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Uncertainties in Measuring Lifetimes of Nuclear Excited States using a Fast Electronic Timing System made of NaI (Tl) Scintillators

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Abstract

Measurements of the lifetimes of nuclear excited states need to be as accurate as possible due to their crucial role in making or verifying various theoretical predictions in the nuclear science and astrophysics. Nevertheless, metrological difficulties in these measurements have brought about controversies that significantly influence the quest to understand the atomic structure, decay processes and nuclear reactions. Among the obstacles are the accuracy and precision of the systems and techniques used for measuring the lifetime of an excited state. It follows that any hypothesis regarding the nuclear behavior can only be considerable upon verification and accountability of stability and uncertainty of the devices used during the experiments for measuring the lifetime of a nuclear excited state. As a step towards accurate measurements of lifetimes of excited states, this work focuses on evaluating the uncertainties of such measurements. In doing so, a fast timing system made of two NaI(Tl) detectors have been developed to measure the lifetime of the 5/2+ excited state of ^{133}Cs using the gamma-gamma coincidences. An uncertainty budget showed the sources of uncertainties in measuring the nuclear excited state's lifetime include timewalk, drift effect, selection of the region of interest, background effects and counting statistics. The total uncertainty in measuring the 5/2+ excited state of ^{133}Cs was found to be 0.024 ns, representing 0.382% of the nominal lifetime value. This implies that the system can accurately measure the lifetimes of excited nuclear states ranging from 1 ns.

Keywords: Excited state lifetime; uncertainty; metrology; gamma-gamma coincidences; fast timing electronic system

Introduction

Lifetime of a nuclear excited state is a fundamental parameter in nuclear physics used as a probe for studying the stability and decay characteristics of nuclei after they absorb energy. Measurement of lifetimes of nuclear excited states provides priceless insight into the nuclear structure, decay mechanisms and nuclear reaction dynamics (Shigekawa et al. 2021). Theoretically, the lifetimes of nuclear excited states are determined by the inverse of transition rates and are known to range from a few femtoseconds to several petaseconds depending on the detailed structure of the nucleus (Bohr and Mottelson 1998). Therefore, measuring the lifetime of a nuclear excited state whose energy is known, provides the reduced matrix elements linking the initial and final quantum states of a particular decay (Lamani et al. 2021). These matrix elements reveal invaluable information about the nuclear structure besides stringent test for the nuclear models which are useful for understanding such complex many bodied quantum ensembles (Voss et al. 2014). Following the importance of nuclear excited states' lifetimes in nuclear

science and technology, several techniques have been developed to measure the lifetimes of excited nuclear states.

Each method used to measure the lifetime of a nuclear excited state has its advantages and limitations (Ko and Kim 2013). This is because experiments for measuring the lifetime of nuclear excited states are influenced by different errors which may arise from variations of experimental conditions such as pressure, temperature, humidity and background radiation (Pommé 2015). Other errors result from instruments used during lifetime measurement, electronics involved in signal processing, experimental setup and human errors (Lugendo et al. 2019). These errors must be quantified to ensure that the nuclear properties that are associated with the excited state's lifetime values are accurately studied.

Quantifying and minimizing whatever experimental uncertainties in measuring the lifetimes of nuclear excited states is unavoidable because accurate values of the nuclear excited state lifetime is crucial for endorsing any claim of changes in the microscopic behavior of the nuclear structure (Heuser and Höfener 2018). All experimental errors in measuring the lifetimes of nuclear excited states must be either compensated or fully reported in the uncertainty budget. However, identifying and quantifying the sources of such experimental errors for a given measurement system requires diligent and systematic analysis of the system. Hence, this study aspires to highlight the important sources of uncertainty in measuring the lifetime of a nuclear excited state when using the gamma – gamma coincidence measurement with NaI(Tl) detectors. The system will be used to measure the lifetime of various nuclear excited states.

To fulfill this purpose, a special lifetime measurement system that is dedicated to measure the nuclear states' lifetime was developed. The system uses two Thallium doped Sodium Iodide (NaI(Tl)) scintillation detectors and employs the fast timing electronic method to measure the lifetime. The experiment uses the coincidence of cascaded gamma rays, which is a suitable technique for measuring nuclear lifetimes ranging from a few picoseconds to several microseconds (Morse 2015). This method takes advantage of the delayed coincidence counting whereby the lifetime of an excited state is determined from the exponential tail of the time distribution spectrum (TDS). This distribution is obtained from the difference in time between the two signals that populates and depopulates the state. The radiation that populates a state of interest is picked by one counter to provide a start pulse for the electronic clock while the radiation that depopulates the state is picked by another counter to provide the stop pulse. The obtained TDS is a combination of the Gaussian distribution known as the prompt peak and the exponential distribution known as the TDS tail. The width of the prompt peak depicts the time resolution while the slope of the tail equals the decay width of the state (ALTO Collaboration et al. 2011). From the TDS, the lifetime of state can be determined by using the slope method which is favorable when the lifetime is reasonably longer than the Full a Width at Half Maximum (FWHM) of the prompt peak (Lugendo 2017). However, despite being considered adequately accurate with wide application, the delayed coincidence method is can be significantly affected by several systematic uncertainties that arise from either the instruments used for the experiment or the experimental procedures.

Typical sources of uncertainties in measuring lifetime of an excited nuclear state via the delayed coincidence counting arise from several sources ranging from the choice of cascaded gamma rays to the electronic devices used in the time picking circuits. These include the time resolution of the system which is influenced by factors like time jitter, time walk and drift. Other uncertainties arise from counting statistics, some secondary effects on electronic devices and data analysis techniques. Furthermore, differences in travelling time of photons from source to detector, spatial distribution of photons interactions within the detection crystal, ambiguities in separation of populating and depopulating radiations,

coincidences with Compton events, cross talk events, different signal velocities in the cables, as well as time calibration and linearity of the digital clock. Moreover, some uncertainty may be caused by the energy resolution of detectors and statistics of the true coincident events (Pommé 2015).

