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ADAPTIVE RAC CODES EMPLOYING STATISTICAL CHANNEL EVALUATION

by

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ABSTRACT

In time varying channels the noise and interference vary randomly. Forward error correction codes (FEC) on such channels are designed to cater for the worst possible state and require a large amount of redundancy at all time. This means that when the channel is relatively noiseless, excessive error control power and hence redundancy is being employed and results in a reduction in the overall information rate. An adaptive encoding technique using row and column array (RAC) codes employing a different number of parity columns that depends on the channel state is proposed in this paper. The trellises of the proposed adaptive codes and a statistical channel evaluation technique employing these trellises are designed and implemented.

Keywords: Adaptive array codes, statistical channel evaluation

INTRODUCTION

Noise and interference in a communication channel cause errors. Error control coding consists of adding structured redundant information to the data prior to transmission and is used to combat the adverse effects of the channel. The decoder in the receiver uses this redundancy to determine the most likely data transmitted. Generally channels are time varying. Examples include multipath radio channels, mobile terrestrial and satellite, HF, UHF, or VHF, telephone links and magnetic recording systems. For these channels forward error correction codes are designed to provide error control for the worst channel state and hence require a large amount of redundancy even when the channel is relatively noisefree. An adaptive coding scheme (Bate, 1992) varies the amount of redundancy according to the state of the channel in order to increase the overall information transmission rate. Such a scheme necessarily yields a higher throughput and a lower bit error rate.

The adaptive array codes proposed in this paper are obtained by using RAC (Farrell, 1979; Farrell, 1990; Farrell, 1992) codes and extending the number of parity check columns. The trellises of the codes are designed. These are based on the modified trellis design proposed by Honary *et al.* (1995). For a given number of information bits arranged in a discrete number of rows and columns the adaptive codes have the same trellis structure independent of the number of parity check columns. This feature makes them very appropriate for an adaptive scheme. Fig. 1 gives a block diagram of the adaptive scheme based on a hybrid ARQ scheme (Lin & Costello, 1984, Lin *et al.* 1984).

ENCODING OF ADAPTIVE ARRAY CODES

Adaptive RAC codes are obtained by incrementing the number of parity check columns to the information block. Extending the number of parity columns of RAC codes having only two information columns causes each parity column (except the first) to be a repeated information or parity column and are therefore not suitable for an adaptive scheme. Instead we analyse RAC codes having a minimum of three information columns, so that four parity columns can be used without any repetition.

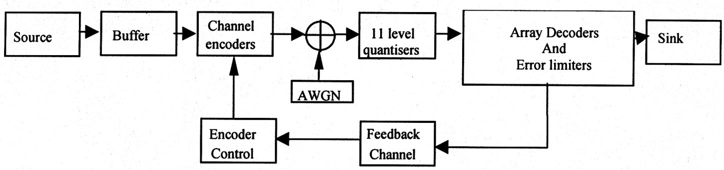

Fig.1 Block diagram of an adaptive scheme employing RAC codes

Encoding procedure:
Consider the *mxn* information matrix *X* given by

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & . & . & x_{1n} \\ x_{21} & x_{22} & x_{23} & . & . & x_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_{m1} & x_{m2} & x_{m3} & . & . & x_{mn} \end{bmatrix} \qquad (1)$$

In an adaptive encoding scheme this information vector *X* can be encoded to give a codeword $\begin{bmatrix} X & C \\ R & D \end{bmatrix}$

where C is a column parity matrix given by

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & . & . & c_{1k} \\ c_{21} & c_{22} & c_{23} & . & . & c_{2k} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ c_{m1} & c_{m2} & c_{m3} & . & . & c_{mk} \end{bmatrix} \qquad (2)$$

R is a row parity vector given by

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & . & . & r_{1n} \end{bmatrix} \qquad (3)$$

D, the check on check vector is given by

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} & . & . & d_{1k} \end{bmatrix} \qquad (4)$$

