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STACK for Computational Science, Mathematics and Engineering e-Learners

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ABSTRACT

E-Learning platforms such as Moodle, Blackboard and Canvas have got reasonable attention in teaching and learning processes. However, when it comes to assessment and interactive learning activities they offer little service to science, engineering, and mathematics e-learners. In this work, we present the application of the System for Teaching and Assessment using Computer Algebra Kernel (STACK) as a plugin in a Learning Management System (LMS) to address the issue. Different features of STACK are demonstrated and discussed. As an LMS plugin, STACK can be used for interactive delivery of content as well as an assessment tool. Here, examples from mathematics, physical sciences and engineering are demonstrated. The use of STACK extends the applicability of LMS for a wider range of subjects to address existing inability to handle higher level mathematical and computational skills. Additionally, the use of STACK in an LMS is useful in handling tutorials for large classes especially when a blended delivery mode is preferred.

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INTRODUCTION

System for Teaching and Assessments using Computer Algebra Kernel (STACK) is an open-source system developed by Chris Sangwin as a development of author's earlier work (Sangwin, 2003, 2013). The STACK plugin, works with a variety of e-Learning platforms such as Moodle, Blackboard and Canvas (Gage, 2017; Ostrow *et al.*, 2017). It is known that STACK can be easily incorporated into other systems (Lowe *et al.*, 2019).

As an open-source learning management system (LMS), Moodle has been widely adopted by many institutions as their LMS of choice. Moodle offers varieties of students' learning and assessment activities. However, these assessment activities are limited to objective type questions which include multiple-choice, short-answer, matching-item, true/false and numerical answer questions. In all these question types, a student submits the final answer which consequently ignores some important aspects of learning, such as mathematical/computational problem formulation and data presentation. The work of Sangwin (2003) employed higher level mathematical skills by creating instances of mathematical objects satisfying certain properties, whose correctness can be assessed competently without the imposition on staff with heavy marking load. In this approach, students can be asked to give their own examples where a computer algebra system (CAS) will automatically check the correctness of the examples, by following a predefined algorithm. It has also been argued that STACK is addressing the need of ignored learners (Sangwin and Grove, 2006), for example those who can hardly face their tutors for discussing their learning difficulty. Henceforth, students can independently achieve different Mathematics learning objectives as detailed in

Bloom *et al.* (1956) and Smith *et al.* (1996).

The flexibility of STACK allows it to be used as either an assessment or interactive delivery tool (Amour, 2023; George, 2019). Despite its considerable potential in enhancing the teaching of Mathematics and related subjects, not many instructors are aware or use STACK to supplement their course delivery or assessment practices in e-learning platforms. In this work, we introduce and demonstrate the use of STACK for enhancing the teaching of Mathematics, engineering and physical sciences. As STACK exploits the power and flexibility of the underlying LMS's quiz features, we focus on practice problems for effective delivery, assessment and student feedback provision. This use case has shown remarkable success when teaching large classes where online tutorials proved a success just as a traditional face-to-face session as demonstrated in the preliminary results.

CONCEPTUAL FRAMEWORK

STACK is a powerful system for online assessment of mathematics and related subjects (Lowe *et al.*, 2019). The installation process and technicalities of authoring STACK questions are best explained in the STACK documentation (STACK, 2022). With the current settings, STACK can be used as a third-party plugin for e-Learning platforms such as Moodle and ILIAS Systems, it can also be incorporated into other systems via the Learning Tools Interoperability protocol (Lowe *et al.*, 2019). Using STACK in an LMS helps to empower the LMS to be useful for a wider range of subjects and audience. The flexibility of STACK to incorporate other engines is also amazing. In Kinnear (2020), for example, the author has shown how STACK successfully used JSXGraph to create advanced interactive plots. The emphasis on authoring STACK questions is to ask a reasonable mathematical question rather than asking easily gradable questions (Gage, 2017). The power of STACK comes from the computer algebra system (Maxima), and the LATEX text processor (rendered with MathJax software), which are all open-source software. STACK is a highly sophisticated system, there is no way to evade some difficulties in authoring robust, meaningful and quality questions (Lowe *et al.*, 2019).

Basic STACK Question Accessories

STACK questions consist of a number of features, namely:

Question name: A STACK question must be given a name, as a rule of thumb, the name has to be simple, readable, even on a small computer screen, and should tell the content/concept of the question.