In order to study the uncertainties arising from measuring the lifetime of a nuclear excited state using the fast electronic timing system based on gamma coincidence counting, tests were performed using the standard radioactive sources (^{60}Co, ^{137}Cs and ^{133}Ba). The time resolution of the system was determined before measuring the lifetime of 5/2+ first excited state of ^{133}Cs which is known to be 6.28 ns (Laboratoire primaire des rayonnements ionisants 1999). Various sources of uncertainties in this measurement were identified and their contributions to the effective uncertainty of the measured lifetime value were evaluated and reported in the uncertainty budget.

Materials and Methods

Experimental setup

The fast timing system used in this work composed of two "2 x 2" cylindrical NaI(Tl) ORTEC 905-3 detectors coupled to 12-stage photomultipliers. These detectors were labelled D1 and D2 and were made to face each other with a separation distance of 5 cm. The radioactive source was stationed at the central point between the detectors' faces. The PMTs attached to the detectors were powered by a Power Supply (PS) module with two separate channels of high voltage (HV) outputs. Signals from each detector were fed to several electronic devices via the signal splitting module as illustrated in Figure 1 and the Computer Automated Measurement and Control (CAMAC) was deployed for data acquisition.

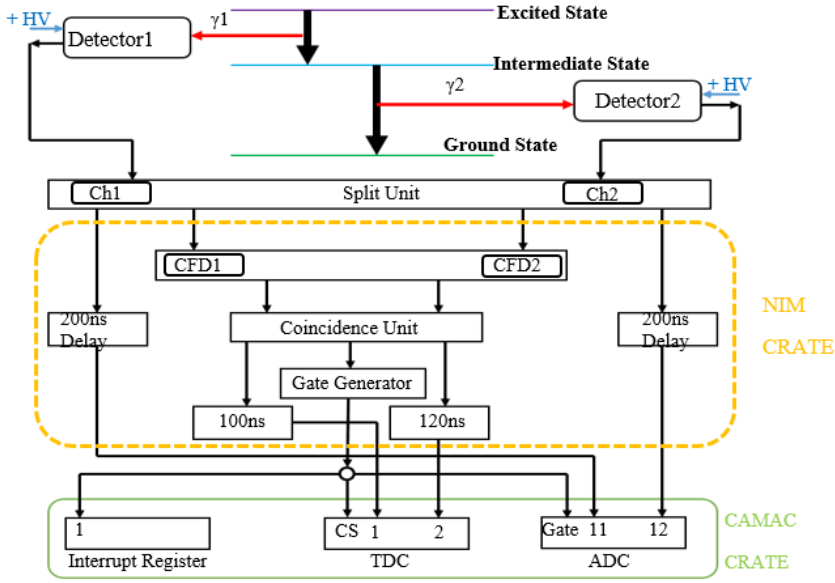

Figure 1: Illustration of the electronic fast timing system that uses γ – γ coincidence technique.

System calibration

The analog signal from each detector was divided into two branches by a splitter (Phillips Scientific 740). One side was discriminated in a 16-channel leading-edge discriminator (Phillips Scientific 706) with two outputs. While one output was fed into a logic coincidence module to realize a two-fold coincidence gate, the other output was digitized in a CAMAC time-to-digital converter (TDC, Phillips Scientific 7186) with a 25-ps resolution and 100-ns dynamic range. The second branch was connected to a 12-bit CAMAC analog-to-digital converter (ADC, REPIC RPC-022), gated with a 1-μs event trigger gate. The coincidence signal

was used as the Common Start (CS) signal for the TDC and the gate signal for the ADC. Both the ADC and TDC were first calibrated in order to get accurate results. The TDC was calibrated using signals from the clock generator and the ADC was calibrated using standard radiation sources. The system's count-rate was ensured to have a maximum of 1 kHz and a minimum of 0.5 kHz so that the pile up and dead time effects could be negligible.

Energy calibration of the timing system was performed using the 356 keV, 662 keV, 1173 keV and 1332 keV gamma lines from the decays of ^{133}Cs, ^{137}Cs and ^{60}Ni which are the daughters of ^{133}Ba, ^{137}Ba and ^{60}Co radiation sources respectively. Data were recorded for each source with the gamma spectra recorded for a duration of 1 hour in each case. From the recorded gamma spectra, centroid ADC channel of each peak of interest was used to plot a graph of energy against channel number in order to obtain the energy calibration equation. This was done for each detector spectrum. The energy resolution of the system was studied using the same gamma lines used in energy calibration. In this case, the corresponding peak for each line in the spectrum was fitted to a Gaussian distribution. From the fitting equation, the centroid (μ) and the standard deviation (σ) were recorded and used to calculate the respective energy resolution using Equation 1.

$$Resolution(\%) = \frac{2.35\sigma}{\mu} \times 100\% \quad (1)$$

Measuring the time resolution of the system

Measuring the lifetime of an excited state requires a good understanding of the timing resolution of the measuring system. To determine the time resolution of the developed system, the lifetime of the 2+ state of ^{60}Ni was measured by placing the ^{60}Co radiation source at the center of the detectors such that the source is at a distance of 2.5 cm from the face of each detector. The ^{60}Ni nucleus is known to have four energy levels (4^+, 2^+, 2^+ and 0^+) with the dominant decay branches being $4^+ \rightarrow 2^+$ and $2^+ \rightarrow 0^+$ as shown in Figure 2.

