

COMPARISON OF DIRECT RETURN AND BIRTH-DEATH RETURN RECEIVERS

HN Kundaeli

Department of Physics, University of Dar-es-Salaam,
P.O. Box 35063, Dar-es-Salaam, Tanzania

ABSTRACT

The performance of frame synchronized communication systems is governed largely by the design chosen for the receiver. Some systems employ a structure in which the receiver returns directly to the sync phase when a valid marker is encountered while in the loss verify state. These have been greatly studied. In some systems, the return path in the loss verify phase is implemented as a birth-death process. These systems have not received much attention. In this report, a receiver which employs a birth-death like structure in the loss path is investigated. Expressions for the performance parameters are derived and compared with those obtained for direct return systems. It is found that both types of receivers have the same performance at lower error rates and low loss verify number, M . At high error rates and high values of M , however, the birth-death receiver outperforms the direct-return receiver. To the knowledge of the author, this is the first time that performance expressions have been reported in this form.

INTRODUCTION

The architecture of receivers for frame synchronized systems depends on a number of parameters. These parameters include the structure and length of the frame, the structure and length of the marker, the allowed error threshold in both the recovery and the loss states of operation, and the window size used for error checking. Receivers for a system employing a single marker, an error threshold of 0 and no windowing have been analyzed by Kundaeli (1995, 1996). Likewise, receivers employing variable error thresholds have been analyzed by Munhoz *et al.* (1980) and Al-Subbagh and Jones (1988), while a receiver employing windowing have been analyzed by Nilsson *et al.* (1991). In these systems, if the receiver encounters a single error free marker when it is about to lose synchronization, it returns directly to the sync phase. On the other hand, in some kinds of systems, part of the receiver is implemented as a

birth-death process. In this case, if the receiver encounters an error free marker while in the loss verify state, it does not return directly to the sync phase, but rather, it advances one step towards it. An example of such a system is the one analyzed by Dodds *et al.* (1985), which, apart from allowing variable error thresholds, has 3 states implemented as a birth-death process. The study of Eu and Rollins (1991) also analyzed receivers with birth-death structures in the loss path, which in addition allow both windowing and variable error thresholds. As an extension to the analysis of these receivers, Kang *et al.* (1992) have also derived the statistical performance parameters.

In the current paper, a receiver consisting of a birth-death structure in the loss path is analyzed. It resembles the ATM receiver analyzed by Dodds and Du (1993), but unlike in that report, this analysis goes on to derive general expressions for the performance of the receiver. These are then compared with those obtained for the direct return receivers.

METHODS

Analysis for Transition Probabilities

The analyzed system is represented by the transition diagram given in Fig. 1. The transition diagram has been split into the true (states 1, 2, 4 and 6) and false (states 1, 3, 5 and 7) paths. When the receiver is reset or when it has lost synchronization, it enters state 1 and proceeds to search for the marker on a bit by bit basis. If it encounters the error-free marker, it goes through the true path, whereas if it encounters any other bit sequence resembling the error free marker, it goes through the false path. If the receiver enters state 2, it tests the marker in each received frame and if it encounters $N-1$ consecutive frames with error-free markers, it transits to state 4. If it encounters a single mismatch however, it returns to state 1 to start the search again. While in state 4, the receiver performs tests on the marker in each received frame and it stays in this state collecting data from the received frames as long as the frames contain valid markers. When an invalid marker is received in state 4, the receiver transits to state 61, and advances in single steps towards state 1 with each received false marker. If it receives a valid marker while in states 61 to 6M, it advances one step towards state 4. Because of the symmetry of the transition diagram, the operation of the receiver in the false path is exactly the same as that in the true path.