Question variables: This is in an online text editor useful for writing Maxima code. The variables defined in this place are global, therefore can be accessed everywhere within the questions.

Question text: This is the actual question text seen by the student, it can pull question variable values defined in the question variables section by using the {@...@} command. Mathematics typesets are included by using the $\LaTeX$ commands \(... \) for inline and \[... \] for display styles, respectively. An answer field for student to enter a response is included in this part, for example, [[input:ans1]] means student answer will be assigned to the variable ans1.

Specific feedback: This is usually generated through the potential response tree (PRT) marking algorithm (Lowe *et al.*, 2019). To display the output from the PRT, the tag [[feedback: prt1]] is included. However, for the multipart question, it is meaningful to put specific feedback just next to the student's input (Lowe *et al.*, 2019).

General feedback: Here the author gets an opportunity to provide a worked solution or some useful explanation or concepts about the question regardless of the student responses.

Input: This is a field where STACK provides a section to define the input type, model answers for giving feedback to students, answer input window's size, syntax hints, and restricting some functions or inputs.

Potential Response Tree (PRT): This provides a way to assess and grade students' work by implementing nested if-else-if statements, graphically, by creating an acyclic directed graph (Lowe *et al.*, 2019). This is a good design as it minimizes the amount of code an author has to write for grading solution steps or grading alternative solutions. It includes the nodes for the branches of the if-else-if statements, each examines a true/false test on the students' answers, and updates the scores and feedback given as a result (Lowe *et al.*, 2019). The PRT allows grading with partial scores of a multi-input answers through PRT nodes. An example of single PRT node is shown in Figure 1.

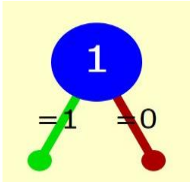


Figure 1: A simple PRT node.

Table 1: STACK's Student Input Types

Input type	Student input
Algebraic	An algebraic expression which strictly/technically agrees with Maxima syntax.
Checkbox	Checking the box(es) against the correct responses from a list of possible responses.
Drop-down list	A correct response from a drop-down list of possible responses.
Equivalence Reasoning	A series of Maxima statements corresponding to a solution step of a mathematical problem.
Matrix	Numerical/algebraic expressions as a matrix with predefined dimension.
Numerical	A numerical value with imposed restriction, for example, number of decimal places, integer or floating type on it.
String	A text as a response.
True/False	True or False value as a response from a list of True and False possible responses.
Unit	Unit of a given physical quantity.

Student Inputs

An author will likely require varieties of student inputs depending on the nature of the question. STACK supports a number of input types as shown in Table 1.

STACK Answer Tests

STACK offers varieties of tests that cover a wide range of mathematical tests. To comprehend what does this mean, consider two cases:

(1) The case of forming an equation of a tangent line to the curve $y = 5x^2$ at $x = \frac{1}{2}$. The answer to this problem is a unique line, however it can be written in a number of ways, such as:

$$y = 5x - \frac{5}{4}, y - 5x = -\frac{5}{4}, 4y - 20x$$

Thus $y = -5, 4y - 20x + 5 = 0$

STACK provides perfect means to test any of these possibilities.

(2) The general solution of a second order differential equation must contain two arbitrary constants. The student's answer to this question can contain any two symbolic variables for the arbitrary constants. STACK provides a way to test equivalence of the two expressions by checking if they are algebraically equivalent by examining the existence of the interchanged variables/constants. For example, the two general solutions $y = A + Be^x + 3\sin x$ and $y = k_1 + k_2e^x + \sin x$ of the differential equation are equivalent.

Basic STACK student's answer tests are summarised in Table 2.

Table 2: STACK Answer Tests (sans and tans are student answers and teacher answers, respectively)

Answer Test	Role
AlgEquiv	To test if two mathematical expressions are algebraically equivalent. For example ATAlgEquiv(y=5*x-5/4, 4y-20*x=-5) returns true.
SubstEquiv	To test if two Mathematical expressions are Algebraic equivalent, up to substitution. For example ATSubstEquiv (y=A+B*exp(x)+3*sin(x), y=k1 + k2 * exp(x) + 3*sin(x)) returns true.
EqualComAss	To test if two Mathematical expressions are equivalent up to commutativity of addition and associativity of multiplication. For example ATEqualComAss(y=5*x-5/4, 4y-20*x=-5) returns false.
NumDecPlaces	To test if sans is given to n decimal places and sans = tans to n decimal places. It also counts the trailing zeros for example 1.5 and 1.50 are different, that is 1.5≠ 1.50.
EquivReasoning	To test whether all lines with sans are equivalent.
EquivFirst	To test whether all lines with sans are equivalent and the first line of the student's and teacher's answers are equivalent, up to commutativity and associativity.