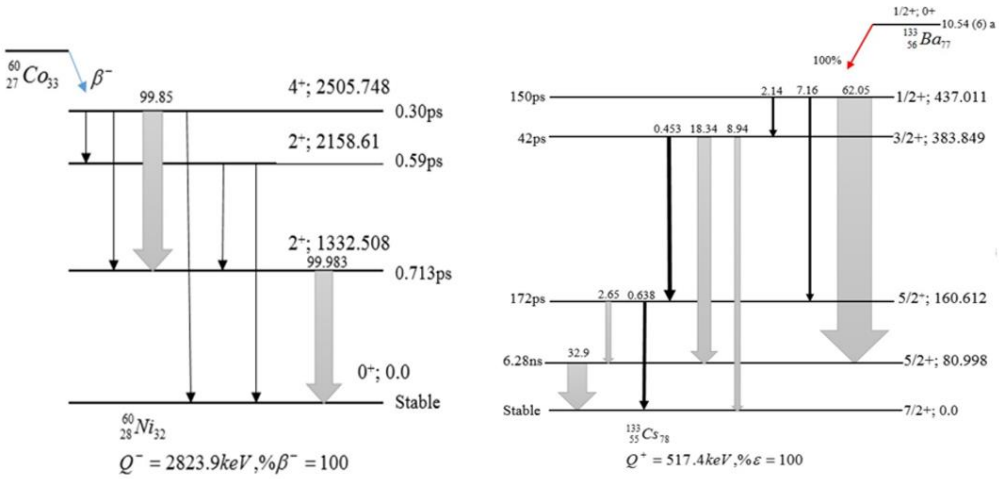

Figure 2: The decay schemes of ^{60}Co and ^{133}Ba radiation sources showing the intermediate nuclear states and the coincident gamma rays.

The cascaded gamma transition $4^+ \rightarrow 2^+ \rightarrow 0^+$ is dominant and has the intermediate state whose lifetime is 0.713 ps. To measure this lifetime, energy windows were set such that D1 detects only the 1172 keV gamma line as the first stop signal while D2 detects only the 1332 keV gamma line as the second stop signal, since we use the common start TDC module. However, since the lifetime of the 2^+ state is very short, the 1173 keV gamma ray

which populates the state and the 1332 keV gamma ray which depopulates the state appear to occur simultaneously. Hence the obtained TDS is a Gaussian distribution that represents the PTS of the system (Lugendo 2017). The time resolution of the system was then determined by evaluating the Full Width at Half Maximum (FWHM) of the PTS.

Lifetime measurement

To measure the lifetime of a 5/2+ excited state of ^{133}Cs, a 189-kBq ^{133}Ba radioactive source with a diameter of 5 mm and wrapped in a 50-μm polyester film was placed at the position of the ^{60}Co source. ^{133}Ba nucleus decays by electron capture to the $1/2^+$ (437.01 keV) excited state of ^{133}Cs, which then decays by gamma radiation via several branches as shown in Figure 2. In this case, the dominant cascaded transition is the $1/2^+ \rightarrow 5/2^+ \rightarrow 7/2^+$ which was used to measure the lifetime of the 5/2+ excited state as the intermediate state in this transition. Therefore, energy gates were set such that 356 keV gamma ray provided the first stop signal on the TDC while the 81 keV gamma ray provided second stop signal of the TDC. The obtained TDS was then used to determine the lifetime of the 5/2+ excited state using the specified C++ programs along with ROOT V5.34 accompanied by the ROOFIT package. During analyses, the 356 – 81 keV cascade events were observed by gating energy windows of 340 – 400 keV for the 356 keV line and 65–85 keV for the 81 keV line (Lugendo et al 2019). The obtained data were arranged into tridimensional coincidence matrices named $M\left(E_{D1}, E_{D2}, T_{D1} - T_{D2}\right)$ with the first and second axes made of energy detected by D1 and D2 respectively while the third axis is the time difference distribution for D1 and D2 signals (Lugendo 2019). The time spectra were analyzed using the shape deconvolution technique whereby the lifetime of a nuclear state is calculated from the slope of the exponential tail of the distribution.

Analysis of uncertainty sources

The process of measuring lifetime of a nuclear excited state may be influenced by several systematic effects including the finite intrinsic time resolution of the timing system, the timewalk effect of the TDC, the background contamination in the signal window, asymmetry of the prompt peak, drift of the prompt peak centroid, choice of the cascade events, width of the region of interest, coincidence resolution and counting statistics. The contributions of each sources of uncertainties in this experiment were quantified and the corresponding uncertainty budget was developed.

Results and Discussion

Energy calibration of the fast timing system

In order to use the ADC effectively, its channels were calibrated to represent the specific gamma energies. This is a crucial procedure as energy calibration is required for selecting the desired gamma lines when constructing the time difference spectra. To perform energy calibration, the energy of each detected gamma line from the deployed sources was plotted against the corresponding centroid channel number recorded by the detector. This was done for each detector and the obtained graphs were fitted to a linear function so as to derive the calibration equations shown in Figure 3.

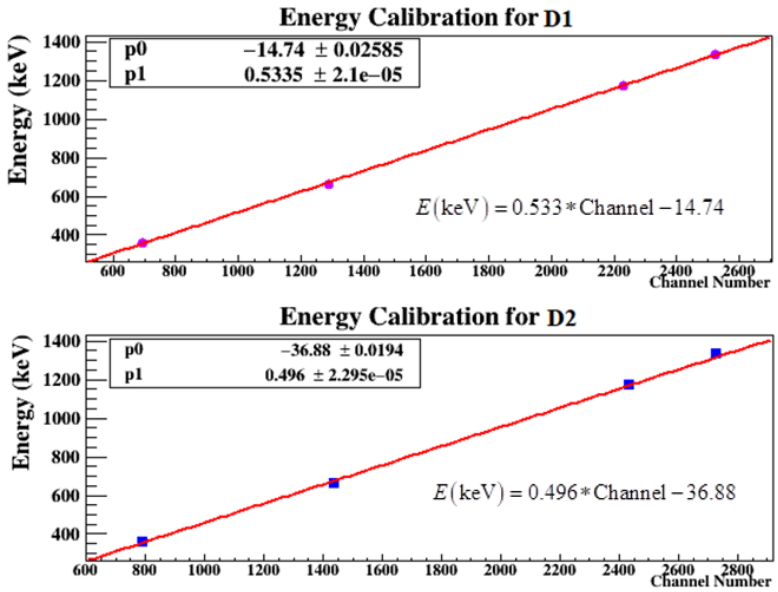

Figure 3: Energy calibration of the detectors. The red line represents the fitting function that is displayed within the figure canvas.

