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Application of Time-Varying Mortality Rate Model in Pension System: A case of Tanzania.

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Abstract

The current increase in the ageing population, driven by a decrease in mortality rates, has impacted most pay-as-you-go defined benefit pension schemes. The resulting increase in the dependency ratio places a greater burden on these schemes. Many pension funds currently use static lifetables to project future obligations, overlooking the gradual decline in mortality rates. This study explores the application of a time-varying mortality rate model to forecast Tanzanian mortality rates. The time-varying model was successfully employed together with multi state method to project and analyse the funding ratio and cashflow to asset value ratio of the mandatory Tanzania Public Service Social Security Fund for the years 2020–2070. Numerical results indicate a decreasing trend in mortality rates over time, leading to a decline in the funding ratio from 92.92% in 2020 to 21.12% in 2070. Additionally, a decrease in cash flow will result in a 0.25% depletion of assets by 2060. Therefore time-varying mortality rate model is an effective tool for pension systems to forecast mortality rates and project their future financial obligations.

Keywords: Life expectancy; Mortality rate model; Pension fund system; Transitional matrix; Funding Ratio

Introduction

Nowadays, the world’s demographics are changing dramatically. People live longer than in the past which causes the dependency ratio of the ageing population to increase (Bloom and Luca 2016). The improvement of technology and medical services to support good healthcare has posed a challenge to the country’s development by contributing to a decrease in the population’s mortality rates and an increase in life expectancy (Antolin 2007, Papademos 2009, Chand and Tung 2014, and Vollset et al. 2020). In developing countries such as Tanzania, the challenge holds an important agenda, particularly in the context of struggles for socio-economic progress, for example, in sectors like insurance corporations, healthcare, education, and pension systems (Mondal and Shitan 2014).

Moreover, this decrease in mortality rates and increase in life expectancy is necessary to be considered in pension fund projections since they determine the total number of individuals who may be eligible for benefits in the future (Galasso 2019). This paper assesses the Tanzania pension fund system’s sustainability by applying a mortality rate model that varies with both age and time. This approach captures the decrease in mortality rates and increase in life expectance of each year which is the advantage over the use of static lifetables. Lifetables use average mortality rates reported over some years and ignore the fact that life expectancy and mortality rates change over time.

Pension fund systems are designed with the main objective of collecting contributions and providing benefits (Mukalazi et al. 2021). There are two main plans of pension fund systems. Defined Contribution (DC) and Defined Benefit (DB). DC plan is a pension system in which the benefit provided to the retirees is not predetermined but is contingent and depends on the sum of contributions made by the members (Childs et al. 2002). DB plan is a retirement scheme where the payout to retirees is predetermined based on factors such as employee's salary, life expectancy, and years of service (Kisser et al. 2013).

This study focuses on the Tanzanian Public Service Social Security Fund (PSSSF) pension system as a case study, which practices a mandatory DB plan. Tanzania has two pension funds, National Social Security Fund (NSSF) for the private sector and PSSSF for the public sector. PSSSF was established in Tanzania in 2018 by the Act, Cap 371 (PSSSF Act) after merging other public service retirement schemes (Bachuba 2018). PSSSF members pay a contribution of 5% from their salaries and 15% from their employer. These funds pay various benefits including death, invalidity, maternity, unemployment, education, and retirement benefits. Retirement benefits are distributed in two main payments; annuities, which are monthly benefits paid throughout of the remaining time of retirees and commuted benefit, which is a lump sum paid at retirement.

Various research has been done about the pension system by using different techniques. Melis and Trudda (2012) employed an autoregressive model to examine both the asset return and the variation rate of entrants in pension plans in Italy. Jing et al. (2015) developed a risk measure for the PAYGO pension scheme, determining the total asset requirement necessary to sustain the pension plan. Godínez-Olivares et al. (2016) designed an optimal strategy using nonlinear dynamic programming to guarantee the required level of liquidity in PAYGO pension systems. Furthermore, Mukalazi et al. (2021) introduced an asset-liability management stochastic programming method to analyse assets for the parliamentary pension fund system in Uganda.