Table 3: Some STACK Randomization functions

Function	Argument type	Output
rand(n)	integer	random integer between 0 and integer n-1.
rand(L)	list	random element from the elements of a list L.
rand (n.0)	float	random floating-point number between 0.0 and floating-point n.
rand with prohib(a,b,exc)	(integer, integer, list)	random integer between integer a and integer b excluding the integers defined in the list exc.

DEMONSTRATION

The STACK questions authored with STACK version 2023060500 installed in Moodle 4.2 stable version running in Debian 12 Linux server (Lenovo Desktop 8GB RAM, AMD® A10-7800 radeon r7, 12 compute cores 4c+8g × 4) with Maxima 5.46.0 is demonstrated. The following examples are meant to demonstrate the potential of STACK in authoring quality questions; they cover higher level skills in mathematics, guided proofs, and computational science and engineering problems. STACK provides a means of randomly sampling questions' variables values such that a number of variants of the same question is generated. Through *Question Tests & Deployed Variants*, each question variant was inspected and tested for its grading behaviour. Some useful STACK randomization functions are shown in Table 3.

Higher Level Skills in Mathematics

Assessing higher skills in mathematics is a challenge in LMS default settings. For example, asking a student to provide data values which satisfy some conditions. The resources to access mathematical conditions are not available, in which the answers are dynamic and cannot be predefined. In this work we demonstrate a case where student has to find any two arbitrary points that lie on the line formed by two randomly sampled points. The Maxima code implementing the STACK question is shown in Code 1. Figure 2 shows a preview of the question generated by Code 1 together with the student's feedback.

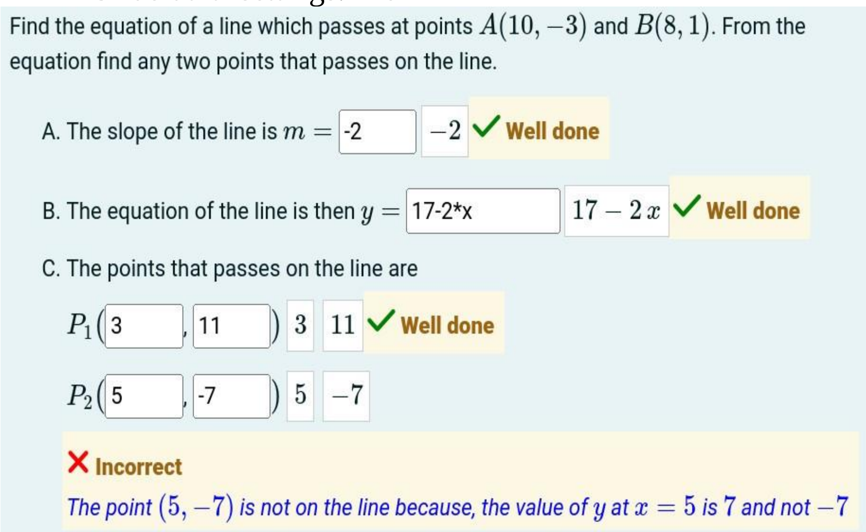


Figure 2: A question variant of the infinitely many answers with specific feedback.

Code 1: Question variables for randomly sampling the points A and B (line 1 to 5) and creating model answers (line 6 to 13).

```
1  /*   Generate random coordinates for points A and B */
2  x1: rand_with_prohib ( -10 ,10 ,[0]);
3  x2: rand_with_prohib ( -10 ,10 ,[0,x1]);
4  y1: rand_with_prohib (-10 ,10 ,[0]);
5  y2: rand_with_prohib(-10,10,[0,y1]);
6  /*   Teacher ' s ( model ) answer */
7  m: (y2-y1) / ( x2-x1 ) ;          /* slope of the line*/
8  y : expand (m*(x-x1)+y1 ) ;   /* Equation of the line */
9  /*   Sample points on the line */
10 spx1: rand with prohib ( -10 ,10 ,[0 , x1 , x2 ] );
11 spy1: ev(y, x=spx1 ) ;          /* Point 1 (spx1, spy1) */
12 spx2: rand with prohib ( -10 ,10 ,[0 , x1 , x2 , spx1 ] ) ;
13 spy2: ev(y, x=spx2 ) ;          /* Point 2 (spx2, spy2) */
```

The PRT question variables code accessing and processing student's input is shown in Code 2. The points are tested if they are on the line by substituting the values of *x* and that of *y* into the equation of the line defined in line 8 in Code 1.