Using these equations, the channel numbers in the energy spectra from each detector were converted to the respective energies and the spectra were plotted as shown in Figure 4. This figure shows the expected energy spectra for the radiation sources used to calibrate the system meaning that the developed system can correctly detect gamma photons of different energies.

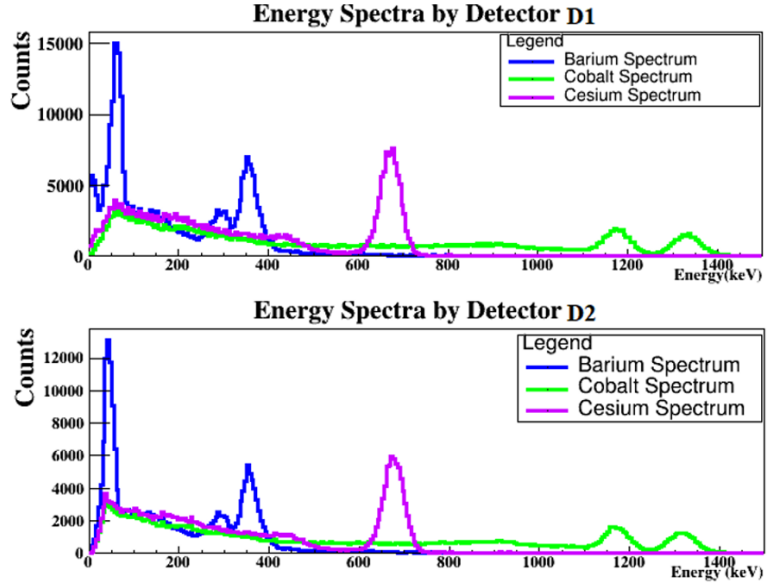

Figure 4: Energy spectra of gamma rays from the measured radiation sources. The spectra show peak centroids at the expected energies showing the system was well calibrated.

Energy resolution of the system

The ability of the detector to clearly distinguish close gamma energies is essential for selecting the desired energy gates. This is because poor energy resolution can result to overlapping of the full energy gamma peaks which negatively affects the ability to identify and distinguish the desired gamma lines during the construction of time difference spectra. Therefore, energy resolution of the detectors is an important parameter to consider when designing a fast timing electronic system. In this work energy resolution detectors D1 and D2 were studied using the 356 keV, 662 keV, 1173 keV and 1332 keV gamma lines. The obtained results are presented in Figure 5.

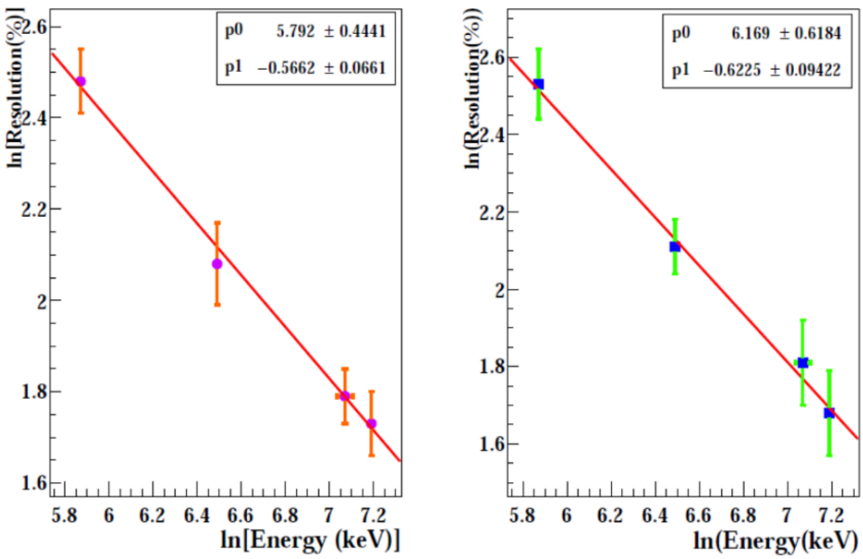

Figure 5: Variation of energy resolution with the photon energy for detectors D1 and D2

According to literature, energy resolution (R) varies with the photon energy (E) according to the equation; $R = \frac{k}{\sqrt{E}}$. Thus, it follows that $\ln R = \ln k - \frac{1}{2}\ln E$, which means the slope of the graph of $\ln k$ against $\ln E$ should be 0.5. However, the fitting equation in Figure 5 shows that the slope is 0.56 for detector D1 and 0.62 for detector D2. This implies that, the energy resolution capability of the system may be suffering from the effect of various factors including electronic noises, charge collection statistics, fluctuation in PMT gain, and variations in the detector response over its active volume as well as drifts in parameters over the course of the measurement (Triambak et al 2012). These errors can result to uncertain choice of energy gates used for deriving the time difference spectrum. Hence their effect on the measurement of lifetime of a state must be accounted. This is done by varying the energy gates and observing the resulting time difference spectra.

Measurement of time resolution and lifetime of a nuclear excited state

The PTS spectrum obtained from the decay of ^{60}Ni was observed to follow the bell shaped distribution and therefore was fitted to a Gaussian function as shown in Figure 6(a). The Gaussian function was used to extract the standard deviation of the PTS, which was in turn used to determine the FWHM as the estimate for the time resolution of the system. It was observed that the time resolution was 853 ps with the prompt peak centroid located at 60.49 ns. Since the time resolution is well

below one nanosecond, then the current system meets the criteria for being a fast timing system that is capable of measuring the lifetimes of nuclear states that are longer than a nanosecond. Now, the 1/2+ → 5/2+ →0 transition was selected for deriving the TDS to determine the lifetime of the 5/2+ excited state of ^{133}Cs. The obtained TDS was observed to be a combination of the Gaussian shaped PTS with an exponential tail to one of its sides. Therefore a Gaussian and exponential convolution function was developed using ROOFIT and used to fit obtained TDS. In the fitting process, the gauss mean and standard deviation were respectively allowed to vary just about 60.49 ns and 0.36 ns in accordance to the values obtained from the prompt peak. Meanwhile the slope of the convolution function was left to vary freely and the best fitting values were obtained from the fitted spectrum shown in Figure 6(b).

