf{W}_1$ is the weight matrix of hidden layer

$$\mathbf{X}_1 = [\Gamma(Y_{1,1}),\Gamma(Y_{1,2}),\ldots,\Gamma(Y_{1,N1})]^T \quad (10)$$

where $\Gamma(x) = \tanh(\beta x)$, and β is arbitrarily chosen to 1.

$\mathbf{X}_1$ is the output vector of the hidden layer and therefore input to the output layer. The function f(.) is then given by:

$$f[x(n),x(n-1),\ldots,x(n-N)] = \mathbf{W}_2.\mathbf{X}_1 \quad (11)$$

where $\mathbf{W}_2$ is the weight matrix of the output layer.

In model I, II and IV there is only a single nonlinear function and therefore a single neural network is required. In model III, there are two nonlinear functions and therefore two neural networks are necessary. The model output y(n) in this case is the sum of each network output.

The system identification problem consists of choosing suitable orders M and N, finding the weight matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ for the feedforward approximators, and the coefficients a_i and b_i where necessary. The system parameters were to be found using appropriate algorithms. The physical system was excited with a broadband signal (white noise in all cases) and the error e(n) between its output d(n) and model output y(n) evaluated. In each case, the modelling error was found based on the series-parallel identification approach described by Narendra & Parthasarathy (1990). The weight matrices were iteratively updated with the enhanced back-propagation (EBP) algorithm proposed by Scalero & Tepedelenlioglu (1992). For models I and II the a_i and b_i coefficients were found using the normalised LMS algorithm.

As an example consider the identification of the control branch based on model II. The physical system is excited with a broadband signal x(n) (white noise) and the response d(n) is measured. In the series-parallel identification process, both x(n) and d(n) are used as reference signals. Let $y_1(n)$ be defined by:

$$y_1(n) = \sum_{i=0}^{N} b_i x(n-i) \quad (12)$$

and $y_2(n)$ defined by:

$$y_2(n) = f[d(n-1), d(n-2), \ldots, d(n-M)] \quad (13)$$

The aim is to find coefficients b_i and function f(.) so that the model output y(n), defined in eqn. 4, tends to $y_1(n) + y_2(n)$. For each pair of samples (x(n), d(n)), the model error e(n) is evaluate as follows:
Tapped delay lines are used to form the vectors:

$$Tdl\#1 = [x(n), x(n-1), \ldots, x(n-N)] \quad (14)$$
$$Tdl\#2 = [d(n-1), d(n-2), \ldots, d(n-M)] \quad (15)$$

Eqn. 12 is modelled by an FIR filter whose input is x(n) and output is $y_1(n)$. Eqn. 13 is modelled by a feedforward neural network NN1 whose input is Tdl#2 and output is $y_2(n)$. $y_2(n)$ is evaluated according to eqns. 7-11. The model output $\hat{y}(n)$ is given by $y_1(n) + y_2(n)$. The error e(n) is given by :

$$e(n) = d(n) - \hat{y}(n) \quad (16)$$

The coefficients b_i of the FIR filter are updated using the normalised LMS algorithm based on reference signal x(n) and error signal e(n). The weights of NN1 are found according to the EBP algorithm based on input pattern Tdl#2 and error e(n). Note that, since d(n) is used to approximate y(n) in eqn. 13, the model will be biased by uncorrelated noise present in d(n). However, the advantages of using this approach are faster convergence and lower risk of local minimum.

RESULTS OF NONLINEAR SYSTEM IDENTIFICATIONS

System identification was performed for all the four models described in the previous section. For model I, identification was performed with M = 2 and N = 127. The neural network representing function g(.) has 128 input points in the input layer, and 64 neurones in the hidden layer. After 45,000 iterations, during which the adaptation rates was gradually reduced, an error-to-signal ratio of -34 ± 3 dB was reached. Further iterations did not improve this figure. Notice that the error using this nonlinear model is not better than the linear recursive model. Figs. 5 a – 5 b. show a block of the system output signal d(n) and the identification error signal e(n) at the end of the identification

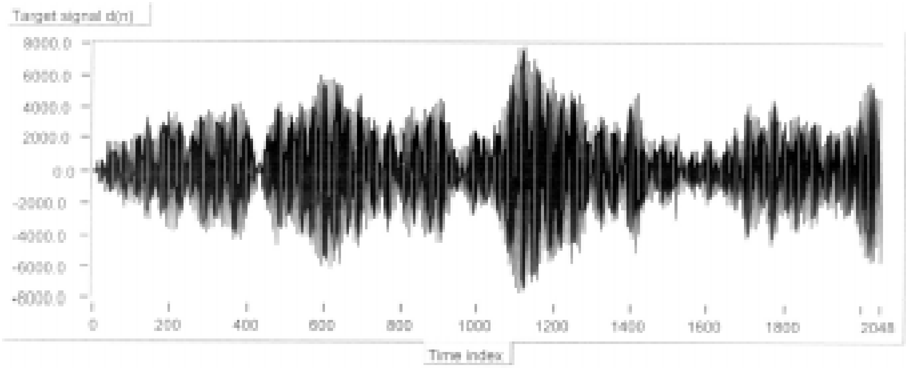

Fig. 5 a. A block of target signal d(n) (*see note below)