Code 2: Question variable for testing if the input point (ans3, ans4) is on the line, defined in Code 1, line 8.

```
/* Examine if point (ans3, ans4) is on the line */
sa: if is(ev(y, x=ans3) = ans4) then 1 else 0
```

Guided Proofs

Mathematics proofs pose a big challenge for online assessments in mathematics. Free style structure of the proof is a major setback. In this work we demonstrate a case of guided proofs, where the structure is designed and student has to input the correct facts and reasoning for the missing fields of the proof. The reasoning facts are implemented by STACK objective type questions such as strings (for short answer texts), true/false or multiple choice as drop-down list or radio button. The advantages of STACK objective questions are clear, they support mathematics and randomization (Bach, 2020). The key point is to give no or minimal clue to the fields' responses. Consider an example of proving a limiting value using the precise definition of limit as shown in Figure 3. The student is guided to complete the proof in a step-by-step fashion, while getting immediate feedback at each step. Similar techniques can be applied to analytical questions, as it is demonstrated in Amour (2023), where convergence of fixed-point iteration formulae analysis has been demonstrated.

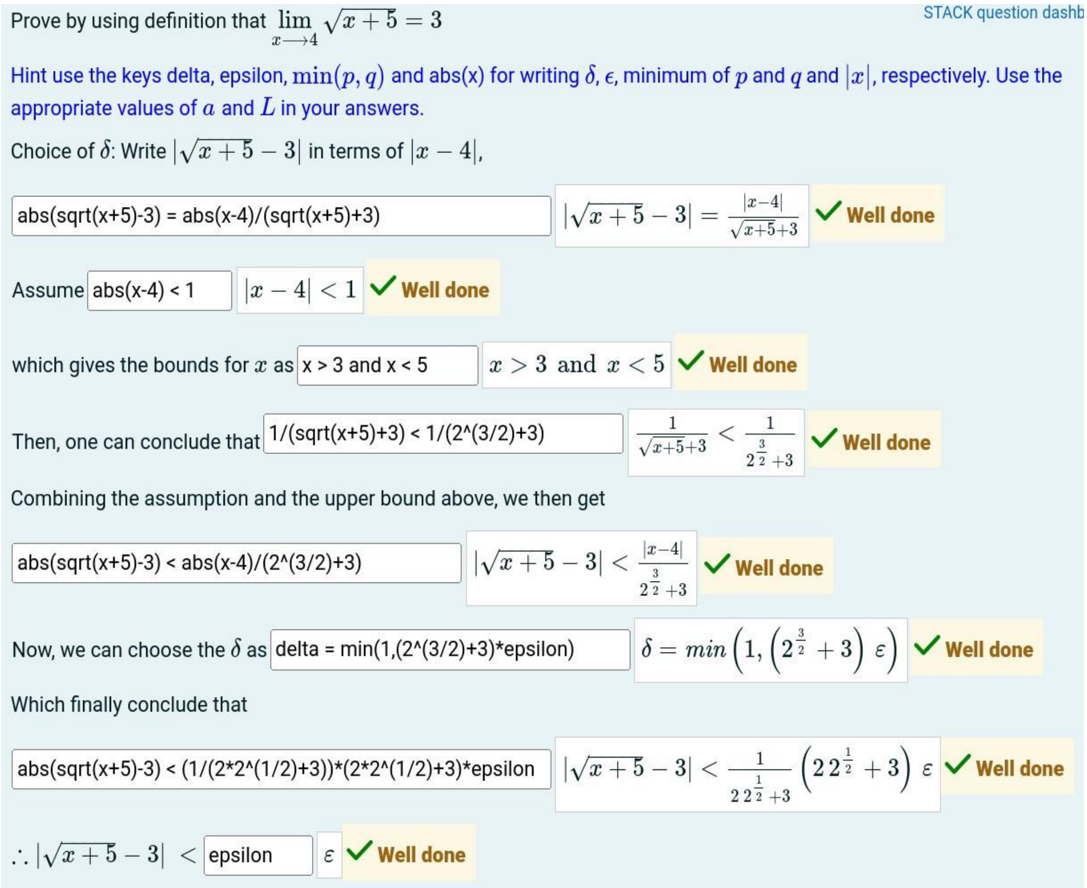


Figure 3: A guided proof question variant with specific feedback.