The elements of C, R and D are given by

$$c_{i1} = \sum_{l=1}^{n} x_{il} \quad i = 1,2,......,m \tag{5}$$

$$c_{i2} = \sum_{l=2}^{n} x_{il} \quad i = 1,2,......,m \tag{6}$$

$$c_{i3} = \sum_{\substack{l=1 \\ l\neq 2}}^{n} x_{il'} \quad i = 1,2,......,m \tag{7}$$

$$c_{ik} = \sum_{\substack{l=1 \\ l\neq k-1}}^{n} x_{il} \quad i = 1,2,......,m \tag{8}$$

$$r_{1i} = \sum_{j=1}^{m} x_{ji} \quad i = 1,2,......,n \tag{9}$$

and

$$d_{1i} = \sum_{j=1}^{m} c_{ji} \quad i = 1,2,......,k \tag{10}$$

and $\sum$ denotes mod 2 addition.

TRELLIS DESIGN OF THE ADAPTIVE RAC CODES

Let the parameters of the adaptive array code be given by $(n_1, k_1)(n_2, k_2)$ where k_1, k_2 are the number of information rows and columns and n_1, n_2 are the total number of rows and columns in the code respectively.
The trellis structure proposed for the adaptive array codes is as follows:

(i) The number of states, N_s, in the trellis is given by

$$N_s = q^{k_2} \tag{11}$$

(ii) The trellis depth (number of columns in the trellis) N_c is then given by

$$N_c = k_1 + 2 \tag{12}$$

(iii) Identify each state at depth p (where p can take all values between 0 and N_c - 1 inclusive) by a k_2 tuple q-ary vector

$$S^p(A) = S^p(a_1, a_2, \ldots\ldots, a_j, \ldots\ldots, a_{k2}) \tag{13}$$

where a_j is an element of GF(q), and j = 1,2,......,k_2.

(iv) Mark the trellis diagram starting at depth p=0 and finishing at p=N_c - 1 with $S^0(0_1, 0_2, \ldots, 0_{k_2})$ and $S^{k_1+1}(0_1, 0_2, 0_3, \ldots\ldots\ldots 0_{k2})$ states respectively.

(v) The trellis branches at each depth p are labelled by a k_2-tuple q-ary vector C_A^p, where C_A^p is determined using

$$C_A^p(B) = S^p(A) \oplus S^{p+1}(A) \tag{14}$$

and A and B are all possible k_2-tuple q-ary vectors that represent the current(p) and the next (p+1) states of the trellis respectively, and Å is mod q addition.

(vi) Each branch is labelled by two particular values:

$C_A^p(a_1, a_2, \ldots\ldots, a_j, \ldots\ldots, a_{k2})$, which represents the input information symbols and

$D_A^p(a_1, a_2, a_3, \ldots\ldots a_{k_1+1}, a_{k_1+2}, \ldots\ldots a_{n2})$ which represents the output vector where a_i for i=1,2,...,k_2 represents the information bits and the parity bits are given by equations 15-18:

$$a_{k_1+1} = \sum_{i=1}^{k_1} a_i \tag{15}$$

$$a_{k_1+2} = \sum_{i=2}^{k_1} a_i \tag{16}$$

$$a_{k_2+3} = \sum_{\substack{i=1 \\ i\neq 2}}^{k_2} a_i \tag{17}$$

$$a_{k_2+4} = \sum_{\substack{i=1 \\ i\neq 3}}^{k_2} a_i \tag{18}$$

and so on.