In Tanzania, research on the projection of pension fund systems includes Kyando (2014), who used a spreadsheet to assess the pension system's contribution to the development of the capital market in Tanzania. Nyangarika and Bundala (2020) examined the influence of retirement benefits and their impact on the socio-economic development of retirees using simple statistical regression methods. They found that, due to the increase in life expectancy, retirees get a little pension, which is not enough to cover their life expenses. Andongwisye et al. (2017) projected Tanzania's pension system by using fixed mortality rates over a period of 10 years. However, the approach fails to capture the decline in mortality rate and the rise in life expectancy.

Despite extensive research done on pension systems in Tanzania, to the best of my knowledge, no study has projected pension systems using an age and time-varying mortality rate model or different transitional matrices for each year. This study fills the gap by applying a developed mortality rate model by Suleiman et al. (2023) to forecast mortality rates of each year and generate different transitional matrices over 50 years. The objective is to analyse pension fund sustainability in terms of funding ratio and cash flow to asset ratio.

This paper is organized as follows: Section 2 gives an overview of the data and methods used in this work. Section 3, gives the results of the projections of liabilities, asset values, funding ratios, and cashflow to asset value ratios for each year using modelled mortality rate data and gives the discussion of the results. Section 4 gives conclusions remarks and recommendations for the study.

Materials and Methods

Materials

The data used in this study are generated mortality rates from regional-wise National Bureau of Statistics (NBS) population projections and total population projections reports (NBS 2002, NBS 2003). Initial values of active members, retirees, contributions, benefit payout, cash flow, assets, and liabilities were obtained from the annual statement reports of the Tanzania PSSSF (PSSSF 2020).

Methods

Time-Varying Mortality rate model

Consider the mortality rate model $\mu_{x,t}$ as in Suleiman et al. (2023), which varies with both ages x and years t, is given as,

$$\mu_{x,t} = \frac{c_1(t)e^{c_2(t)x}}{x} + a_1(t)\left(xe^{-a_2(t)x}\right)^{a_3(t)}. \qquad (1)$$

The parameters $c_1(t)$ and $c_2(t)$ represent the change in mortality rates for ages before adolescence and old age over time t respectively. Parameters $a_1(t)$, $a_2(t)$ and $a_3(t)$ describe the effect of the hump in the mortality rates for ages between adolescence and early adulthood around 15–25 years over time. The model was used to forecast mortality rates from 2020–2070 for ages 1 to 78+. The forecasted mortality rate was used to produce transitional matrices.

Population projection

Transitional Matrix

A transitional matrix is a square $(n \times n)$ matrix describing the transition of a stochastic system, where the size n of the matrix is related to the total number of elements in each set of the state that describes the system being transited (Aktas 2015).

Let Π_t be a transitional matrix at each year t given as

$$\Pi_t = \begin{array}{c|ccccccc|} x & 25 & 26 & \ldots & 59 & \text{Re} & \text{old} & \text{death} \\ \hline 25 & s_{25,t} & 0 & \cdots & 0 & 0 & 0 & d_{25n,t} \\ 26 & 0 & s_{26,t} & 0 & 0 & 0 & 0 & d_{26n,t} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 59 & 0 & 0 & \cdots & s_{59,t} & 0 & 0 & d_{59n,t} \\ \text{Re} & 0 & 0 & \cdots & 0 & s_{\text{Re},t} & 0 & d_{\text{Re}n,t} \\ \text{old} & 0 & 0 & \cdots & 0 & 0 & s_{\text{old},t} & d_{\text{old}n,t} \\ \text{death} & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \end{array}$$

The transitional matrix consists of diagonal survival probabilities $S_{x,t}$ and a last column represents death probabilities at the final state n as $d_{xn,t}$ and zero elsewhere. The variable x represents ages from 25 to 59 at states 1 to 35, the age of retiree 60 as (Re) at state 36, and an average of ages from 61 and above as (old) at states 37 and t are years such that $t = 1 \cdots 50$. That is, an individual will have a survival probability $S_{x,t}$ at a certain age x at the states 1 to 37 and zero elsewhere, since to transit to another age at the same year is not possible then he/she will transit to death at the final state $n = 38$ at each year t.

