COMPARISON OF ACCURACIES IN TECHNIQUES FOR EVALUATING WIND POWER DENSITY

[1] Reuben Kainkwa and John Kumwenda
[1] Department of Physics, University of Dar es Salaam, Dar es salaam, Tanzania

[*] Corresponding Author. E-mail: kainkwa@yahoo.com, Tel: +255754886864

ABSTRACT
Wind power is among important renewable sources of energy. In order to know the magnitude of wind power, appropriate techniques for evaluating it need to be investigated. Apart from the standard formula that can be used to evaluate wind power density at a site there are two more techniques namely power law exponent and Weibull two-parameter density function. However the accuracy of the latter two methods relative to the standard one has not yet been investigated. The main objective of this paper is to compare accuracies of these techniques. The wind power densities were calculated using standard formula, Weibull model as well as exponential factor of power law. The annual average of wind power density evaluated using the standard formula at the test site is 623 W/m^2 while the values estimated using power law exponent and Weibull model are respectively, 629 W/m^2 and 615 W/m^2. The correlation coefficients between the standard method and the other two methods were found to be respectively, 0.996 and 1.000. These correlations are high enough to warrant that each one can be used to evaluate wind energy density at site with sufficiently high accuracy.

Key words: Annual average, Wind power density, Power law exponent, Weibull model

INTRODUCTION

The traditional method of estimating available wind power at a site is the one in which the energy is calculated from the relation that the value is directly proportional to the air density and the cube of wind speed. There are however, other techniques of evaluating available wind energy density at a site namely the power law exponent and the Weibull two-parameter density function. Nonetheless, the accuracy of the latter methods relative to the standard one has not yet been investigated. The main objective of this article is therefore to compare the accuracy of determining the available wind energy density at a site using the power law exponent and the Weibull two parameter density functions relative to the standard technique.

THEORETICAL BACKGROUND

Vertical Variation of Wind Speed

The power law is an empirical relationship between wind speeds at two heights that can be expressed as (Gipe 1999):

$$\frac{V_x}{V_r} = \left(\frac{Z_x}{Z_r}\right)^{\alpha} \tag{1}$$

where V_X is the wind speed (m/s) at height Z_X (m), and V_r is the known wind speed at a reference altitude, Z_r and α is the power law exponent at the location. Site's specific values of the power law exponent may be determined for sites with two levels of wind data by solving equation (1) for α to get:

$$\alpha = \frac{ln\frac{V_x}{V_r}}{ln\frac{Z_x}{Z_r}} \qquad (2)$$

The Weibull Distribution Function
The wind speed probability distributions functions are the main tools used in the wind related literature. Their use includes a wide range of applications, from the techniques used to identify the parameters of distribution functions to the use of such functions for analyzing the wind speed data and wind energy economics (Christofides and Pashardes 1995). The first statistical studies of wind speed as a discrete random variable began 50 years ago with the Gamma distribution (Lambert and Seguro 2000). Over this period, different distribution function have been suggested to represent wind speed, such as those of Pearson, Chi-2, Weibull, Rayleigh and Johnson functions (Conradsen *et al*. 1984). Several non normal distributions have been suggested as appropriate models for wind speed such as inverse Gaussian, long normal, Weibull and squared normal (Kaptouom and Tchinda 2003). Among these the Weibull distribution function is still the most commonly used in wind energy distribution.

Furthermore, wind speed probability distribution function reveals the character of the apparently random variation. This is the statistical description of the wind climate at any site. The data would be more useful if it could be described by mathematical expression. The two-parameter Weibull distribution is often used to characterize and estimate probable future wind regimes for selected project sites, because it has been found to fit a wide collection of the recorded wind data (Celik 2004). In Weibull distribution function, the cumulative distribution function *F(V)*, for the wind speed *V* is expressed as (Hennessey 1978):

$$F(V) = 1 - \exp\left(-\left(\frac{V}{c}\right)^k\right) \qquad (3)$$

where *V* is the wind speed, *k* is a dimensionless shape factor, and c is a scale parameter with the same units as *V*.