Consider for example the (28,9,8), (24,9,6), (20,9,4) and (16,9,4) family of codes having 9 information bits each. The (28,9,8) code can be written as

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & c_{11} & c_{12} & c_{13} & c_{14} \\ x_{21} & x_{22} & x_{23} & c_{21} & c_{22} & c_{23} & c_{24} \\ x_{31} & x_{32} & x_{33} & c_{31} & c_{32} & c_{33} & c_{34} \\ r_{11} & r_{12} & r_{13} & d_{11} & d_{12} & d_{13} & d_{14} \end{bmatrix}$$

where x_{ij} for i=1,2,3 and j=1,2,3 represents the information bits,

$$c_{i1} = \sum_{l=1}^{3} x_{il} \quad i=1,2,3 \tag{19}$$

$$c_{i2} = \sum_{l=2}^{3} x_{il} \quad i = 1,2,3 \tag{20}$$

$$c_{i3} = \sum_{\substack{l=1 \\ l\neq 2}}^{3} x_{il} \quad i=1,2,3 \tag{21}$$

$$c_{i4} = \sum_{\substack{l=1 \\ l\neq 3}}^{3} x_{il} = \sum_{l=1}^{2} x_{il} \quad i=1,2,3 \tag{22}$$

$$r_{1i} = \sum_{j=1}^{3} x_{ji} \quad i=1,2,3 \tag{23}$$

$$d_{1i} = \sum_{j=1}^{3} c_{ji} \quad i= 1,2,..,4 \tag{24}$$

where $\sum$ denotes mod 2 addition. Successfully dropping the last column of the code gives the (24,9,6), (20,9,4), and the (16,9,4) codes. These four codes respectively have the generator matrices G_1, G_2, G_3 and G_4 where

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{25}$$

$$G_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{26}$$

$$G_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{27}$$

and

$$G_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{28}$$

In order to obtain the code block from the information block we first compute the column parities of the information block. If X represents the information block then the information with column parities become $\begin{bmatrix} X \\ R \end{bmatrix}$ and the products

$\begin{bmatrix} X \\ R \end{bmatrix} G_1$, $\begin{bmatrix} X \\ R \end{bmatrix} G_2$, $\begin{bmatrix} X \\ R \end{bmatrix} G_3$, $\begin{bmatrix} X \\ R \end{bmatrix} G_4$

respectively give the codes with four, three, two and one parity columns. Any code having three information columns like the (21,6,8), (18,6,6), (15,6,4), (12,6,4) or the (35,12,8), (30,12,6), (25,12,4), (20,12,4) codes have the same generator matrices G_1, G_2, G_3 and G_4. The trellis structure of these four adaptive array codes of nine information bits is obtained as follows:

$$N_s = 2^{k_1} = 2^3 = 8 \qquad (29)$$

$$N_c = k_1 + 2 = 3 + 2 = 5 \qquad (30)$$

Fig. 2 gives the trellis of the adaptive codes with 9 information bits.

PERFORMANCE ANALYSIS OF AN ARRAY CODE IN AN ARQ SYSTEM

Theoretical expressions for the throughput efficiency and reliability of an embedded array code of dimension (n,k,4) have been derived (Darnell et al. 1988). We consider the case when the array code is used in a selective-repeat ARQ arrangement (Lin & Costello, 1983; Lin et al. 1984) in a Binary Symmetric Channel (BSC) with bit transition probability P and noiseless feedback channel. The following probabilities can be obtained:

(i) P_c is the probability that a received block is error free
(ii) the P_e is probability that the received block contains undetectable errors
(iii) P_d is the probability that the received block contains detectable errors,