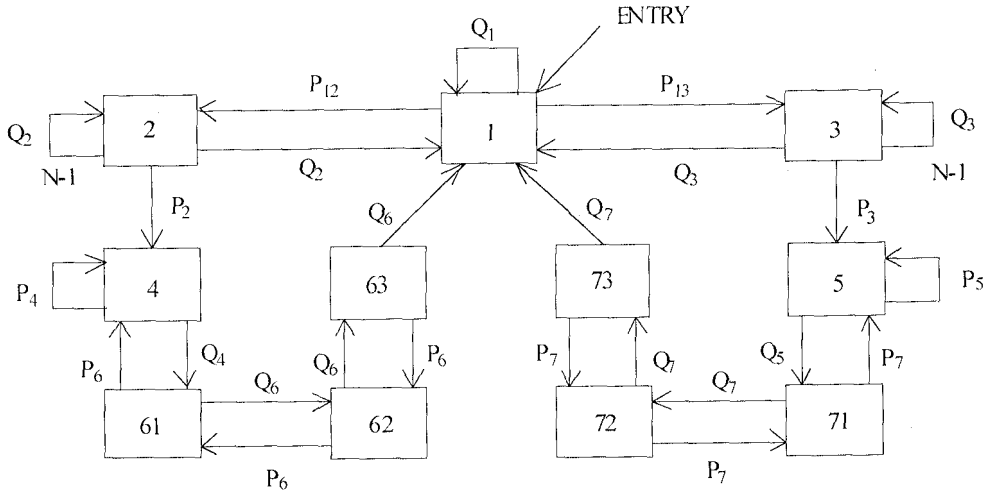


Fig. 1: The transition diagram of a receiver with a birth-death structure and $M = 3$.

The relevant transfer functions for this receiver can be derived by first reducing the transition diagram using graph reduction techniques (Sittler 1956) to the form shown in Fig. 2.

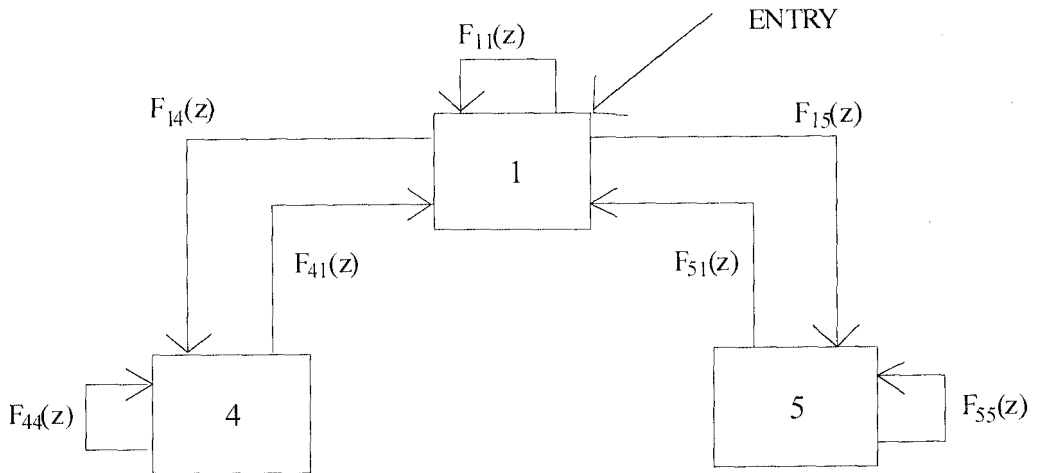


Fig. 2: The reduced transmission diagram of Fig. 1.