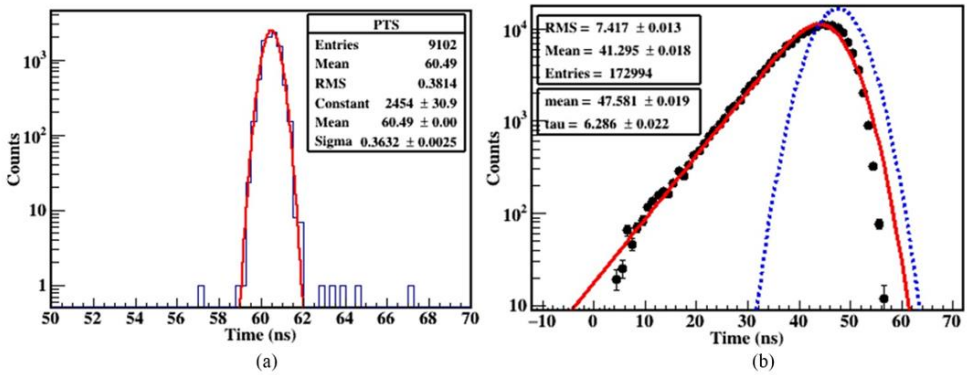

Figure 6: (a) The PTS formed by the timing system (b) The TDS for determining the lifetime of 5/2+ state of ^{133}Cs.

The lifetime of the 5/2+ state is given by the parameter tau (τ) whose value was 6.286 (22) ns. This value is deemed correct since the known lifetime of 6.28 ns is within the error margins of the measured value. Yet, the reported error that reaches 0.35 % of the measured value implies that several factors may be affecting the lifetime measurement process. Based on the role of nuclear excited state's lifetime in the quest to understand several nuclear properties, efforts are always set to improve accuracy and precision of measuring this parameter. This can be done by either deploying detectors of better energy and time resolution like LaBr(Ce) detectors or by striving to minimize the errors that affect the lifetime measurement process. This work focuses on the second case whereby the important sources of uncertainty in measuring the nuclear excited state's lifetime using NaI(Tl) detectors are identified and quantified. This will in turn enhance the ability of the NaI(Tl) system to accurately measure the lifetimes for different applications including understanding the nuclear structure, mechanisms of nuclear reactions, nucleosynthesis process in the stars as well as testing various nuclear models.

Uncertainties in measuring lifetime of a nuclear excited state

Several factors have been identified as the major contributors of uncertainties during the lifetime measurement process using the fast electronic timing system based on NaI(Tl) detectors. These factors were analyzed to determine their contribution to the overall measurement uncertainty of the developed fast timing system.

Timewalk

The TDC timewalk effect arises from the variations in the measured time of arrival of a signal due to the differences in the characteristics such as shape and amplitude of a signal entering the TDC (Harter et al 2023). For example, a lower amplitude signal might take longer to reach the threshold of the constant fraction discriminator (CFD) compared to a higher amplitude signal hence

affecting the prompt peak width. The timewalk effect was studied by setting the energy gate of D1 on the 1173 keV photopeak for providing the first TDC stop signal while the second stop signal was provided by D2 with energy gate set on the 1332 keV peak. Different TDS were then obtained by shifting the 80 keV wide gate of D1 along the Compton continuum with the mean points separated by 100 keV. The time resolution for each TDS were then determined and plotted against the energy as shown in Figure 7(a). The equation in this figure was used to correct the timewalk effect on the TDS and the new time resolution was obtained to be 0.795 ns. The procedure was then repeated by using different points along the Compton continuum to obtain several timewalk correction functions. After TDS corrections using different functions, the values of time resolution were plotted in Figure 7(b). The standard deviation, 0.005 ns was considered as the uncertainty in lifetime measurement caused by the timewalk effect.

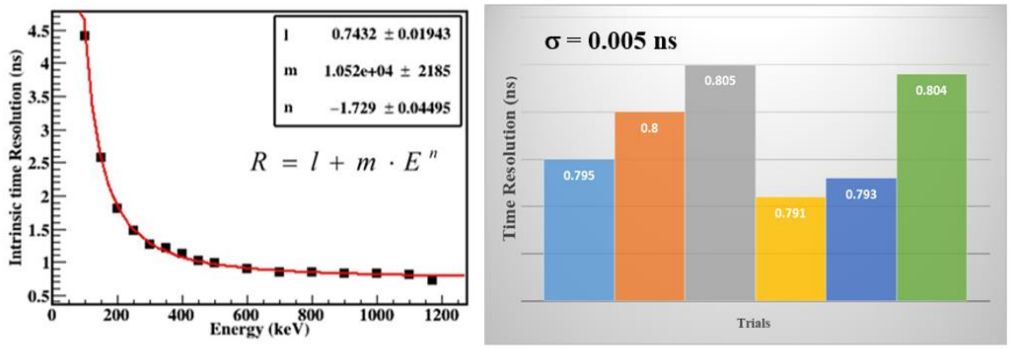

Figure 7: (a) Variation of the time resolution due to the timewalk effect (b) Uncertainty due to timewalk effect

Asymmetry of the prompt peak

The prompt peak distribution is significantly affected by some secondary effects arising from factors like geometrical imperfections of the detectors, different photon interaction in the detection crystals, phototubes response, signal speeds in the Lemo cables as well as the occasional instabilities of the system. These factors cause the prompt peak to be asymmetric leading to errors in determining its FWHM which in turn affects the estimation of the lifetime. To quantify such error, different values of the FWHM (the sigma parameter in the convolution function) were used to estimate the lifetime. Figure 8 shows the variation of lifetime as the sigma parameter in the fitting function is slightly varied. The average value of the measured lifetimes was found to be 6.289 ns with the standard deviation of 0.011 ns which is taken as the uncertainty due to the asymmetry of prompt peak.

