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Bayesian Estimation of Regression Quantiles in the Presence of Autocorrelated Errors

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Abstract

This is a study of Bayesian quantile regression that broadly considered the estimation of regression quantiles in the presence of autocorrelated error. Regression models are based on several important statistical assumptions upon which their inferences rely. Autocorrelation of the error terms violates the ordinary least squares regression assumption that error terms are uncorrelated which invalidate Gauss Markov theorem. This study designed schemes for estimation and making inference of regression quantiles in the presence of autocorrelated errors using Bayesian approach. Bayesian method to quantile regression models regards unknown parameters as random variables and the parameter uncertainty was taken into account without relying on asymptotic approximations.The empirical analysis used the data set from Central Bank of Nigeria bulletin which comprised of Nigeria GDP growth, export rate, import rate, inflation rate and exchange rate from the period of 1985–2018. Bayesian inferences with autocorrelated error in the framework of quantile regression accounted better for the variability in the distribution of autocorrelation and gave robust and less biased estimates in dealing with non normality and non constant variance assumptions, the results of the research reported minimal Mean Square Errors in Bayesian approach than classical approach across the entire distribution.

Keywords: Bayesian Estimation, Regression Quantiles, Autocorrelated Errors, Regression Analysis.

Introduction

In regression analysis, the researcher is interested in analyzing the conduct of a dependent variable y_t given the information contained in a set of explanatory variables x_t, however, performing a regression does not spontaneously yield a reliable relationship between the variables but selecting an estimator that gives best parameter estimates.

Regression analysis seeks to find the relationship between a dependent variable and one or more independent variables, certain widely used strategy of regression such as ordinary least squares method has applauding properties if the underlying assumptions are true, but can give inappropriate inference and misleading decisions if those assumptions are not true; thus ordinary least squares is not robust to violations of its assumptions (Andersen 2008).

Autocorrelation of the error terms breaches the ordinary least squares regression assumption that error terms are uncorrelated, hence ordinary least squares no longer have the minimum variance property, hence invalidate Gauss Markov theorem. Regression models with correlated errors have been thecentre of significant attention in econometrics and statistics. Gujarati (2003) pinpointed several ways in which autocorrelation may be introduced; which include inertia, specification bias, excluded variables, transformation of the original model and manipulation of data. Shadish et al. (2013) worked on Bayesian estimate of autocorrelation in single case designs, the article proposed procedures of obtaining empirical Bayes estimates of autocorrelation only in mean regression model. The central location, the scale, the skewness, and other higher-order properties not central location alone characterize a distribution, thus mean models are inherently ill-equipped to depict the relationship between a response distribution and predictor variable. Since the groundbreaking work of Koenker and Bassett (1978), quantile regression models have been increasingly used in applied areas in economics due to their flexibility to allow researchers to investigate the relationship between economic variables not only at the centre, but also over the entire conditional distribution of the dependent variables. Theoretical results established that ordinary least squares regression models could be deficient if the probability distribution of the observed response variables do not follow a symmetric distribution (Min and Kim 2004). Quantile regression was able to tackle this problem since it turns out to be a better alternative for accommodating outliers and misspecification of the error terms. Bind (2016) studied quantile regression analysis of distributional effects of air pollution on blood pressure heart rate variability, blood lipids, and biomarkers of inflammation in American men. Koenker and Machado (1999) discovered the linkage between the quantile regression loss function and assymetric Laplace distribution.

Application of quantile regression appears in Yu and Moyeed (2001) and Tsionas (2003), which specify a Bayesian quantile regression model with independent and identically distributed asymmetric Laplace error terms, the posterior means were simulated using the Mefropolis Hasting algorithm which invalidate the estimates to be best linear unbiased estimates. Alhamzawi et al. (2012) worked on Bayesian regularized quantile regression with lasso by allowing different penenalization parameters for different regression coefficients. This study will fill the vacuum in the literature by examining the estimation of Bayesian quantiles regression models with serially correlated error using Gibb's sampling techniques. Bayesian inference in the context of quantile regression was achieved by adapting the problem to the framework of the generalized linear model using the asymmetric Laplace distribution for the error terms with $y = x^T \beta$ as proposed by Yu and Moyeed (2001).

Bayesian methods do not need to be tested for their sampling properties (Gelman 2008), instead they are concerned with the facts that the correct likelihood and prior are employed for Markov Chain Monte Carlo (MCMC) methods converge to the implied posterior distribution. This current research adopted a Bayesian approach to estimate the regression quantiles with correlated errors across the entire distribution.