By differentiating equation (3) with respect to *V*, the wind speed probability density function *f(V)* can be written as (Ulgen and Hepbasli 2002**,** Christofides and Pashardes 1995):

$$f(V) = \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} \exp\left[-\left(\frac{V}{c}\right)^k\right] \qquad (4)$$

The mean wind speed of the Weibull distribution is expressed as (Johnson 2006):

$$\overline{V} = \int_0^\infty \frac{Vk}{c}\left(\frac{V}{c}\right)^{k-1} \exp\left[-\left(\frac{V}{c}\right)^k\right] dV \qquad (5)$$

Taking:

$$x = \left(\frac{V}{c}\right)^k \quad , \qquad \text{then}\,\frac{dV}{dx} = \frac{c}{k}x^{1/k-1} \qquad (6)$$

Substituting for *dV* in equation (5) then the mean wind speed can be written as (Mathew 2006):

$$\overline{V} = c\int_0^\infty x^{1/k} e^{-x} dx \qquad (7)$$

Equation (7) can be expressed in the form of another mathematical function, the gamma function. The gamma function Γ is expressed as (Johnson 2006):

$$\Gamma(y) = \int_0^\infty e^{-x} x^{y-1} dx \qquad (8)$$

Equations (7) and (8) have the same integrand if $y = 1 + 1/k$.

Therefore the two parameters *c* and *k* are linked to the average wind speed by the relation:

$$\bar{V} = c\Gamma\left(\frac{1}{k}+1\right) \qquad (9)$$

Any Weibull distribution can therefore be portrayed by the average wind speed and the Weibull *k* value. The Weibull *k* value is an indication of the breath of the distribution of wind speeds and Weibull *c* value indicates how 'windy' a location under consideration is. The lower *k* value corresponds to the broader distribution. In addition, lower average wind speed corresponds to lower Weibull *k* values (Foxon and Weisser 2003).

In circumstances where *k* is equal to 2 the Weibull distribution is referred to as Rayleigh distribution (Bowden 1983). Often wind energy conversion turbine manufactures provide standard performance figures for their turbines using this special case of the Weibull distribution (Weisser 2003). The main limitation of the Weibull density function is that it does not accurately represent the probabilities of observing zero or very low wind speed (Persaud *et al*. 1999).

Average Wind Speed and Variance

Evaluation of the average wind speed and variance is very important in the study of Weibull distribution specifically in the determination of Weibull parameters. The mean wind speed $\bar{V}$, and variance σ^2, are expressed as (Foxon and Weisser 2003), respectively:

$$\bar{V} = \frac{1}{n}\sum_{i=1}^{n} V_i \qquad (10)$$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(V_i - \bar{V}\right)^2 \qquad (11)$$

where n is defined as the number of wind speed recordings.

The variance of wind velocity can be calculated on the basis of the Weibull parameters as (Jamil *et al*. 1995):

$$\sigma^2 = c^2\left[\Gamma\left(1+\frac{1}{k}\right) - \Gamma^2\left(1+\frac{2}{k}\right)\right] \qquad (12)$$

Determinations of Weibull Parameters

There are several methods available for the determination of Weibull parameters *c* and *k*. In previous studies the least square fit to the observed distribution, Median and quartile wind speed, WAsP and Weibull probability paper methods were used to determine Weibull parameters. In this study Mean wind speed will be used to determine Weibull parameters c and k.

Once the mean wind speed, $\bar{V}$ and the variance, σ^2 of the observed wind speed data are known, the following approximation can be used to calculate parameter *c* and *k* (Abbaszade *et al*. 2009):

$$k = \left(\frac{\sigma}{\bar{V}}\right)^{-1.086} \qquad (1 \le k \le 10) \qquad (13)$$

And from equation (9) the scale parameter *c* is given as:

$$c = \frac{\bar{V}}{\Gamma\left(1+\frac{1}{k}\right)} \qquad (14)$$

The dimensionless shape factor (*k*) can be estimated even when the variance is not known. It is found that *k* appears to be proportional to the square root of the mean wind speed and is expressed as (Justus *et al*. 1978):

$$k = d_1\sqrt{\bar{V}} \qquad (15)$$

where proportionality constant d_1 is site specific constant with an average value of

0.94 when the mean wind speed is given in meter per second (Johnson 2006).

Wind Power

Power from wind can be characterized as either the available wind power or the extractable wind power. While the available wind power density is considered to be the power available in a cross section area perpendicular to the wind stream of density ρ moving with a speed V, the extractable wind power density take into consideration the operating characteristics of the Wind Energy Conversion System (WECS).