Also, $S_{x,t} + d_{xn,t} = 1,\ S_{x,t}, d_{xn,t} \geq 0$ and the element at the last row and column is a probability of death which is 1 since it is the state where a person has no probability of surviving when he/she is dead.

Survival probabilities

Survival probabilities are calculated to form transitional matrices using mortality rate results from Equation (1) and the actuarial formula of survival and death probability as presented in Dickson et al. (2009),

$$S_x(t) = exp\left[-\int_x^{x+t} \mu_{x+s} ds\right] \text{ and } d_x(t) = 1 - S_x(t),$$

where μ_{x+t} is the mortality rate for ages x and each year t. The survival probabilities were calculated for all 50 years from $2020-2070$.

Population size

The population model that describes Tanzania's pension fund system is an open system dynamic model. This model allows new members to join every year (Mettler 2005). To describe the pension funds population, let the size of the population in the pension system be P_t which varies due to the transitional matrix Π_t, a growth rate ω_t, and P_{t-1} as the previous population size. The pension population size is given as,

$$P_t = P_{t-1}(\Pi_t^T + \omega_t). \qquad (3)$$

Growth rate values in this study were assumed by focusing on both general Tanzanian growth rates stated by NBS (2013) and employee growth rate according to Natalie (2024) the growth rate for Tanzania's working population in 2020 is around 15%. So, the 5 different growth rates of 10 years intervals from 2020–2070 15%, 10%, 6%, 4%, and 4% respectively are assumed to a higher rate at the beginning to match the growth of employees, followed by a slow decrease due to the general decrease in Tanzanian growth rates.

Analysis of sustainability of the pension fund system

The most important variable in analysing the sustainability of the pension system is the funding ratio. It expresses the proportion between available assets and liabilities. Also, cashflow to asset values ratio assesses what fraction of asset values will be utilized to cover the deficit in pension systems.

The funding ratio is obtained as follows,

$$FR_t = \frac{A_t}{L_t} \times 100\%. \qquad (4)$$

A_t is the total asset and L_t is the liability at the time t. The total asset value is given by,

$$A_t = CF_t + (1+r)A_{t-1}, \quad (5)$$

where, $r = 4\%$ is the investment growth rate (PSSSF 2020) and cashflow at the time t is CF_t.

Also, the total liability is defined by the following equation.

$$L_t = \sum_{x=25}^{59} \frac{BP_{x,t}}{(1+d)^{60-x}}, \qquad (6)$$

where BP_{xt} is the expected total benefit for a person aged x in year t. The discount rate d is assumed to be 8%. The expected total benefit is calculated by,

$$BP_{x,t} = CB_{x,t} + MB_{x,t},$$

given that, CB_{xt} and MB_{xt} are commuted and monthly benefits for a member aged x in year t.

According to Tanzania pension regulations, CB_{xt} and MB_{xt} are given as

$$CB_{x,t} = {}^{1}\!/_{580} \times n_x \times m_x \times 12.5 \times S_{xt(ave)} \times 33\% \times S_x(60-x),$$
$$MB_{x,t} = {}^{1}\!/_{580} \times n_x \times m_x \times S_{xt(ave)} \times 67\% \times S_x(60-x) \times e_{p_{60(t)}},$$

where m_x is the total population of members aged x in the year t and n_x is the total number of months a member has contributed to the pension system. The final average employee's salary at age x and year t is defined by $S_{xt(ave)}$. The survival probability of a member of age x to survive until retirement, i.e., $t = 60 - x$ years is represented by $S_x(60-x)$ and $e_{p_{60(t)}}$ is the remaining life expectancy of a member who is of age x in years $t = 60 - x$ when he reaches the age of 60. Projections of $S_x(60-x)$ and $e_{p_{60t}}$were calculated using mortality rates from equation (1) with x ages from 25–59 and $t = 1$ to $t = (60-x)$ as follow,

$$S_x(60-x) = \exp\left(-\int_t^{t=60-x} \mu(x+s)ds\right) \text{ and } e_{p_{60(t)}} = \int_1^{t=60-x} S_x(s)ds. \qquad (7)$$

Furthermore, the study analysed the pension system using cashflow to asset values ratio which is calculated as follows.

$$CAS_t = \frac{CF_t}{A_t} \times 100\%. \qquad (8)$$

A cashflow to asset values ratio assesses what fraction of asset values are utilized to cover the lesser amount of benefit to contribution.