Materials and Methods

In this section, the methods involved in the estimation of regression quantiles and Bayesian estimation of regression quantiles with autocorrelated error in the model are described.

Method of estimating regression quantiles

Let $$y_t = x_t^T \beta_\tau + \varepsilon_t, \quad t = 1, \ldots, n \qquad (1)$$

Where y_t be response variable and x_t, a $k \times 1$ vector of covariates for the t^{th} observation. ε_t is the error term whose distribution is restricted to have τ^{th} quantile equal to zero, that is

$$\int_{-\alpha}^{0} F_{\tau}(\varepsilon_t)d\varepsilon_t = \tau \qquad (2)$$

y_i, i = 1,…, n be a response variable and x_i, a k x 1 vector of covariates for the i^{th} observation.

let $Q_{\tau}(x_t)$ denote t^{th} $(0<\tau<1)$ quantile regression function of y_t given X_t. The relationship is

$$Q_{\tau}(x_t) = x_t^{'}\beta \qquad (3)$$

where β_{τ} is a vector of unknown parameters of interest, quantile regression estimation of β_{τ} proceeds by minimizing

$$\hat{\beta}_{\tau} = \arg\min_{\beta\in R^k} \sum_{t=1}^{n} \rho_{\tau}(y_t - x_t^{'}\beta_{\tau}) \qquad (4)$$

Where the loss function ρ is simplified as

$$\rho_{\tau}(u) = [\tau - I\{u<0\}] \qquad (5)$$

and the model's residuals are formulated as an indicator function with

$$\mathrm{I}\,\{u\} = \begin{Bmatrix} 1 & for\; u < 0 \\ 0 & otherwise \end{Bmatrix} \qquad (6)$$

The quantile $\hat{\beta}_{\tau}$ is the τ^{th} regression quantile.

The loss function is not differentiable; solutions to the minimization cannot be derived explicitly. Linear programming method in 'R' was designed to obtain quantile regression estimates for $\widehat{\beta}_{\tau}$ using the simplex iteration procedure of Koenker and d'Orey (1994), the minimum was obtained at the vertices of the feasible region.

Bayesian estimation of regression quantiles with autocorrelation

Considering the regression model

$$Y_t = X_t^T \beta(\tau) + \varepsilon_t \qquad (7)$$

Where: $$\varepsilon_t = \sum_{j=1}^{p} \rho_j \varepsilon_{t-j} + u_t \qquad (8)$$

For t = 1,…, n, X_{it} is the q dimensional predictors, u_t follows independently identical normal with mean 0 and variance $\sigma^2.\rho_j$, j = 1,…,p is the autocorrelation coefficient of order p which determines the dependency of the error term ε_t.

In contrast to the standard linear regression model, the error terms are correlated. Estimating the parameters in the model (7) is to transform it as follows:

$$y^{*}_{t(\tau)} = X^{*T}_{it}\, \beta(\tau) + u^{*}_{t} \qquad (9)$$

Where y_t^* and X_{it}^* represent the following transformed variable:

$$y_t^* = \sqrt{1-\rho^2\, y_t} \qquad (10)$$

for t = 1,2,3 …, n

and

$$X_t^* = X_t - \rho X_{t-1} \qquad (11)$$

for t = 2, 3, 4,…, n

the model inference theoretically requires the initial values $(y_0, \dots, y_{t-p})$ and $(x, \dots, x_{t-p})$, noting that error term u_t are independent normal, the assumption that errors are independent over all individual and time periods implies that the transformed model simply reduces to the standard linear regression framework, the density can be expressed as

$$f(y_t^* \big| X_{t-1}^*, \alpha, \beta, \rho)$$

$$= \frac{1}{(2\pi\sigma^2)^{(t-p)/2}}\, exp\left[\frac{-(y_t^* - X_t^*\beta)^T(y_t^* - X_t^*\beta)}{2\sigma^2}\right] \qquad (12)$$

Where ρ is the autocorrelation coefficient of order one and $X_t^*$and y_t^* are the (t-p) x q dimensional matrix and t-p dimensional vector, respectively which depend on autocorrelation coefficient ρ.