The partial transfer functions in Fig. 2 are then given by

$$F_{11}(z) = \left[I - (I - z) \sum_{n=0}^{N-1} (P_{12} P_2^n + P_{13} P_3^n) z^n - (P_{12} P_2^N + P_{13} P_3^N) z^N \right] z^\alpha \quad (1)$$

$$F_{14}(z) = P_{12} P_2^N z^{N \cdot \alpha} \tag{2}$$

$$F_{15}(z) = P_{13} P_3^N z^{N \cdot \alpha} \tag{3}$$

$$F_{41}(z) = \frac{Q_4 Q_6^M z^{M \cdot l}}{T_4(z)} \tag{4}$$

$$F_{51}(z) = \frac{Q_5 Q_7^M z^{M \cdot l}}{T_5(z)} \tag{5}$$

$$F_{44}(z) = \frac{B_4(z)}{T_4(z)} \tag{6}$$

$$F_{55}(z) = \frac{B_5(z)}{T_5(z)} \tag{7}$$

where

$$B_4(z) = P_4 z T_4(z) + Q_4 P_6 z^2 T_6(z) \tag{8}$$

$$B_5(z) = P_5 z T_5(z) + Q_5 P_7 z^2 T_7(z) \tag{9}$$

$$T_4(z) = \sum_{m=0}^{Mc} (-1)^m A_m (P_6 Q_6 z^2)^m \tag{10}$$

$$T_5(z) = \sum_{m=0}^{Mc} (-1)^m A_m (P_7 Q_7 z^2)^m \tag{11}$$

$$T_6(z) = \sum_{m=0}^{Mc} (-1)^m C_m (Q_6 P_6 z^2)^m \tag{12}$$

$$T_7(z) = \sum_{m=0}^{Mc} (-1)^m C_m (Q_7 P_7 z^2)^m \tag{13}$$

$$A_m = \frac{(M - m)!}{(M - 2m)! m!},$$

$$C_m = \begin{cases} 0, & M = 0, \\ \frac{(M - m - 1)!}{(M - 2m - 1)! m!}, & M > 0. \end{cases}$$

(14)

$M_c = \text{floor}(M/2)$, $K_c = \text{floor}(K/2)$, $K = M - 1$, $\alpha = (\text{duration of 1 bit})/(\text{duration of 1 frame})$ and $\text{floor}(x)$ is the largest integer which does not exceed x .

Derivation of performance parameters

Using the partial transfer functions derived in the previous section, the transfer function from states 1 to 4 is given by

$$\phi_{14}(z) = \frac{F_{14}(z)(1 - F_{55}(z))}{1 - F_{55}(z) - F_{11}(z)(1 - F_{55}(z)) - F_{15}(z)F_{51}(z)} \quad (15)$$

We can rewrite (15) in short form as

$$\phi_{14}(z) = \frac{X(z)}{Y(z)} \quad (16)$$

and express the recovery time from state 1 to 4 as

$$L_{14} = \left. \frac{d}{dz} \phi_{14}(z) \right|_{z=1} = \left. \frac{d}{dz} \left(\frac{X(z)}{Y(z)} \right) \right|_{z=1} = \frac{X'(1)Y(1) - X(1)Y'(1)}{[Y(1)]^2} \quad (17)$$

We then use the notations F_{ij} , T_j and T_j' in place of $F_{ij}(1)$, $T_j(1)$ and $T_j'(1)$ respectively to obtain

$$X(I) = F_{14} [I - P_5] T_5 - Q_5 P_7 T_7 = P_{12} P_2^N Q_5 [T_5 - P_7 T_7] \quad (18)$$

We then perform the expansion

$$\begin{aligned} T_5 - P_7 T_7 &= \sum_{m=0}^{Mc} \frac{(-1)^m (M-m)!}{(M-2m)! m!} \sum_{n=0}^m \frac{(-1)^n m!}{(m-n)! n!} Q_7^{m-n} \\ &\quad - \sum_{m=0}^{Kc} \frac{(-1)^m (M-m-1)!}{(M-2m-1)! m!} \sum_{n=0}^m \frac{(-1)^n m!}{(m-n)! n!} Q_7^{m-n} \\ &\quad + \sum_{m=0}^{Kc} \frac{(-1)^m (M-m-1)!}{(M-2m-1)! m!} \sum_{n=0}^m \frac{(-1)^n m!}{(m-n)! n!} Q_7^{m-n-1} \end{aligned} \quad (19)$$