Results and Discussions

Mortality rates

Figure 1 illustrates the results of the logarithm of mortality rates for ages 1 to 78+ and years 2020, 2030, 2040, 2050, 2060, and 2070. Since mortality rates vary greatly over an individual's lifetime, it can be easier to see the qualitative behaviours of the mortality rate by looking at its logarithm. Negative values in the logarithm of mortality rates arise from the transformation of mortality rates from the model of equation (1) to logarithm to get the general statistical pattern of how mortality rates vary with age and time. Mortality rate values were small positive values between 0 and 1 in which 0 and 1 excluded, the logarithm became negative. Therefore, negative values do not represent the negative values of mortality rates.

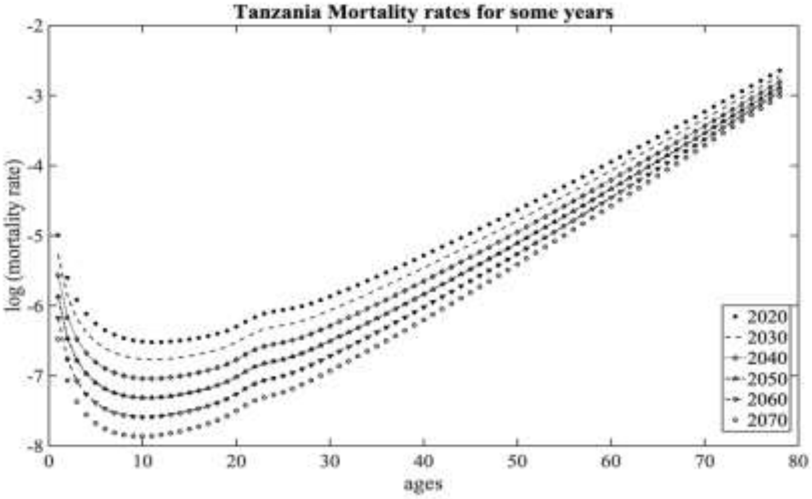

Figure 1: Tanzania mortality rates for years 2020, 2030, 2040, 2050, 2060, and 2070.

Figure 1 generally shows a decrease in mortality rate with age for all years. The mortality rate for ages 1 to 5 is high compared to ages 6 to 10. Then start to increase with a small hump at adolescent ages around 15 to 25 which is usually caused by the traffic accidents or unsafe lifestyle examples, involving the use of drugs or unsafe working conditions. Thereafter, mortality rates increase exponentially from the period of adulthood around ages 30 to older ages.

In terms of the pension system, as it is observed in Figure 1, there is a decrease in mortality rates trend for all ages in each year. For example, at age 60 as shown in Table 1, the mortality rate decreases with the decrease in growth rates of mortality between each ten years.

Table 1: Mortality rates and mortality growth rates for age 60 in years 2020, 2030, 2040, 2050, 2060, and 2070.

Years	2020	2030	2040	2050	2060	2070
Mortality rates	-3.9496	-4.0745	-4.2082	-4.3364	-4.4589	-4.5756
Growth rates	3.1%		2.9%		2.6%	

This decrease affects pension systems since it shows that as years increase, mortality rates for all ages will continue to decrease making individuals live longer which indicates high longevity risk and high liability payments in terms of pension fund systems. The government should establish the need for careful financial planning to support an ageing population. Also, the pension funds to use the mortality rate model to forecast Tanzanian mortality rates. As mortality model can capture the real trend of a decrease in mortality of the elderly in Tanzania and help in maintaining sustainability.

Pension Fund system

To discuss the results of the pension fund projection obtained, the study assumed the initial time for projection to be 2020. According to the PSSSF report in the year 2019/2020, the fund had a total of 697,677 members. This study used it as the initial total population size to calculate population size P_{t-1} age wise from 25–59 using population data from NBS (2013). The initial number of retirees aged 60 years was assumed to be zero at the beginning of the horizon. Also, an initial number of pensioners was obtained from (PSSSF 2020). The total assets and liabilities were then obtained.