Bayesian implementation with quantile regression begins by erecting a likelihood, the error term in equation (9) is assumed to follow the asymmetric Laplace distribution. Adopting the method of Kozumi and Kobayashi (2011), the asymmetric Laplace distribution was allowed to be represented as a location of scale mixture of normal distribution

$$y_t = X'_{it}\,\beta(\tau) + \theta\, Z_i + \delta\sqrt{(z_i u_i)}. \quad (13)$$

where the mixing distribution follows an exponential distribution in equation (13) for effective and easy draw.

where $\theta\, Z_i + \delta\sqrt{(z_i u_i)}$. is the error term expressed as a location of scale mixture of normal distribution.

A scale parameter σ was introduced into the model in equation (13) given as

$$y^*_{t\,(\tau)} = X^{*T}_{it}\,\beta_i(\tau) + \sigma\theta Z_i + \delta\sqrt{Z_i u_i} \quad (14)$$

Re-expressing equation (14) in terms of the parameter for easy sampling gives;

$$y^*_{t(\tau)} = X^{*T}_{it}\,\beta_i(\tau) + \theta v_i + \delta\sqrt{\sigma\, v_i u_i} \quad (15)$$

This leads to the likelihood function

$$f(y^*_t | X'_t\beta, v, \sigma, \rho, \tau)\exp\frac{-\sum_{t=1}^{n}(y^*_t - X'_t\beta(\tau) - uv_i^2}{2\delta^2\sigma v_i}\prod\frac{1}{\sigma v_i} \quad (16)$$

To proceed in the Bayesian analysis, conjugate prior for β, σ, ρ and v was chosen separately.

Prior of $\beta \sim N(\beta_\tau, B_\tau)$ (17)

where β_τ, B_τ are the hyperparameters of β prior chosen from normal distribution at various chosen quantiles.

For the prior on σ, inverse gamma distribution I G(a,b), inv Gamma(shape = $n_{0,}$ scale = $s_{0,}$ was chosen with density

$$f(x|n_0, s_0) = \frac{s_0^{\,n_0}}{\Gamma(n_0)}x^{-n_0-1}\exp\left(-\frac{s_0}{x}\right) \quad (18)$$

The posterior distribution for σ follows an inverse Gamma distribution

$$\sigma\ |y_t, \beta, v, \rho \sim IG(\frac{n^*}{2}\frac{s^*}{2}) \quad (19)$$

The prior of v_i follows a generalized inverse Gaussian distribution

$$v_i\ |y_t, \beta, \sigma, \rho \sim GIG(\frac{1}{2}, \alpha_i, \gamma_i) \quad (20)$$

where the probability density function of $GIG(v, \alpha, \gamma)$ is given by

$$f(x|v, \alpha, \gamma) = \frac{(\gamma|\alpha)^v}{2kv(\alpha\lambda)}x^{v-1}\exp\left(-\frac{1}{2}(\alpha^2 x^{-1} + \gamma^2 x)\right) \quad (21)$$

$x > 0$, $-\infty < v < \infty$, $\alpha, \gamma \geq 0$, and $kv(\alpha\gamma)$ is a modified Bessel function of the third kind.

However, the posterior distribution for v_i still follows a generalized inverse Gaussian distribution

$$v_i\ |y_t, \beta, \sigma, \rho \sim GIG(\frac{1}{2}, \alpha_i, \gamma_i) \quad (22)$$

The posterior of ρ depends upon its prior that reflects the research's non data information. The improper prior of ρ follows a multivariate normal truncated to the stationary region

$$f(\rho) = I(\rho \in \emptyset) \quad (23)$$

The probability distribution is denoted by

$$\frac{1}{(\emptyset(b^*) - \emptyset(a^*))}\frac{1}{\sqrt{2\pi\sigma^2}}\,e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\times I(a \leq x \leq b\right)} \quad (24)$$

where $I(\rho \in \phi)$ is the indicator function which equals 1 for the stationary region and zero otherwise and ϕ denotes stationary region for the model.

Hence the conditional posterior density of ρ given β, σ, v, is written as

$$\text{`TN}\,(\hat{\rho}_n^{\,*}\sigma^2 V_n^*\phi) \quad (25)$$

Where: $\hat{\rho}_n^* = (E^T E)^{-1}E^T\varepsilon$ (26)

and

$$V_n^* = (E^T E)^{-1} \quad (27)$$

E is a (n-p) x k matrix, the n[th] row given by $\varepsilon_{n-1} \ldots \varepsilon_{n-p}$ and ε denotes an n-p dimensional vector.