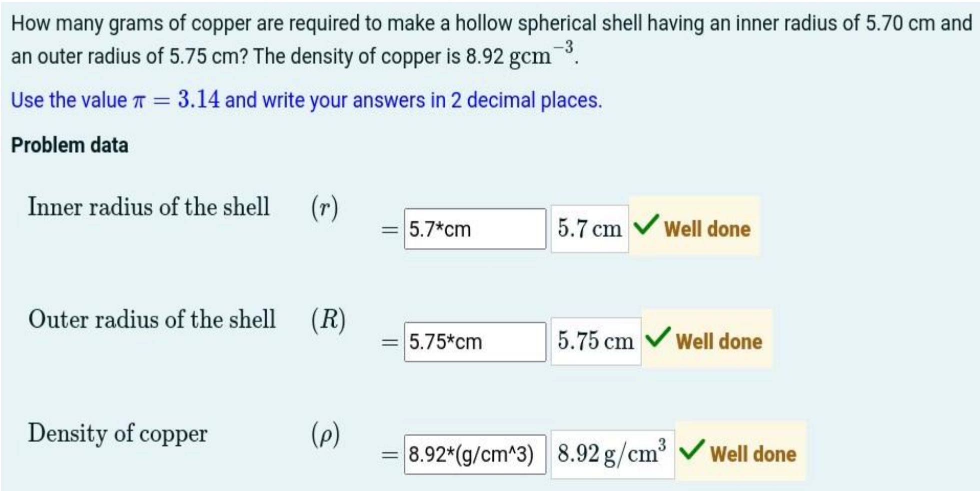


Figure 4: Grading of problem data gathering for the engineering problem.

Computational Science and Engineering

Engineering problems require apart from others gathering of problem's data, data presentation and calculations. In this work we demonstrate a case of an engineering problem of determining the mass of a copper required to make a specified hollow spherical shell. The STACK question preview for the problem is shown in Figures 4 and 5. Figure 4 shows fields input for gathering problem data, while Figure 5 shows fields input for calculation part of the problem.

Support for quantities' units in STACK is consistent with the requirements of physical science, and engineering problems computation and presentation. Consider a case of a projectile motion shown in Figure 6, where the projectile is fired at a point above the ground. The preliminary information for the solution of the problem is gathered in Figure 7. The grading of student's solutions for part A and part B are shown in Figure 8.

PRELIMINARY RESULTS

The randomization of the variables has been carried out by analysing the domain of problem's dynamic variables (Amour, 2023). Table 4 shows a summary of the random variables for the STACK questions in Figures 2 – 8.