where

$$P_c = (1-p)^n \qquad (31)$$

$$P_d = \sum_{i=1}^{3} \binom{n}{i} P^i (1-P)^{n-i} \qquad (32)$$

$$P_e = 1 - P_d = P_c \qquad (33)$$

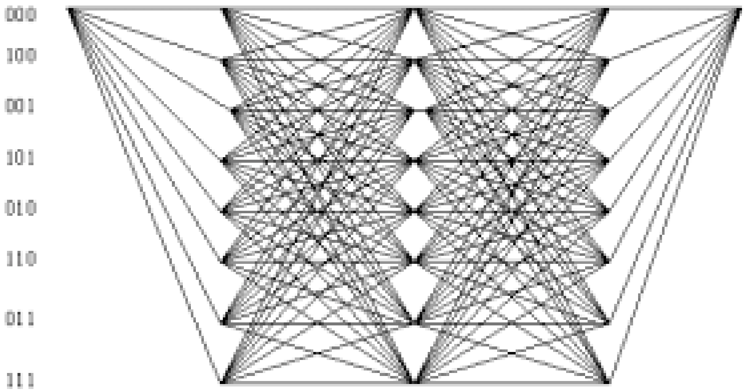

Fig. 2 Trellis structure of the (28,9,4), (24,9,4), (20,9,4) and (16,9,4) RAC array code.

The lower bound on the throughput efficiency is defined as the ratio of the number of error-free blocks accepted by the receiver to the total number of blocks transmitted (Lin & Costello, 1983). For a block to be successfully accepted by the receiver, the average number of retransmissions needed (Lin & Costello, 1984; Yu & Lin, 1981) is

$$M = 1P_c + 2(1-P_c)P_c + 3(1-P_c)^2 P_c + \ldots + (m+1)(1-P_c)^m P_c \quad + \ldots\ldots\ldots\ldots$$

$$= \frac{1}{P_c} \qquad (34)$$

$$\text{throughput efficiency} = R \times \frac{1}{M} = RP_c \qquad (35)$$

where R is the coderate, $\left(\frac{k}{n}\right)$, and efficiency is the overall throughput of the system.The reliability of an ARQ system (Lin & Costello, 1984), *P(e)*, is defined as the ratio of the number of digits in error at the output to the total number of digits at the output and is given by

$$P(e) = \frac{P_e}{P_a} = 1 - \frac{p_c}{p_a} \tag{36}$$

where P_a is the probability of acceptance, given by

$$P_a = P_e + P_c \tag{37}$$

finally

$$P(e) = 1 - \frac{(1-p)^n}{\left[1 - \sum_{i=0}^{3}\binom{n}{i}p^i(1-p)^{n-i}\right]} \tag{38}$$

STATISTICAL CHANNEL EVALUATION EMPLOYING THE TRELLIS OF THE RAC CODES

In soft decision maximum likelihood decoding of the RAC codes used in this paper, the received signal voltage is divided into 11 levels. This is done as follows: all voltages less or equal to -1.0 volt are converted to 0.0 and voltages greater or equal to 1.0 volt are converted into 1.0. The voltage interval between -1.0 and 1.0 volts is equally divided into steps of 0.1 in such a way that there are 11 levels altogether. The Euclidean distances between the branch values and the quantised received data are evaluated for each branch in the trellis during trellis decoding. This gives N_s Euclidean distances at depth $p=1$, that is, one at each node. The Euclidean distance from a node at depth 1 is separately added to the Euclidean distance between a branch value and the received quantised signals for each branch emanating from that node. At depth *2, 3, 4* ....N_c*-2*, there are N_s cumulative Euclidean distances at each node. We select the minimum distance at each node and proceed to the next depth in the same way. Hence there are only N_s Euclidean distances left at the final depth N_c*-1*. At this node we select the minimum distance in order to trace out the maximum likelihood path. The next higher Euclidean distance corresponds to the maximum likelihood error path. The maximum likelihood error path differs from the maximum likelihood path in at least two branches. The difference in the Euclidean distance between these two paths is used as an error detecting metric. Obviously this error detection metric depends on the noise present in the channel. As the channel becomes noisier the error detection metric decreases. Under very noisy conditions this metric will fall to zero. This means that the decoded bits are totally unreliable. Fig 3 gives the probability density function of the error detection metric at the output of the decoder for the (16,9,4) RAC code for channel signal to noise ratio 2, 4, 6, and 8 for 100000 blocks transmitted. It can be seen that the mean values are quite distinct. Fig. 4 shows the variation of the mean error detection metric and its standard deviation for the (16,9,4) RAC code with respect to the channel SNR. The error detection metric is a good indication of the noise level in the channel. Since the error probability depends on the channel signal to noise ratio an estimation of this ratio enables the output probability of error from the decoder to be obtained. The statistical channel evaluation is done as follows: The channel evaluator averages the error detection metrics over a certain number of blocks, say *b* blocks, where *b* depends on the standard deviation of the error detection metric about its mean. This average is compared with a set of previously determined thresholds. The thresholds are arranged so that, the error probability is kept below a certain maximum specified level.