However, drawing ρ from truncated multivariate normal was done by drawing from untruncated normal $N(\hat{\rho}_n^* \sigma^2 V_n^*)$ and discard the draws which fall outside the stationary region. Combining the likelihood density in (16) with the prior specification $for\ \beta, v, \sigma\ and\ \rho$ in equations (17), (18), (20) and (23). The joint posterior distribution of $(\beta, v, \sigma\ and\ \rho)$ becomes $\pi(\beta, \sigma, v, \rho | y_t, X_t, \tau) \propto$ L($(y_t | X'_{it}\beta, v, \sigma, \rho, \tau)$ x joint priors of $(\beta, v, \sigma\ and\ \rho)$.

This yields the following full conditional posteriors

$$\begin{bmatrix} \beta | y, v, \sigma, \rho \sim N(\beta_p, B_p \\ \sigma | y, \beta, v, \rho \sim IG\left(\frac{n^*}{2}, \frac{s^*}{2}\right) \\ v | y, \beta, \sigma, \rho \sim GIG\left(\frac{1}{2}, \alpha_i \gamma_i\right) \\ \rho | y, \beta, \sigma, v \sim {}'TN(\hat{\rho}_n^* \sigma^2 V_n^* \emptyset) \end{bmatrix} \quad (28)$$

The full conditional posterior distribution of β, σ, v and ρ is not of tractable form, therefore MCMC method is employed using Gibb's sampling to draw samples from the posterior. The Gibb's sampler is an iterative Monte Carlo scheme designed to extract conditional posterior distribution from intractable joint distribution. Gibb sampler was run for 120,000 replications and discarded the first 20,000 as burn-in period. The Bayesian test considered the relevant posterior interval estimate. The test was performed separately for each τ, in the presence of autocorrelated error, which is then used for posterior inferences. MCMC sampling was carried out in R (R Development Core Team 2016), the Monte Carlo simulation was implemented by taking random draws from the posterior distribution of β and then averaging the appropriate functions of these draws across the quantiles range Model comparison was done between the frequentist and Bayesian methods.

Empirical study

To illustrate the estimation method of Bayesian quantile regression method empirically with autocorrelated error, the performance of the MCMC scheme proposed was checked considering the data set from Nigeria CBN bulletin which comprised of Nigeria GDP growth, export rate, import rate, inflation, and exchange rate from the period of 1985-2018. The response variable is the GDP growth, while the explanatory variables are the export, import, inflation and the exchange rate, using the model

$$y_t = \beta_o + \beta_1 x_t + \beta_2 x_t + \beta_3 x_t + \beta_4 x_t + \varepsilon_t \quad (29)$$

Where y_t = GDP growth, X_{1t} = import rate at time t, X_{2t} = export rate at time t, X_{3t} = inflation rate at time t and X_{4t} = exchange rate at time t, posterior estimates for $\beta_{it}(\tau)$ for τ = 0.05, 0.10,…, 0.95 quantiles using the Gibb's sampling were obtained, where ε_t was generated based on the assumption of $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$ where $u_t \sim$ N(0, σ^2), where σ_ε^2=1,the value of ρ is an AR(1) parameter autocorrelation coefficient which was determined through the estimation procedures from Cochrane Orcutt approach of estimating autocorrelation coefficient in regression model.

Quantile regression model with autocorrelated error

In most cases, time series data inherits autocorrelation, this property was verified in the quantile regression models with Ljung-Box test, this test was applied to residual from the fitted parametric quantile regression model in equation (29) at lag 1. The Ljung–Box test examines the null of independently distributed residuals, it was derived from the idea that the residuals of a correctly specified model are independently distributed. The test statistic is computed using the residuals of the regression quantiles estimates, the regression quantiles estimates for $\widehat{\beta}_{\tau}$. were estimated using the method of simplex iteration procedure of Koenker and d'Orey (1994). Mean square error was used as a criterion of validation to measure the relative effectiveness of Bayesian quantile regression with autocorrelated error and classical quantile regression with autocorrelated error in exploring the data at τ^{th} quantile.

Results and Discussion

Table 1 below presents the posterior means and their standard deviations in parenthesis for the selected quantiles of Bayesian quantile regression with autocorrelated error (BQRWA) parameter estimates, the result obtained was based on the procedures itemized in the method of estimating regression quantiles with autocorrelated error using bayesian approach.