Asset values calculated by Equation (5) are shown in Figure 2(a), the initial asset value is 6.436 trillion TSh in 2020. Equation (6) was used to calculate total liabilities given in Figure 2(b) using the initial value of liability is 6.927 trillion TSh in 2020.

The fast increase in the fund's total liability compared to the increase in assets is shown in Figure 2, which explains how projections indicate a decrease in mortality rates for pension members.

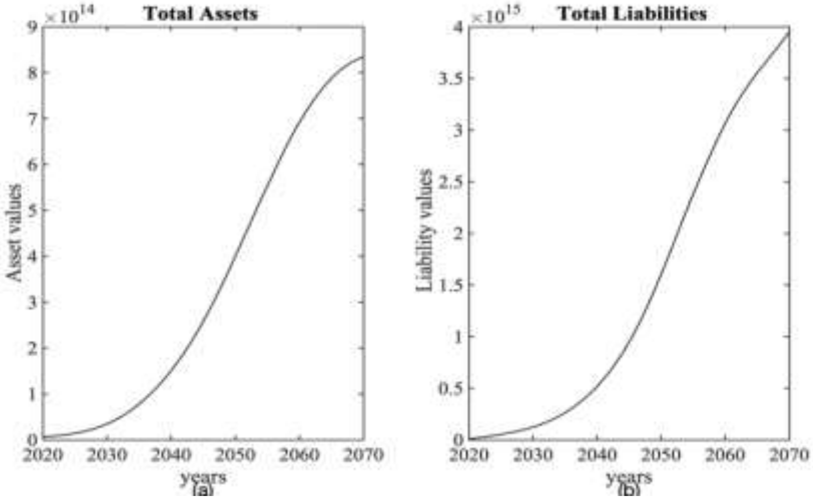

Figure 2: Total Assets and Liability values for years from 2020 to 2070.

The difference in their increase gives a higher increase in liability by 3943.7 trillion Tshs shown in 2(b) with a growth rate of 99.82% and an increase in asset values by 633.59 trillion Tshs in 2(a) with a growth rate of 99.22% from 2020 to 2070. The variability between total liabilities and total assets emphasizes the importance of the using mortality rate model that varies with time. Helps pension funds for better anticipation of the financial implications due to changes in demographic factors and make informed decisions to ensure the fund's long-term sustainability. Underestimating mortality rates can result in misleading the system projection of total assets and liability.

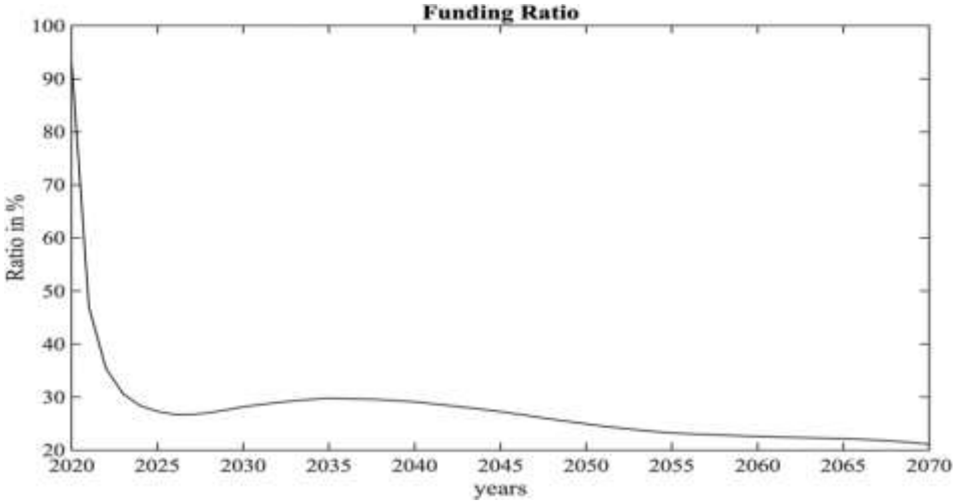

Figure 3: Funding ratio values for years from 2020 to 2070.