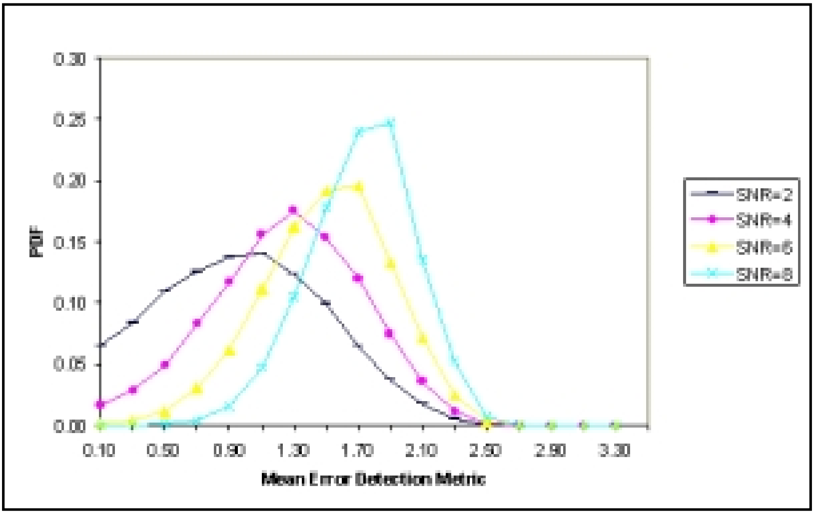

Fig. 3 PDF vs Mean error detection metric for the (16,9,4) RAC code.

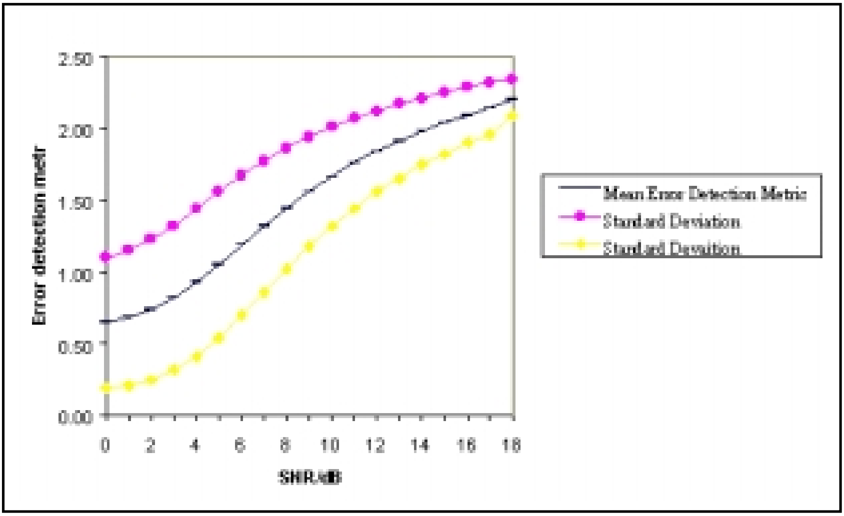

Fig. 4 Mean error detection metric and standard deviation vs SNR for the (16,9,4) RAC code.