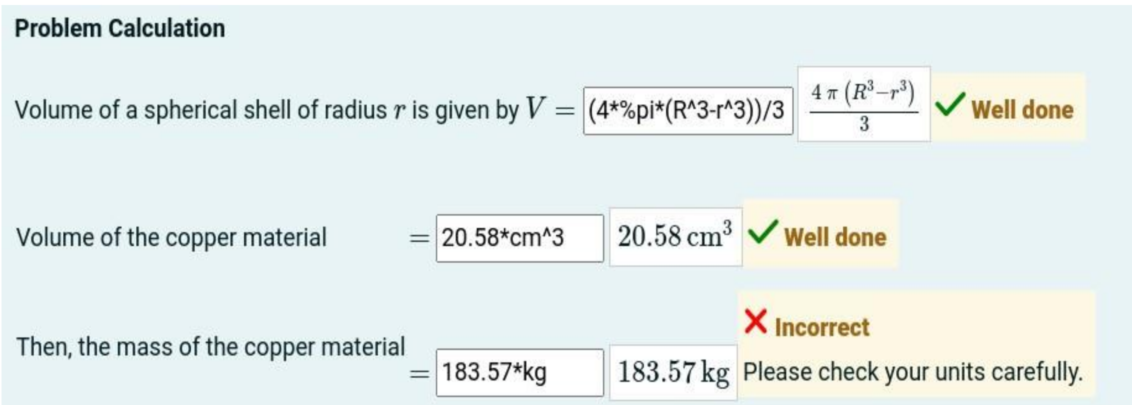
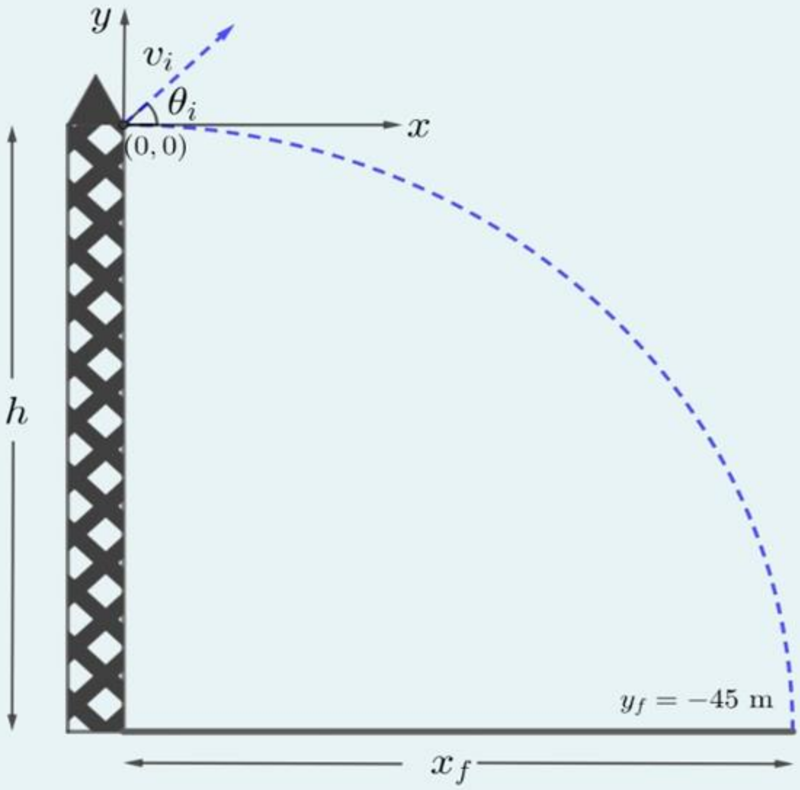


Figure 5: Grading of mass of copper based on gathered data in Figure 4.

A stone is thrown from the top of a building at an angle of $\theta_i = 30.0^0$ to the horizontal and with an initial speed of $v_i = 20.0$ m/s.

A. If the height of the building is $h = 45.0$ m, how long is the stone "in flight"?
B. What is the speed of the stone just before it strikes the ground?



Use $g = 9.8$ m/s^2. Express your answers in 2 decimal places.

Figure 6: Projectile motion.

The initial x and y components of the velocity are v_{xi} and v_{yi} which are given by the formular

	Expression	Value
v_{xi}	cos(theta_i)*v_i $\cos(\theta_i)\, v_i$ ✓ Well done	17.32*(m/s) $17.32\,\mathrm{m/s}$ ✓ Well done
v_{yi}	sin(theta_i)*v_i $\sin(\theta_i)\, v_i$ ✓ Well done	10.0*(m/s) $10.0\,\mathrm{m/s}$ ✓ Well done

Figure 7: Physical science problem's data.

PART A

Assume top of the building as the origin, then the equation for vertical motion is

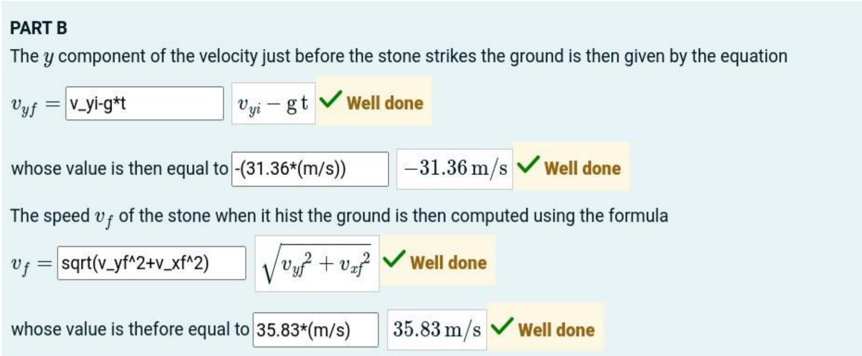
$y_v =$ y_i+t*v_yi-(g*t^2)/2 $y_i + \mathrm{t}\, v_{yi} - \frac{\mathrm{g}\,\mathrm{t}^2}{2}$ ✓ Well done