Furthermore, the projection of the funding ratio is shown in Figure 3. The funding ratio started with a percentage of 92.92% in 2020, then declined to 21.12% by the end of the horizon. This decline implies a faster increase in liability values compared to the increase in asset values. Makes asset values to only be sufficient to cover 21.12% of the total liabilities in the system. The figure shows a concerning decline in the pension system's finances which resulted from considering the improvement in mortality rates and life expectancy obtained by the mortality rates model of Equation (1). Results imply the need for strategic measures in forecasting Tanzanian mortality rates to adequately manage the challenges posed by an ageing population.

Finally, Figure 4 shows the cashflow to asset ratio obtained from equation (8). Initially it indicates a negative value of 3% in the year 2020 which was caused by the contributions being less than the paid-out benefit as reported by (PSSSF 2020).

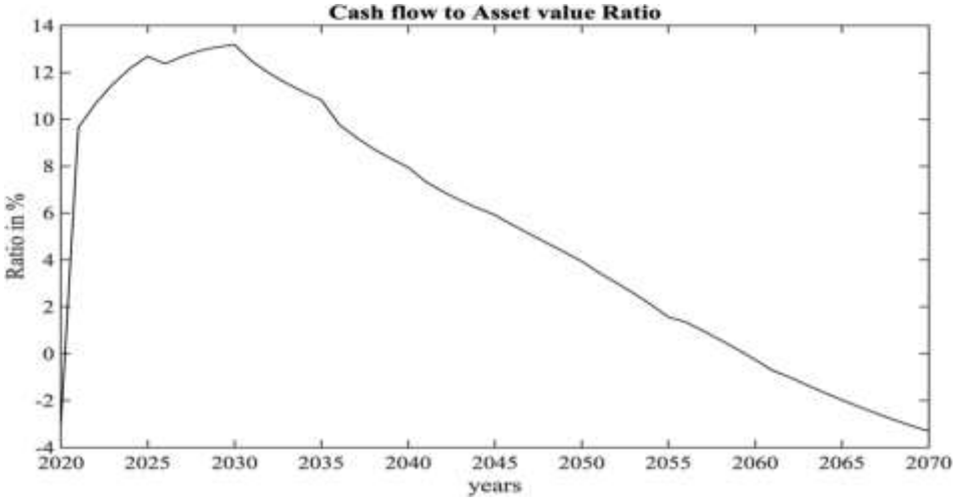

Figure 4: Cashflow to asset value ratios for years 2020 to 2070.

Then the system was projected to maintain the positive cashflow to asset value ratio until 2060, when the ratio started to deplete to 0.25% and continued to deplete to 3.3% at the end of the horizon. The result means that the system is at risk of becoming unsustainable in the future. Additionally, projection using different transitional matrices from the mortality rate model shows the depletion will start twelve years before the time stated by PSSSF (2020), which is in 2072. This implies the motivation of using the time-varying mortality rates model in pension systems to capture the real depletion of Tanzanian pension systems.

Conclusion and Recommendation.

The rise in life expectancy presents a significant challenge for pension systems worldwide. The issue arises from the decrease in mortality rates over time, especially for the old population who are expected to live longer. Consequently, they will consume more benefits than what was expected.

In this study, a new mortality rate model that varies over both time and ages was applied to forecast mortality rates for the years 2020 to 2070 and project pension system obligations. The study managed to differentiate pension projection method by applying the time varying mortality model. The applied model gave different mortality rates for each year over the period of 50 years. Having different mortality rates have an advantage of getting better results rather than using constant mortality rates for each year.

Results showed a decline in funding ratio as time increases. Hence, asset values will not be able to cover the total liability in the future. Also, the model addressed a 12-years early depletion of assets in 2060 compared to the one stated by the PSSSF, which is in 2072. The earlier depletion alert demonstrates that the decrease of mortality rates and increased life expectancy that is captured by the proposed model can have a noticeable effect on the sustainability of the pension system. Furthermore, the model can be used by demographers and statisticians for population projection yearly as mortality rate is one of the main inputs in developing population projection results.

Future research areas of this study will be addressing the underfunding and negative cashflow-to-asset ratios in Tanzanian pension funds.

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Declaration

The authors state that they have no conflicts of interest related to this research work.

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