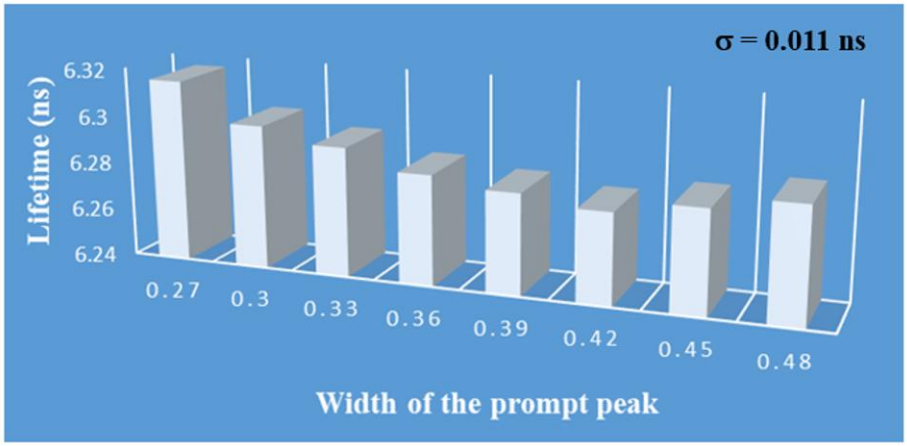

Figure 8: The uncertainty due to the prompt peak asymmetry

Drift of the prompt peak

Electronic devices are normally affected by the changes in temperature during the experiment. Therefore, the time distribution spectrum was recorded after every 30 minutes for the whole 4 hours of lifetime measurement. From each spectrum obtained, the centroid of the prompt peak and the corresponding lifetime value were recorded. A graph of lifetime against the prompt peak centroid was then generated as shown in Figure 9. The standard deviation (σ = 0.009 ns) of the lifetime distribution was then taken as the uncertainty in lifetime measurement due to the drift in centroid of the prompt peak.

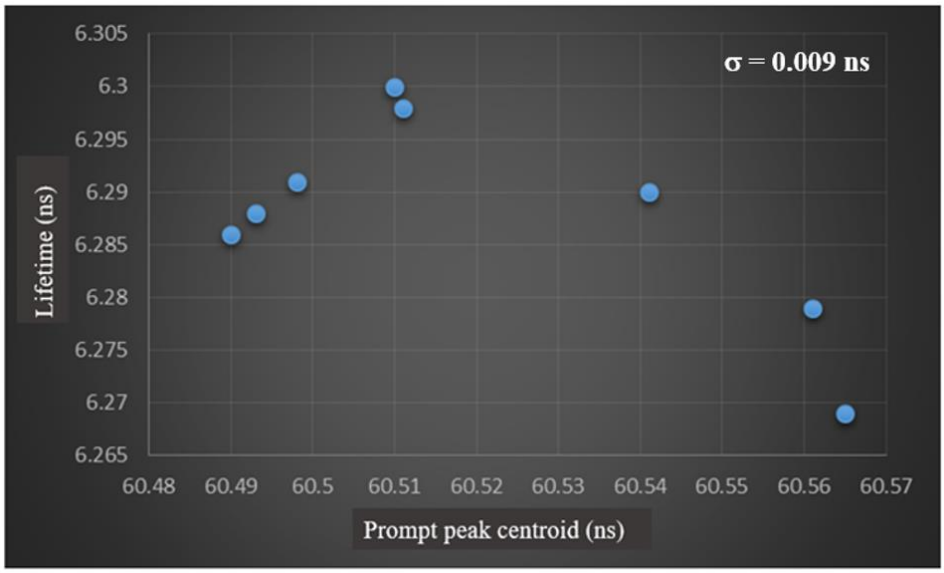

Figure 9: Variation of lifetime with the prompt peak centroid

Feeding effect

The 5/2+ (81 keV) state of ^{133}Cs, is populated via the decay of 437 keV state, the 383 keV state as well as the 160 keV state. In this work, the lifetime of the 81 keV state has been extracted using the 437 keV – 81 keV – 0 keV gamma cascade. However, the decay of 383 keV and 160 keV states keeps on populating the 81 keV leading to a feeding effect. This effect slightly elongates the lifetime of the 81 keV state leading to errors in measurement of the state's lifetime. As

feeding is a natural process, one can only quantify and account for its effect in the final measurement of the lifetime. The quantification of this error involves choosing different gamma cascades with the same intermediate state, which is the state whose lifetime is to be measured. The lifetime of this intermediate state is then measured several times using different cascades and the average lifetime value is assigned as the lifetime of the state with the standard deviation as its corresponding uncertainty. In the case of ^{133}Cs, there are only two measurable gamma cascades with the 5/2+ state as their intermediate state, hence the lifetime values measured from each cascade were averaged to give 6.284 ns. This average deviates from the literature value of 6.28 ns by 0.004 ns, which was considered to be the uncertainty due to the feeding effect.

Width of the region of interest

The time distribution is obtained by setting the energy gates around the mean energy of interest. However, events under the selected region of interest may be contaminated by some noise that arise due to some random coincidences and the finite energy resolution of the detectors. Therefore, choice of the ROI widths during time spectrum analysis can lead to uncertainty in estimating the lifetime of a state. In order to quantify the effect of ROI widths in lifetime measurements, energy windows with widths varying from σ (68% confidence level), 1.645σ (90% confidence level), 1.96σ (95% confidence level), 2.576σ (99% confidence level) and 3.291σ (99.9% confidence level) were used to study the lifetime of the 81 keV excited state of ^{133}Cs. The measured lifetime was observed to vary with the energy gate widths as shown in Figure 10 with the standard deviation of 0.016 ns. This value was considered as the uncertainty in lifetime measurement due to the effect of ROI width.