To compute the time of flight, we use the height of the building, substitution gives

10.0*t-4.9*t^2 = -45 $10.0\,\mathrm{t} - 4.9\,\mathrm{t}^2 = -45$ ✓ Well done

Solving the above equation gives $t =$ 4.22*s $4.22\,\mathrm{s}$ ✓ Well done

(a) Grading distance function and time of flight.



(b) Grading velocity components and speed.

Figure 8: Physical science problem solution and grading.

Table 4: Randomization of problems' data

Figure	Problem data	Fixed variables	Random variables	Data type
2	$A(x_1, y_1), B(x_2, y_2)$	None	x_1, y_1, x_2, y_2	Integers
3	$\lim_{x\to a}(px + q) = L$	None	a, p, q, L	Integers
4	Radii r, R	Density of copper	r, R	Floats
6	θ_i, v_i, h	None	θ_i, v_i, h	Floats

STACK questions in Figures 2 – 8. For the last two years at the University of Dar es Salaam (UDSM) in Tanzania, STACK has played a central role in handling large Mathematics classes. Among the beneficiaries are the first-year engineering students whose classes enroll around 1000 students, and Science students whose class size is reasonably normal. Total of three courses have been fully digitised with STACK. The courses cover the contents in Linear algebra, differential calculus, integral calculus, numerical methods, complex numbers and Fourier series. In these courses, students were able to perform their online tutorials and quizzes with immediate feedback on their scores as well as their mistakes (Amour, 2023; Amour *et al.*, 2023), which served the purpose of assessment for learning.

The quality of majority of the authored questions were reasonable in terms of difficulty level, explained by facility index (P), and ability to distinguish students of different learning abilities, explained by discriminative efficiency (d), (Amour *et al.*, 2023). The difficulty levels are divided into high ($P < 30\%$), medium ($30\% < P < 80\%$) and low ($P > 80\%$) difficulty (Amour *et al.*, 2023). These measures informed about the quality of the developed questions and their appropriateness to replace the traditional Moodle's objective type questions. It is evident from the left three bars in Figure 9 that few number of high difficult questions ($d < 30\%$) were authored. The medium difficulty questions with discriminative efficiency $30\% < d < 50\%$ do not appropriately reflect learning, therefore not a desired property (Gamage *et al.*, 2019). Consequently, this is suggesting that majority of the developed questions were able to assess intended learning, which is usually ignored or difficult to achieve with the default LMS assessment tools.

Performance in the numerical analysis class which used STACK for online tutorials was similar to the results of the same course taught using face-to-face tutorials in the previous years. Results from continuous assessment collected from online quizzes together with timed tests correlated with the final examination results as shown in Figure 10 (Amour, 2023).