All results were based on 120,000 replications with 20,000 burn-in replications discarded and 100,000 replications retained. Furthermore, Bayesian estimation approaches were employed to simultaneously estimate the quantiles parameters and the serially correlated residual parameters.

Table 1: The Bayes Estimates of the Bayesian quantiles regression parameters with auto-correlated error

τ	β_0	β_1	β_2	β_3	β_4	ρ
0.05	0.4847 (0.03)	0.1410 (0.16)	–0.2461 (0.07)	–0.188 (0.49)	0.2030 (0.02)	0.0483 (0.19)
0.10	0.7503 (0.24)	0.1088 (0.05)	–0.1088 (0.10)	–0.1467 (0.29)	0.0743 (0.04)	0.0619 (0.18)
0.15	0.7838 (0.06)	0.0742(0.025)	–0.101(0.013)	–0.0439 (0.04)	0.0965 (0.17)	0.0587 (0.08)
0.20	0.8539 (0.09)	0.0767 (0.11)	–0.0916 (0.04)	–0.0317 (0.165)	0.0620 (0.04)	0.0656 (0.122)
0.25	0.8692(0.017)	0.0615 (0.01)	–0.080(0.155)	–0.0245 (0.03)	0.0599 (0.29)	0.0524 (0.006)
0.30	0.8720 (0.5)	0.0507 (0.09)	–0.0823 (0.26)	–0.0121 (0.11)	0.0754 (0.39)	0.0472 (0.007)
0.35	0.8913 (0.04)	0.0434 (0.08)	–0.074 (0.017)	0.0019 (0.016)	0.0767 (0.002)	0.0524 (0.010)
0.40	0.899(0.009)	0.0438(0.003)	–0.083 (0.007)	0.009 (0.001)	0.0799 (0.025)	0.0481 (0.05)
0.45	0.8985(0.012)	0.0380(0.057)	–0.081(0.001)	0.0015 (0.048)	0.0887 (0.029)	0.0447 (0.04)
0.50	0.8994 (0.03)	0.0243(0.018)	–0.076 (0.021)	0.0129 (0.093)	0.1003 (0.008)	0.0431 (0.014)
0.55	0.9016(0.027)	0.0182(0.061)	–0.0792 (0.09)	0.0144 (0.028)	0.1117 (0.018)	0.0342 (0.063)
0.60	0.8984(0.018)	0.0055(0.030)	–0.070 (0.076)	0.0214 (0.024)	0.1237 (0.015)	0.0345 (0.094)
0.65	0.9147(0.036)	0.0076 0.075)	–0.081 (0.027)	0.0252 (0.052)	0.1190 (0.014)	0.0354 (0.078)
0.70	0.9052(0.020)	–0.019(0.069)	–0.061 (0.051)	0.0306 (0.045)	0.1445 (0.011)	0.0227 (0.027)
0.75	0.908 (0.036)	–0.028(0.005)	–0.073 (0.031)	0.0412 (0.024)	0.1558 (0.048)	0.0274 (0.099)
0.80	0.9329(0.085)	–0.035(0.011)	–0.069(0.045)	0.0648 (0.081)	0.1735 (0.176)	0.0230 (0.029)
0.85	0.9914(0.022)	–0.056(0.063)	–0.042 (0.010)	0.0636 (0.055)	0.1363 (0.028)	0.0109 (0.017)
0.90	1.0534(0.013)	–0.094(0.072)	–0.005(0.084)	0.1448 (0.062)	0.0051 (0.081)	0.0368 (0.031)
0.95	1.2091(0.046)	–0.111(0.008)	0.0068 (0.015)	0.2478 (0.025)	0.0462 (0.07)	0.0411 (0.165)

Table 2: Quantiles parameter estimates using quantile regression with auto-correlated error