SIMULATION TESTS AND DISCUSSION

100 000 channel code symbols using bipolar signalling technique were transmitted over a discrete noisy channel employing a selective repeat adaptive ARQ encoding scheme in each test. For all cases perfect bit and block synchronisation were assumed. In each test the average energy per information bit was fixed, and the variance of the additive white Gaussian noise was adjusted for a range of average bit errors. The noise level now provides a measure for the comparison of the various systems. The results of the simulation tests are shown in Figs. 5 to 8. Fig. 5 gives the error performances of the adaptive RAC codes transmitting 9 information bits. It can be seen that at a bit error rate of 0.01 the code with 4 parity columns has about 2 dB advantage over the single parity column code. Fig. 6 gives variation of the output BER against the channel BER of the adaptive RAC codes. It is observed that at high channel bit error rate the code with four parity columns has a higher reliability than that with a single column. Fig. 7 shows the variation of the throughput of the same codes against the channel bit error rate. As expected, at lower bit error rate the single parity column code gives the higher throughput whereas at higher bit error rate the throughput of the code with four parity columns is better. Fig. 8 gives the variation of the overall throughput against the signal to noise ratio for the adaptive RAC code with 9 information bits codes.

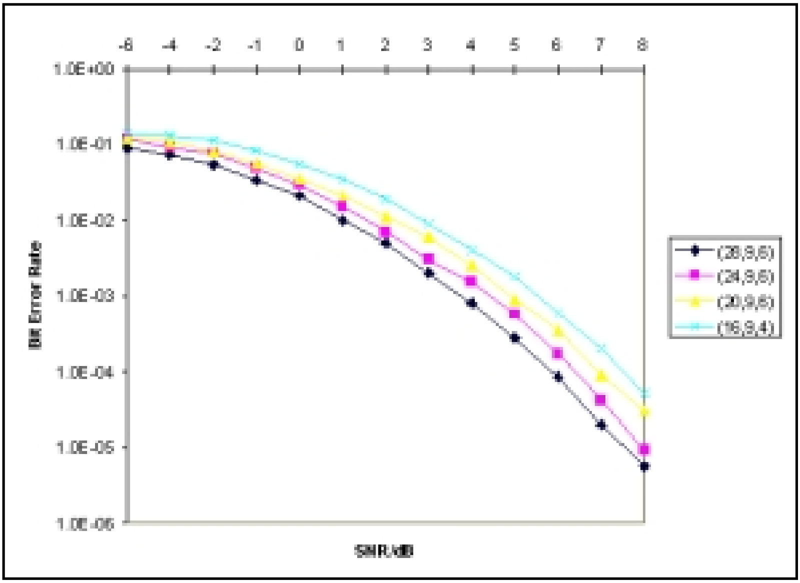

Fig. 5 BER vs SNR for array codes with 9 information bits.

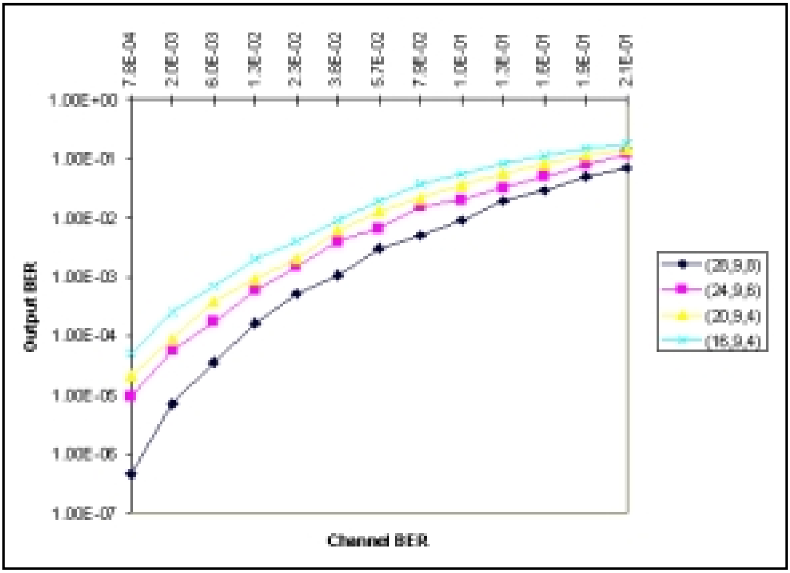

Fig. 6 Output BER vs Channel BER for 9 information bits.