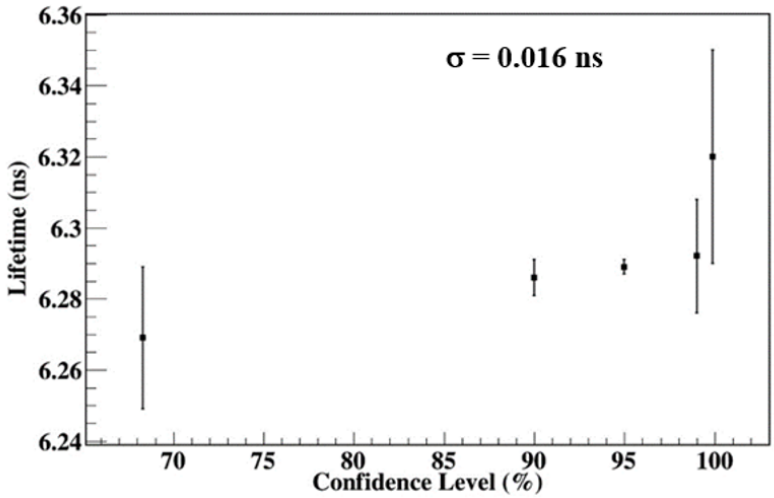

Figure 10: Variation of the measured lifetime with the ROI widths

Background effect

In all cases, the time spectrum lies on top of significant background contributions which mainly comes from the Compton-scattered events associated with the higher energy photons. Background could also arise from other radiations, both natural and man-made radiations, as well as from electronic noises. Although much of these effects are limited by the coincidence setup and use of relatively narrow coincidence windows, it is worthy to estimate the contribution of background radiation on the lifetime measurement uncertainty. The process of evaluating the effect of background radiation was done by deploying the sideband analysis as explained in literature (Lugendo et al 2019). In this case, energy gates were shifted towards the right or

left side of ROI for both 81 keV and 356 keV peaks. Lifetime values were then determined from the raw TDS and the background free spectrum that was obtained after subtraction of all background spectra shown in Figure 11. The lifetime value obtained after background subtraction from the raw TDS improved by 0.007 ns. This value was therefore adopted as the error due to background events.

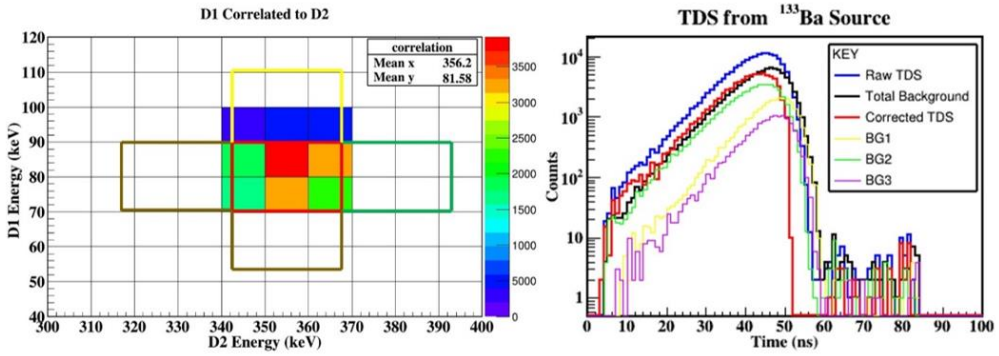

Figure 11: Illustration on how to determine the background events from the time spectrum using the sideband analysis as shown on the left figure. The figure on the right shows the time spectra obtained for each shift of energy gate (BG1, BG2 and BG3), the total background spectrum (Total Background), the raw time difference spectrum (Raw TDS) and the background free spectrum (Corrected spectrum).

Coincidence resolution

Several practical limitations can cause the timing system to record events that are not truly coming from the same nucleus (Gurgi et al 2017). Such events are referred as accidental coincidences while coincident events from the same nucleus are called true coincidences. Accidental coincident events may lead to errors in measuring the lifetime a nuclear state. The magnitude of accidental coincidence events can be determined by using Equation 2.

$$\frac{N_{True}}{N_{Accidental}} = \frac{1}{2 \cdot A \cdot \tau} \qquad (2)$$

Note that, N_{True} is the number of true coincidences, $N_{Accidental}$ is the number of accidental coincidences, A is the source activity (Bq) and τ is coincidence resolution (s). The coincidence resolution was set to 100 ns and the source activity was 8510 Bq. Thus, the ratio of accidental to true coincident events was found to be 1:588. It follows that out of 172,994 coincident events recorded by the system, only 294 events seem to be accidental coincident events. To establish how much coincidence resolution affects the measured lifetime, we generated Monte-Carlo time distributions with number of events varied from 172,700 to 173,288 and obtained the TDS for several cases. From these spectra, several lifetime values were determine as illustrated in Figure 12. The standard deviation of these lifetimes, which was found to be 0.002 ns, was considered as the uncertainty in lifetime measurement due to coincidence resolution.