τ	β_0	β_1	β_2	β_3	β_4	ε_t
0.05	1.2093	2.1594	–2.1440	–1.6620	–1.9562	3.3111
0.10	1.2093	2.1594	–2.1440	–0.16620	–0.1956	3.0692
0.15	1.1057	0.5990	–1.1185	–0.0663	0.0518	–1.1185
0.20	0.9253	0.0768	–0.0941	–0.0352	0.0632	0.0617
0.25	0.8814	0.0595	–0.0816	–0.0272	0.0584	0.0522
0.30	0.8736	0.0512	–0.0831	–0.0137	0.0735	0.0475
0.35	0.8853	0.0418	–0.0790	0.0026	0.0684	0.0516
0.40	0.8754	0.0427	–0.0800	0.0006	0.0783	0.0460
0.45	0.8618	0.0325	–0.0862	0.0025	0.0837	0.0482
0.50	0.8792	0.0217	–0.0780	0.0173	0.1162	0.044
0.55	0.8936	0.0210	–0.0752	0.0152	0.1219	0.0381
0.60	0.8973	0.0050	–0.0715	0.0225	0.1305	0.0364
0.65	0.9086	0.0072	–0.0831	0.0243	0.1475	0.0390
0.70	0.9257	–0.0218	–0.0613	0.0307	0.1511	0.0261
0.75	–0.019	–0.7163	0.0398	0.1610	0.1581	0.0199
0.80	0.9408	–0.0316	–0.0597	0.0629	0.1742	0.0217
0.85	0.9735	–0.0553	–0.0460	0.0613	0.1514	0.0162
0.90	1.1446	–0.0901	–0.0018	0.1377	0.1426	0.0052
0.95	1.0037	–0.1125	0.0624	0.2548	0.0481	0.0074

The Ljung Box test statistic for the model in equation (29) is 165.0825 which has a *p* –value of $1.8e^{-14}$. since the *p* –value is close to zero for the test statistic, it is concluded that the economic data has significant autocorrelation. The final estimate of ρ obtained from the Cochraine Orcutt procedure is 0.70 which lies between 0 and 1 as expected for a ρ when autocorrelation is present. After fitting a parametric model to the numerical data, Table 2 comprises the frequentist estimates of the model obtained using the method of estimating regression quantiles highlighted above and resampled error terms from the autocorrelated residuals using the empirical data.

Table 3 above reports the MSE of Quantile regression model with autocorrelated error (QRWA) and Bayesian Quantile regression model with autocorrelated error (BQRWA) at various selected quantiles. Comparing the mean square error of the frequentist approach with the Bayesian approach in Table 3, it is revealed from empirical results that Bayesian approach produced minimal MSE which implies that the Bayesian approach in estimating regression quantiles in the presence of serially correlated error outperformed the frequentist approach in terms of MSE.

Table 3: MSE of Quantile Regression Model with Autocorrelated Error (QRWA) and Bayesian Quantile Regression Model with Autocorrelated Error (BQRWA)

Quantiles	QRWA	BQRWA
0.05	0.0258	0.0051
0.10	0.0153	0.0046
0.15	0.0607	0.0049
0.20	0.0315	0.0026
0.25	0.0720	0.0618
0.30	0.0552	0.0049
0.35	0.0376	0.0085
0.40	0.0295	0.0068
0.45	0.0361	0.0077
0.50	0.0826	0.0020
0.55	0.0945	0.0018
0.60	0.0312	0.0025
0.65	0.0364	0.0099
0.70	0.0266	0.0012
0.75	0.0436	0.0026
0.80	0.0297	0.0032
0.85	0.0610	0.0085
0.90	0.0294	0.0011
0.95	0.0233	0.0072

Conclusion

This study expatiated the estimation of quantile regression models using the Bayesian approach. The estimation of coefficients in a simple regression with autocorrelated errors is an important problem that has received a great deal of attention in econometrics. The research work measured quantile relations after allowing serial correlated error, using likelihood–based approach. It explored the predictive ability of a model on a data set that has autocorrelated errors which were used to fit the model.

The work develops a practical framework for Bayesian analysis of regression models with autocorrelation. Compared to the frequentist estimate, the Bayesian method still performs better even when the error distribution assumption is violated. This research gives an insight into the methods of estimating regression quantiles in the presence of autocorrelated error. It is observed that when dealing with non-normality and non-constant variance assumption, the performance of Bayesian quantile regression does not depend on autocorrelation level as the research reported minimal MSE across the entire quantiles with autocorrelated errors. The smaller mean square error in Bayesian estimation of regression quantiles in the presence of autocorrelated errors proves that the data values are dispersed closely to its central moment and produces minimal errors. The Bayesian approach in the framework of quantile regression gives robust and less biased estimates, the research justified the results of Shadish et al., (2013) that worked on the estimation of Bayesian estimates of autocorrelation in a single case design that concluded that Bayesian estimation reduces the role of sampling error.

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