and use the relation

$$\sum_{m=0}^M \alpha_m z^m \sum_{n=0}^m \beta_n z^n = \sum_{k=0}^M \sum_{n=0}^{Kc} \alpha_{k-n} \beta_n z^k + \sum_{k=M-1}^{2M} \sum_{n=k-M}^{Kc} \alpha_{k-n} \beta_n z^k \quad (20)$$

with $kc = \text{floor}(k/2)$ to obtain

$$\begin{aligned}
 T_5 - P_7 T_7 = & \sum_{k=0}^{Mc} (-1)^k Q_7^k \sum_{n=0}^{kc} \frac{(M-k+n)!}{(M-2k+2n)!(k-2n)!n!} \\
 & + \sum_{k=Mc+1}^{2Mc} (-1)^k Q_7^k \sum_{n=k-Mc}^{kc} \frac{(M-k+n)!}{(M-2k+2n)!(k-2n)!n!} \\
 & - \sum_{k=0}^{Kc} (-1)^k Q_7^k \sum_{n=0}^{kc} \frac{(M-k+n-1)!}{(M-2k+2n-1)!(k-2n)!n!} \\
 & - \sum_{k=Kc+1}^{2Kc} (-1)^k Q_7^k \sum_{n=k-Kc}^{kc} \frac{(M-k+n-1)!}{(M-2k+2n-1)!(k-2n)!n!} \\
 & + \sum_{k=0}^{Kc} (-1)^k Q_7^{k-1} \sum_{n=0}^{kc} \frac{(M-k+n-1)!}{(M-2k+2n-1)!(k-2n)!n!} \\
 & + \sum_{k=Kc+1}^{2Kc} (-1)^k Q_7^{k-1} \sum_{n=k-Kc}^{kc} \frac{(M-k+n-1)!}{(M-2k+2n-1)!(k-2n)!n!} \tag{21}
 \end{aligned}$$

If we take M as odd and set $M = 2S + 1$ or take M as even and set $M = 2S$ we find that the coefficients of equal powers of Q_7 in (21) cancel out to give the result

$$T_5 - P_7 T_7 = Q_7^M \tag{22}$$

and we therefore obtain

$$X(I) = P_{12} P_2^N Q_5 [T_5 - P_7 T_7] = P_{12} P_2^N Q_5 Q_7^M \tag{23}$$

If we proceed in a similar manner, we obtain

$$Y(I) = P_{12} P_2^N Q_5 Q_7^M = X(I) \tag{24}$$

We therefore obtain the recovery time as

$$L_{14} = \frac{X(I) - Y(I)}{P_{12} P_2^N Q_5 Q_7^M} \tag{25}$$

By employing the same procedure as above, and after a little algebraic manipulation we obtain

$$\begin{aligned}
 X(1) - Y(1) = & \left[\alpha + \sum_{n=0}^{N-1} (P_{12} P_2^n + P_{13} P_3^n) \right] Q_5 Q_7^M \\
 & + P_{13} P_3^N \left[M Q_5 Q_7^M + T_5 - Q_5 T_5^{-1} + Q_5 P_7 T_7 + Q_5 P_7 T_7^{-1} \right]
 \end{aligned}
 \tag{26}$$

and if we apply more algebraic manipulation to (26), we obtain the transfer function from state 1 to 4 as

$$L_{14} = \frac{\left[\alpha + \sum_{n=0}^{N-1} (P_{12} P_2^n + P_{13} P_3^n) \right] Q_5 Q_7^M + P_{13} P_3^N \Delta_{57}}{P_{12} P_2^N Q_5 Q_7^M}
 \tag{27}$$