The available power density due to wind speed V m/s per unit area perpendicular to the wind direction is given as (Anani *et al*. 1988):

$$P_a = \frac{1}{2}\rho V^3 \qquad (16)$$

From equations (1) and (16), the available power in the wind is expressed as (Gipe 1999):

$$P_x = P_r\left(\frac{Z_x}{Z_r}\right)^{3\alpha} \qquad (17)$$

where P_x and P_r are the available power density at height Z_x and Z_r respectively. In this article Z_x = 30 m and Z_r = 10 m, so that equation (17) reduces to:

$$P_x = 3^{3\alpha} P_r \qquad (18)$$

Extractable wind power density, P_e, on the other hand, is the power, which can be extracted from the wind stream depending on the available wind power and on the operating characteristics of the wind turbine. The extractable power density is given by the equation:

$$P_x = 3^{3\alpha} P_r \qquad (19)$$

where C_p is the power coefficient with the maximum value of 0.593, which is the Betz limit or the Betz coefficient (Bansal *et al*. 1990). This is attributed to the fact that the power a wind machine intercepts is not what the machine can produce as it can not capture all of it for if it did, the wind could come to halt at the rotor.

Available wind power can also be estimated using the concept of the Weibull probability distribution**.** If the actual wind data is modeled by the Weibull probability distribution *f(V)* of a wind velocity *V*, then the average power density in the wind is expressed as (Johnson 2006):

$$\overline{P}_a = \frac{1}{2}\rho\int_0^\infty V^3 f(V)dV \qquad (20)$$

It can be shown that if *f(V)* is the Weibull probability density function, the average power density can be calculated by the relation (Abbaszadeh *et al*. 2009):

$$\overline{P}_a = \frac{1}{2}\rho V^3\Gamma\left(\frac{k+3}{k}\right) \qquad (21)$$

If the Weibull density function fits the actual wind data exactly then the power in the wind predicted by equation (21) is expected to be the same as that predicted by equation (20). The greater the difference between the values obtained from these equations, the poorer is the fit of the Weibull density function to the actual data (Johnson 2006).

MATERIALS AND METHODS

Study Site and Equipment

Wind data used in this study were obtained from Kititimo site South East of Singida town. The site is at an elevation of about 1393 m above sea level. The site is located at around longitude 34° 43' E and latitude 5° 40' S in Singida region, Tanzania. The distance from Singida town to Kititimo site is about 5 km.

The main wind data sources that were used in this study were obtained from Tanzania Electricity Supply Company (TANESCO). The meteorological parameters namely wind speed and direction were recorded from May 2005 to June 2006. The wind data were collected at two different heights 10 m and 30 m above ground level using anemometers while the wind direction was measured at 30 m above the surface wind vane. The wind speed and direction were recorded at time interval of 10 minutes.

Methods

The available power density was estimated by the standard formula given by (16). The available power density was also approximated by power-law exponent relation specified by equation (18). The reference power P_r, used in this equation is the available power density at reference height of 10 m. In addition, the available power density for Weibull model was evaluated by (21). The correlations between the powers evaluated by the latter two methods relative to the standard methods were found using the standard correlation analysis.

RESULTS AND DISCUSSION

The monthly mean wind power density based on standard equation, exponential factor of power law and Weibull model at 30 m above the ground were calculated and the results are shown in Table 1. The trends of wind power density estimated using power law exponent from June to October 2005 is higher as compared to the results obtained from the standard formula and the rest of the months are lower. Furthermore power density estimated using Weibull model is lower from June to November 2005 and May 2006 and the remaining months is higher as compared to the values of the standard formula. The overall annual average values shows that the power density obtained using power law exponent is overestimated by 6 W/m^2 while that obtained using Weibull model is underestimated by 8 W/m^2.

Table 2 depicts correlation matrix of wind power density of power law exponent and Weibull model as compared to standard values calculated from standard method. The table shows further that the correlation coefficient between P1 and P2 is 0.996 and that of P1 and P3 is 1. Therefore correlation coefficient of the wind power density is high, which suggests that both methods, the power law exponent as well as the Weibull distribution model methods can be used to evaluate wind power density.

Table 1: Monthly Power Density based on Standard equation (16), Weibull (21) and Wind Shear Exponent (18) at 30 m

Month - Year	**Standard**	**Exponential factor**	**Weibull**
	(W/m^2)	**(W/m^2)**	**(W/m^2)**
Jun-05	750	755	712
Jul-05	673	708	647
Aug-05	900	936	854
Sep-05	1414	1455	1323
Oct-05	1131	1181	1070

Month - Year	Standard	Exponential factor	Weibull
	(W/m^2)	(W/m^2)	(W/m^2)
Nov-05	755	714	742
Dec-05	431	416	448
Jan-06	295	274	314
Feb-06	282	275	299
Mar-06	203	192	221
Apr-06	323	314	334
May-06	1117	1107	1069
Mean	623	629	615

Table 2: Correlations Matrix of Wind Power Densities

	P1	P2	P3
P1	1	0.996	1
P2	0.996	1	0.997
P3	1	0.997	1

CONCLUSIONS

The results of this study demonstrates that the available power density calculated using both methods; power law exponent as well as Weibull distribution model shows high correlations relative to correlation of the standard formula. correlations relative to correlation of the standard formula. This indicates that both two methods can be used to evaluate wind power density instead of the standard method because the difference between the standard method and the other two methods is insignificant.