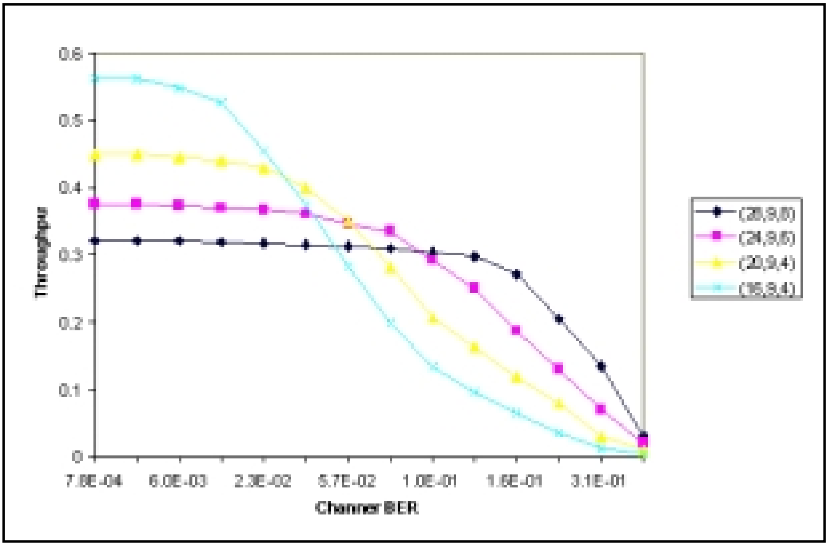

Fig. 7 Throughtput vs Channel BER for 9 Information bits.

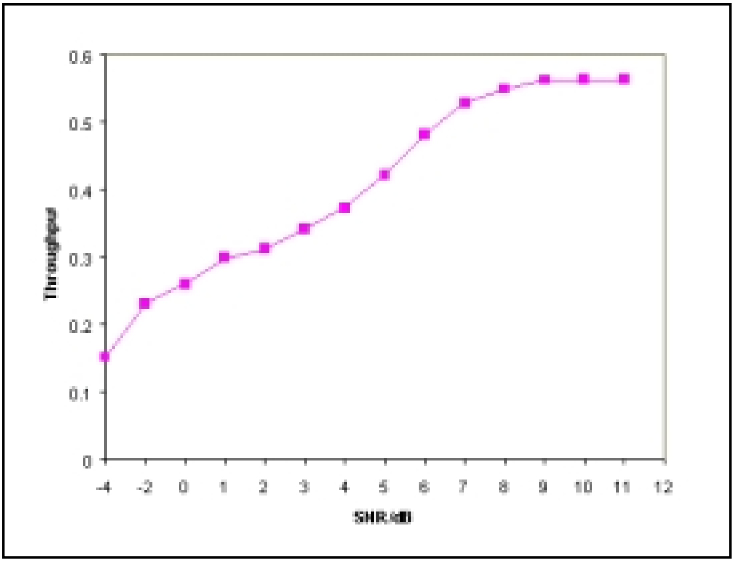

Fig. 8 Throughtput vs SNR for 9 Information bits.

CONCLUSION

A new technique of encoding adaptive array codes, a corresponding trellis decoding structure for the codes and a statistical channel evaluation technique based on the trellis of these codes have been proposed. It has been shown that soft decision trellis decoding of the codes using 11 level quantisation and four parity columns performs significantly better than with a single parity column. The adaptive array codes represents a scheme for transmitting information signals in time varying channels in an ARQ system so as to reduce the bit error rate and improve the overall throughput.

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