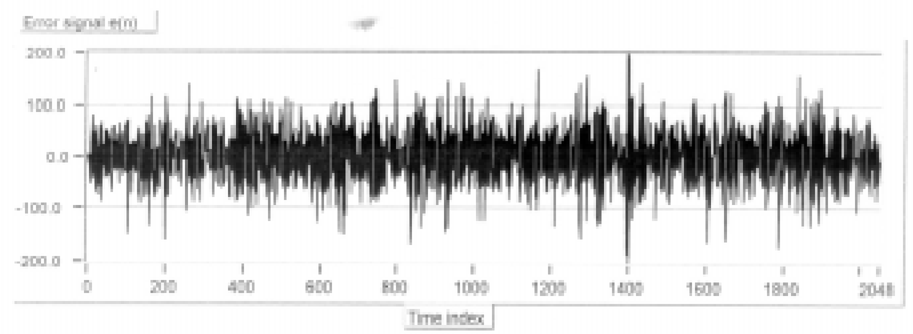

Fig. 5 b. Error signal e(n) at the end of identification process (model I) (*)

*Note : The vertical axis in Fig. 5 a represents the output of the A/D converter. To obtain the measured voltage in μV, the vertical axis must be scaled by a factor of 91, which is the sensitivity of the A/D converter. Also, the horizontal axis gives the time index. To obtain time in seconds, the horizontal axis must be scaled by a factor of 1/120 representing the sampling period. This also applies to Fig. 5 b and Fig. 6.

In a system identification based on model II, the model orders were set to M =8 and N = 159. The neural network modelling function f(.) has 8 entry points and 50 neurones in the hidden layer. After 75,000 iterations the error-to-signal ratio was – 36 ± 3 dB. This value represents a slight improvement on model I.

Identification based on model III required two neural networks: NN1 to represent function g(.) and NN2 to represent function f(.). Values of M = 8 and N = 127 were used. NN1 contained 128 entry points and 48 neurones in the hidden layer. NN2 contained 8 entry points and 48 hidden layer neurones. After 45,000 iterations the error-to-signal ratio was –39 ± 3 dB. Further iterations does not cause significant improvement. We note that this result is much better than with the first two models. This is to be expected as model III is more general than models I and II.

Identification based on model IV was performed with M = 8 and N = 127. The neural network modelling the function f(.) contained 136 entry points and 48 neurones in the hidden layer. After 45,000 iterations the error-to-signal ratio was –40 ± 2 dB which represents a slight improvement over model III. The convergence does not improve with further iterations. The error signal at the end of the identification process is shown in Fig. 6. Notice that the amplitude is considerably less when compared with Fig. 5 b (model I).

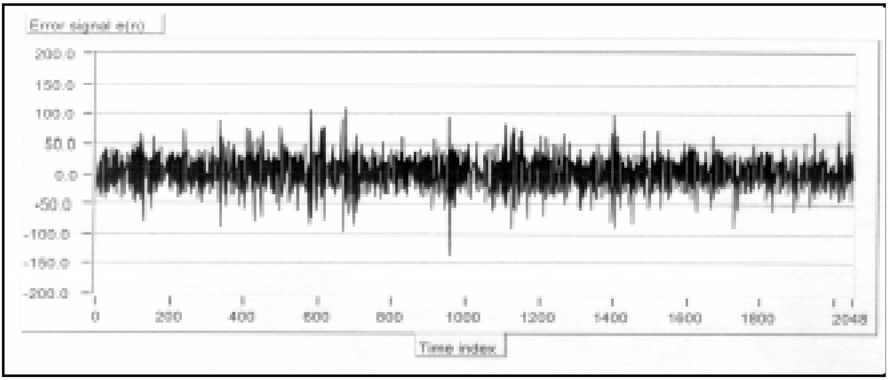

Fig. 6. Error signal at the end of identification process with model IV

In further investigations, model IV was optimised in terms of system order (M and N) and the number of neurones in the hidden layer. Table 1 shows the error-to-signal ratio attainable with the number of hidden layer neurones fixed at 56 and the orders M and N varied. Clearly the orders M = 24 and N = 31 seem more appropriate since they represent the shortest length of tapped delay line.

Table 1. Error-to-signal ratio (dB) when system orders M and N are varied

		N				
		127	63	31	15	7
	32				-39	-3
	24			-40	-39	-3
M	16	X*	-40	X*	-39	
	8	-40	-40	-39	-35	
	4	-38	-37			
	2	-35	-33			

* For some combinations of M and N, the model would not converge during training and these are marked by X in the table.

Table 2 shows the error-to-signal ratio with M = 24, N = 31 and the number of hidden layer neurones (N_1) varied.

Table 2. Error-to-signal ratio with M and N fixed to 24 and 31 respectively and N_1 varied

N_1	**28**	**56**	**112**	**224**
Error-to-signal ratio (dB)	X	-40	-41	-41

We note that there is no significant improvement by using more than 56 neurones in the hidden layer. However, if the number of hidden layer neurones is too small, the model may fail to converge (c.f. N_1 = 28).

CONCLUSION

In this study, identification of the control branch in an active vibration cancellation system employing piezoelectric actuators was investigated. In a system identification using a linear recursive model, the error-to-signal ratio was about –34 dB. Identification of the control branch using nonlinear recurrent models based on neural networks was then considered. It was found possible to reduce the error-to-signal ratio down to about – 40 dB with the most general nonlinear model, that is the error signal power was reduced down to one-quarter of the error signal power obtained with the linear model.

Once an accurate model for the control branch is obtained, the next step is to find an optimum controller for the system. Since the control branch is nonlinear, finding a nonlinear controller is not so simple as in the case of linear control branches. Techniques for finding the optimal controllers must be investigated in future research work.

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