where for the case when M is odd and given as $M = 2S + 1$

$$\begin{aligned}
 \Delta_{57} = & T_5 + Q_5 \left[I - \sum_{k=1}^S (-1)^k Q_7^k \left\{ \sum_{n=0}^{k_c} U_n - \sum_{n=0}^{k_c} V_n - \sum_{n=0}^{k_c^*} W_n \right\} \right. \\
 & \left. - \sum_{k=S+1}^{2S} (-1)^k Q_7^k \left\{ \sum_{n=k-S}^{k_c} U_n - \sum_{n=k-S}^{k_c} V_n - \sum_{n=k-S-1}^{k_c^*} W_n \right\} \right]
 \end{aligned}
 \tag{28}$$

and for the case when M is even and given as $M = 2S$

$$\begin{aligned}
 \Delta_{57} = & T_5 + Q_5 \left[I - \sum_{k=1}^{S-1} (-1)^k Q_7^k \left\{ \sum_{n=0}^{k_c} U_n - \sum_{n=0}^{k_c} V_n - \sum_{n=0}^{k_c^*} W_n \right\} \right. \\
 & \left. - \sum_{k=S}^{2S-2} (-1)^k Q_7^k \left\{ \sum_{n=k-S}^{k_c} U_n - \sum_{n=k-S-1}^{k_c} V_n - \sum_{n=k-S}^{k_c^*} W_n \right\} + S Q_7^{M-1} \right]
 \end{aligned}
 \tag{29}$$

with

$$\begin{aligned}
 U_n &= \frac{(M - k + n)!(2k - 2n)}{(M - 2k + 2n)!(k - 2n)!n!} \\
 V_n &= \frac{(M - k + n - 1)!(2k - 2n + 1)}{(M - 2k + 2n - 1)!(k - 2n)!n!} \\
 W_n &= \frac{(M - k + n)!(2k - 2n - 1)}{(M - 2k + 2n + 1)!(k - 2n - 1)!n!}
 \end{aligned}
 \tag{30}$$

and $k_{c,s} = \text{floor}((k-1)/2)$.

The transfer function through the false recovery path in terms of the partial transfer functions is given by

$$\phi_{15}(z) = \frac{F_{15}(z)[(1 - P_4 z)T_4(z) - Q_4 P_6 z^2 T_6(z)]}{(1 - F_{11}(z))[(1 - P_4 z)T_4(z) - Q_4 P_6 z^2 T_6(z)] - Q_4 Q_6^M z^{M-1} F_{14}(z)} \tag{31}$$

and if we follow the same simplification procedure as above, the recovery time through the false path is obtained as

$$L_{15} = \frac{\left[\alpha + \sum_{n=0}^{N-1} (P_{12} P_2^n + P_{13} P_3^n) \right] Q_4 Q_6^M + P_{12} P_2^N \Delta_{46}}{P_{13} P_3^N Q_4 Q_6^M} \tag{32}$$

where Δ_{46} has the same form as Δ_{57} in (28) and (29) but with Q_5 replaced by Q_4 and Q_7 replaced by Q_6 .

In a similar manner, the transfer function through the false loss path and its corresponding holding time are given respectively by

$$\phi_{51}(z) = \frac{F_{51}(z)}{1 - F_{55}(z)} \tag{33}$$

and

$$L_{51} = \frac{\Delta_{57}}{Q_5 Q_7^M} \tag{34}$$

where Δ_{57} is as given (28) and (29).

Finally, the transfer function and the holding time for the true loss path are given respectively by

$$\phi_{41}(z) = \frac{F_{41}(z)}{1 - F_{44}(z)} \quad (35)$$

and

$$L_{41} = \frac{\Delta_{46}}{Q_4 Q_6^M} \quad (36)$$

where Δ_{46} is as given before.