AKNOWLEDGMENTS

This work was funded by World Bank (C1B1) through College of Natural and Applied Sciences, University of Dar es Salaam and technically supported by Physics Department. We would like also to extend our sincere gratitude to TANESCO for the data support.

REFERENCES

Abbaszadeh R, Ghasemi M, Khanali M, Keyhani A and Varnamkhasti M 2010 An

Assessment of Wind Energy Potential as a Power Generation Source in the Capital of Iran, Tehran. *Energ.* **35**:188-201.

Anani A, Zuamot S, Abu-Allan F and Jibril Z 1988 Evaluation of Wind Energy as a Power Generation Source in a Selected Site in Jordan. *Sol Wind Technol.* **5:** 67-74pp.

Bansal K, Kleemann M and Mellis M 1990 Renewable Energy Sources and Conversion Technology. Tata McGraw-Hill Publishing Company Limited New Delhi, India.

Bowden GJ, Barker PR, Shestopal and Twidell JW 1983 The Weibull Distribution Function and Wind Power Statistics. *Wind Eng*. **7**: 85-99**.**

Celic AN 2004 A Statistical Analysis of Wind Power Density Based on the Weibull and Rayleigh Models at the Southern Region of Turkey. *Ren Energ*. **29** (4):593-604.

Conradsen K, Nielse LB and Prahm LB 1984 Review of Weibull Statistics for Estimation of Wind Speed Distributions. *J Clim Appl Meteorol.* **23**:1173-83.

Christofides C and Pashardes S 1995 Statistical Analysis of Wind Speed and Direction in Cyprus. *Sol Energ.* **55:** (5):405-14.

Foxon TJ and Weisser D 2003 Implications of Seasonal and Diurnal Variations of Wind Velocity for Power Output Estimation of a Turbine: a Case Study of Grenada. *Int J Energy Res.* **27**:1165-1179.

Gipe P 1999 Wind Energy Basics: A Guide to Small and Micro wind System Chelsea Green Publishing Company, White River Junction, VermoTotnes England.

Hennessey JP1978 A comparison of the Weibull and Rayleigh Distributions for Estimating Wind Power Potential. *Wind Eng*. **2**, 156-164.

Jacovides CP, Theophiou C, Tymvios FS and Pashiardes S 2002 Wind Statistics for Coastal Stations in Cyprus University of Athens, Greece. *Theor Appl Climatol*. **72**, 259-263.

Jamil M, Parsa,S and Majidi M 1995) Wind Power Statistics and Evaluation of Wind Energy Density. *Ren Energ*. **6**:623–8.

Johnson GL 2006 Wind Energy Systems, Electronic Ed, Manhattan (KS), USA. Available at http://www.ece.k-state.edu//people/faculty/gjohnson/files/Windbook.pdf. Viewed August 2016.

Justus CG, Hargraves WR, Mikhail A and Graber D 1978 Methods for estimating Wind Speed Frequency Distributions. *J Appl Meteorol*. **17**, 350-353.

Kaptouom E and Tchinda R 2003 Wind Energy in Adamaoua and North Cameroon Provinces. *Energ Convers Manage*. **44**: 845–57.

Lambert TW and Seguro JV 2000 Modern Estimation of Parameters of the Weibull Wind Speed Distribution for wind energy analysis. *J Wind Eng Ind Aerodynamic*. **85**:75-84.

Mathew S 2006 Wind Energy Fundamentals, Resources Analysis and Economics. HAR/CDR ed,

Springer-Verlag, Berlin Heidelberg.

Persaud S, Flynn D, and Fox B 1999 Potential for Wind Generation on the Guyana Coastlands. *J Ren Energ*. **18**(2): 175–89.

Ulgen K and Hepbasli A 2002 Determination of Weibull parameters for wind energy analysis of Izmir, Turkey. *Int J Energy Res* **26**(6):495-506.

Weisser DA 2003 Wind Energy Analysis of Grenada: An Estimation Using the Weibull Density Function. *Ren Energ* **28**: 1803-1812.