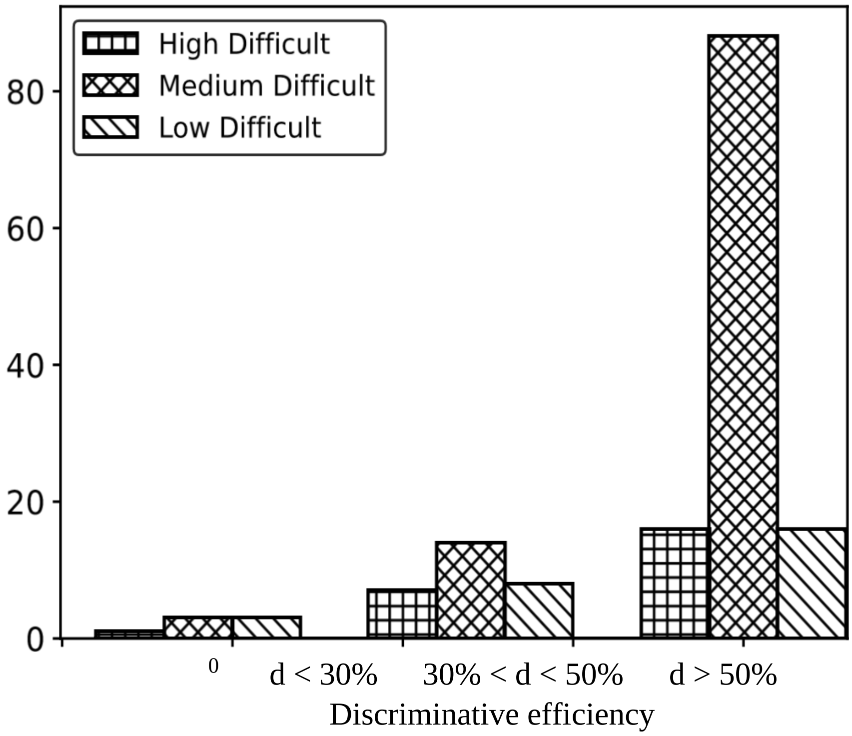


Figure 9: Number of questions with respect to their difficulty level and discriminative efficiency (*d*) for the three categories of questions' difficulty (Amour *et al.*, 2023).

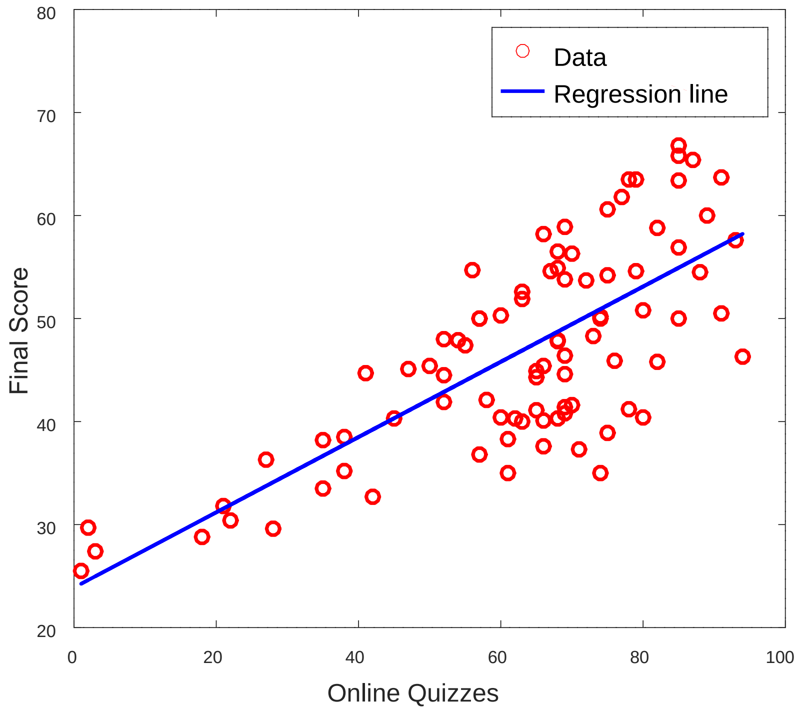


Figure 10: Students' final scores versus online quizzes (Amour, 2023).

CONCLUSION AND RECOMMENDATION

The potential of STACK in teaching and learning computational science, mathematics and engineering has been demonstrated. Questions with high level mathematical skills and reasoning have been presented, which add value to an LMS default assessment tool. The randomization feature in STACK is of great advantage as students can repeat the questions as many times as the Moodle quiz is set. This allows students to work with different variants of the question while they are learning the required concepts and improve their computational skills. Provision of the specific feedback is the most distinguishing feature of STACK as each student gets the feedback according to committed mistake. These provisions shall improve student engagement in interacting with the subject matter as there will be no fear of embarrassment as a result of making mistakes.

This is particularly useful in cases where STACK is used to conduct online tutorials. It is worth mentioning that authoring one high quality STACK question can take a significant amount of time and effort, but with the randomization feature, it pays as it improves the lifespan of the question and keeps the novelty of the question intact. Running of online courses in computational science, mathematics and engineering will be more affordable and effective.

Apart from the development of high-quality questions, it is worth to study on students' behaviour, experiences, affection, usability and affordability of the online practice. This shall inform about the scale-up plan of the practice, as the needs and requirements are analysed, and challenges are addressed.

Conflict of Interests

The author declared to have no conflict of interests in this work.

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