RESULTS AND DISCUSSION

The results obtained from the *birth-death return receiver* can be compared with the ones for the *direct-return receiver*. For example, the recovery time through the true path in both cases can be represented by

$$L_{14} = \frac{\left[\alpha + \sum_{n=0}^{N-1} (P_{12} P_2^n + P_{13} P_3^n) \right] Q_5 Q_7^M + P_{13} P_3^N \Delta_{57}}{P_{12} P_2^N Q_5 Q_7^M} \quad (37)$$

The parameter Δ_{57} has been derived for direct-return receivers by Kundaeli (1995, 1996) as

$$\Delta_{57} = 1 + \sum_{m=0}^{M-1} Q_5 Q_7^m. \quad (38)$$

In this report, this parameter can be expressed in terms of (38) as

$$\Delta_{57} = \left[1 + \sum_{m=0}^{M-1} Q_5 Q_7^m \right] - Q_5 P_7 [\beta + Q_5 \gamma] = \Gamma_1 - \Gamma_2. \quad (39)$$

with

$$\Gamma_1 = \left[1 + \sum_{m=0}^{M-1} Q_5 Q_7^m \right] \text{ and } \Gamma_2 = Q_5 P_7 [\beta + Q_5 \gamma]. \quad (40)$$

Comparing (39) and (40), Γ_2 can be regarded as a correction term in Δ_{57} with both β and γ being functions of Q_7 . As an example, when $M = 2$, we obtain

$\beta = 1$ and $\gamma = 0$ but when $M = 6$ we obtain $\beta = 5 - 6Q_7 + 7Q_7^2 - 2Q_7^3 + Q_7^4$ and $\gamma = 2(2 + 2Q_7^2 + Q_7^3)$. It can be seen that as M increases, the correction term Γ_2 contains long expressions. Various values of Γ_1 and Γ_2 are given in Table 1 for various values of Q_7 and M . These values show that, the correction term Γ_2 introduces a very small error in the value of Δ_{57} . As a follow-up, various simulations have been performed for the recovery and holding times and this was found to be the case also. This is true, however, when lower values of Q_7 and M are employed. At higher values of Q_7 and M the correction term is significant, and can not be neglected. Fortunately, most channel conditions exhibit very low values of Q_7 ($<10^{-3}$ for a single bit marker for example) and the systems therefore employ very low values of M (<5). In such cases therefore, the correction term can be neglected.

Table 1: Various values of Γ_1 and Γ_2 obtained for various values of Q_7 and M

M	Q_7			
	10E-3		10E-6	
	Γ_1	Γ_2	Γ_1	Γ_2
0	1	0	1	0
1	1.001	0	1	0
2	1.001	9.99E-4	1	1.00E-6
3	1.001	2.00E-3	1	2.00E-6
4	1.001	3.00E-3	1	3.00E-6
5	1.001	4.00E-3	1	4.00E-6
6	1.001	5.00E-3	1	5.00E-6
7	1.001	6.00E-3	1	6.00E-6
8	1.001	7.00E-3	1	7.00E-6
9	1.001	8.00E-3	1	8.00E-6
10	1.001	9.00E-3	1	9.00E-6

CONCLUSION

The analysis of a frame synchronized communication system which employs a birth-death structure in the loss path has been performed. The expressions for the performance parameters turn out to be more complicated than those of

direct return systems and therefore demand more complicated programming and computing requirements where numerical evaluations are needed. It has been shown, however, that for systems employing low loss verify numbers and operating at low error rates, the simpler expressions of the direct return system can be used for the birth-death system without any corrections.

Both the direct-return and birth-death receivers can be implemented using standard digital design techniques. They both use counters and some additional combinational circuitry. The direct return receiver, however, needs more counters than the birth-death receiver, and the counters are of the counter/latch type. This makes the circuit of the direct-return receiver more complex. On the other hand, the birth-death receiver can be implemented using a single up/down counter and some additional combinational circuit. Its circuit is therefore less complex. It has to be noted that with recent advances in circuit integration techniques, the manufacture of complex circuits has become less demanding and less expensive. The frame synchronization circuits in the receivers are therefore implemented on the same chip with other circuits of the receiver.

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