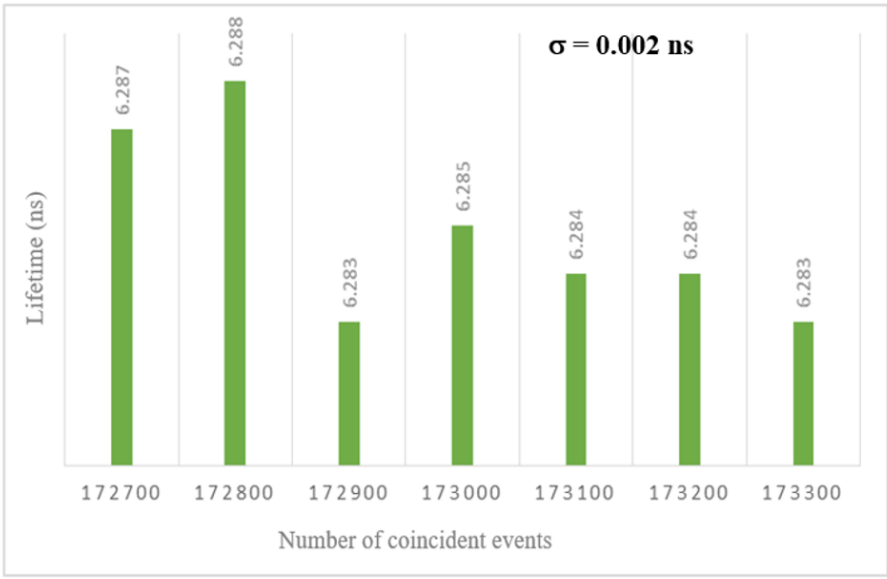

Figure 12: Variation of lifetime with the number of coincident events

Counting statistics

Statistical precision can lower the uncertainties in lifetime measurement experiment (Ertoprak 2017). In order to study the influence of statistical counting on our study, toy Monte-Carlo lifetime distributions were generated with different number of events and fitted using the convolution function that was used to fit the lifetime spectrum this study. Figure 13 shows the variation of lifetime with respect to the number of events generated in the simulation. The uncertainty value associated with statistics was found to be 0.01 ns.

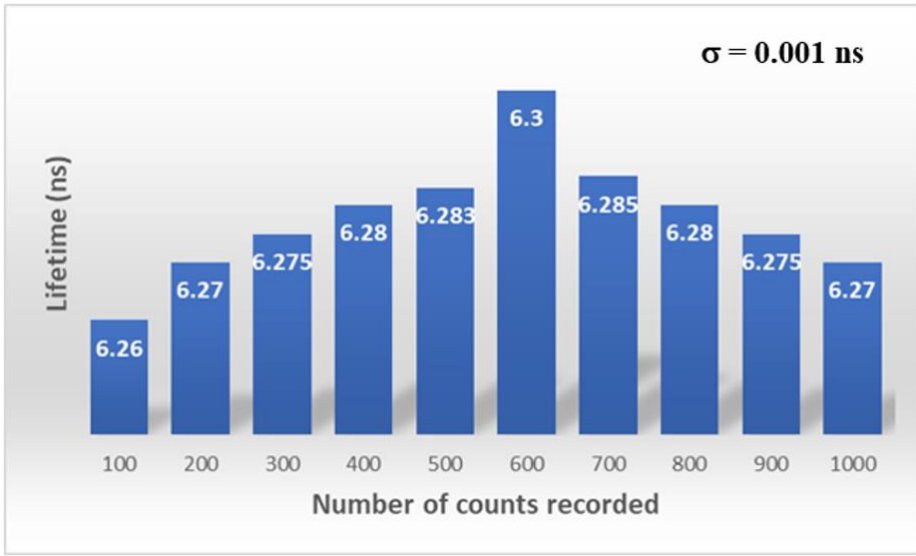

Figure 13: Variation of lifetime value with the number of events recorded.

Estimation of the total experimental uncertainty

It is crucial to determine an effective value that represents the effect of all sources of uncertainties that have been identified in this study. Therefore Table 1 was used to record all sources of errors and their contribution to the experimental uncertainty in measuring the lifetime of 81 keV excited state of ^{133}Cs. Each contribution is presented in nanoseconds and in percentage of the nominal lifetime value (6.286 ns) as measured in this experiment. The effective uncertainty value was then determined using the quadratic sum of all the individual contributions. The obtained effective uncertainties shown in the Table 1, used to present the measurement uncertainty of the developed fast timing system. Therefore, the final value of lifetime of the 5/2+ first excited state of 133Cs was quoted as 6.286 ± 0.024 ns . As the systematic uncertainty in the system is 0.024 ns, it follows that our fast timing system is capable of accurately measure the lifetimes of nuclear excited states to the precision of 0.382%.

Table 1: Systematic uncertainty budget for the measurement of lifetime of 5/2+ first excited state of ^{133}Cs.

Source of Uncertainty	**$\pm\Delta\tau$ (ns)**	**$\pm\Delta\tau$ (%)**
Timewalk effect	0.005	0.079
Asymmetry of the prompt peak	0.011	0.175
Drift of the prompt peak	0.009	0.143
Feeding effect	0.004	0.064
Width of the region of interest	0.016	0.255
Background effect	0.007	0.111
Coincidence resolution	0.002	0.032
Counting statistics	0.001	0.016
Quadratic sum	**0.024**	**0.382**

Conclusion and remarks

This study was set to establish the sources of uncertainties in measuring the lifetimes of nuclear excited states using the electronic fast timing system based on delayed $\gamma-\gamma$ coincidences. To achieve this, a bench-top detector system made of two NaI(TI) detectors was deployed. A total of eight factors have been identified as the significant contributors of uncertainties in measuring the lifetime of the 5/2+ excited state of ^{133}Cs using our bench-top detector system. With the energy resolution of about 7 % at 662 keV gamma peak and time resolution of 603 ps, the system is capable of accurately measuring the lifetime of 5/2+ excited state of ^{133}Cs which was found to be 6.286 ns. However, the established sources of measurement uncertainties for this system amounted to 0.024 ns, indicating that the system can be significantly accurate when used to measure the lifetimes that are longer or equal to 1 ns. The results also show that NaI(Tl) detectors are suitable for measuring lifetimes in nanoseconds range despite their moderate energy resolution. It follows that, detectors with superior energy resolution and timing resolution such as Cerium doped Lanthanum Bromide detectors are recommended for high precision measurements. Since these detectors have different characteristics, a new study is recommended to quantify the uncertainties in measuring nuclear excited state's lifetimes using other detectors that are superior to NaI(Tl) detectors.

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