ELECTRICAL EQUIVALENT METHOD FOR THE CALCULATION OF VIBRATIONS IN TRANSFORMERS

By Dr. D.J. Chambega

ABSTRACT

Noiselevel calculations may be done using an electrical equivalent method. So far a single mass and a double mass vibrating system may easily be represented by equivalent electrical circuits[1] which are in turn useful for analysis. Multimass vibrating systems like transformers do not have straight forward methods which can enable analysis of noiselevels. This paper attempts to develop such a method by extending the method applicable to a double mass vibrating system. Then the validity of the method is tested using a transformer.

INTRODUCTION

Vibrations and therefore noiselevels produced by individual elements in electrical machines may be calculated by using an electromechanical analogy[1]. In general, the method replaces any vibrating system by an equivalent electrical circuit. Then an analogy between the mechanical vibrating system and the equivalent electrical system is done by using differential equations describing the systems. Similar to how mass, energy and friction change in the mechanical system to effect movement of the body and so will inductance, capacitance and resistance affect current in the obtained electrical circuit.

SINGLE MASS VIBRATING SYSTEM

A single-mass vibrating translational system may be:

(a) vibrations produced by an electrical machine flexibly mounted on a massive body; the latter being vibrationless and therefore stationary due to its mass while the former vibrates due to, say, a non-balanced rotor.

(b) vibrations of the stator core in rotating machines or those of transformer lamination due to electromagnetic forces.

Electrical machines having rotors with large moment of inertia are usually considered to have torsional vibrations and hence belong to torsional vibrating systems. The sudden occurence of short-circuits in windings of a machine also cause torsional vibrations.

In a mechanical translational system a moving force P_m acts on mass m connected to a translational elastic element λ_v whose frictional losses r are proportional to speed[2]. In a mechanical torsional system a moving torsional torque m_θ acts on the body whose moment of inertia is J and connected to a rotating elastic element $\lambda_\theta R$ with frictional losses caused by frictional resistance λ_θ also being proportional to speed. Both systems may be analogous to an electrical circuit with an emf source e - comparable to force P_m or torque T_θ - acting on a series connection of inductance L, electrical resistance r and capacitance C.

The unit of measurement for the impedances in the mechanical vibrating systems is termed the "mechanical ohm" which has kgf. sec/cm as the dimension for the mechanical translational system and kgf. cm sec/rad for the mechanical torsional system.

At resonance frequency $\omega = \omega_o$ while the imaginary part of the impedances disappear. At this frequency current i and translational speed y or displacement ϕ in the torsional system have maximum values[3]. In addition, current and voltage, speed and force, angular speed and torque coincide in phase.

In other analogous systems an equality may be drawn between current i and displacement speed y[1]:

(a) For the mechanical translational system

Linear speed of mass m

$$y_3 = \frac{P_M}{r_M + j\omega m}$$

Linear speed at the point of action of force P_m

$$y_1 = \frac{P_M\,(r_M + j\omega m) + \dfrac{1}{j\omega\lambda_M}}{(r_M + j\omega m) + \dfrac{1}{j\omega\lambda_M}}$$

Speed y_2 i.e. the difference between linear speeds at both ends of the spring is given by:

$$y_2 = y_1 - y_3 = P_M j\omega\lambda_M$$

(b) For the mechanical torsional system

Angular speed of the rotating mass with a moment of inertia J

$$\varphi_3 = \frac{m_\theta}{r_\theta + j\omega J}$$

Total angular speed at the point of action of the torque

$$\varphi_1 = \frac{m_\theta \left(r_\theta + j\omega J + \frac{1}{j\omega\lambda_\theta}\right)}{(r_\theta + j\omega J)\frac{1}{j\omega\lambda_\theta}}$$

Angular speed φ_2 i.e. the difference of angular speeds at the two ends of the spring λ_θ:

$$\varphi_2 = \varphi_1 - \varphi_3 = m_\theta j\omega\lambda_\theta$$

(c) For the electrical system
Current i_3 through inductor L and resistor r is given by

$$i_3 = \frac{e}{r_E + j\omega L}$$

Total current

$$i_1 = e\,\frac{\left(r_E + j\omega L\,\frac{1}{j\omega C}\right)}{(r_E + j\omega L)\frac{1}{j\omega C}}$$

Current i_2 through the capacitor equals:

$$i_2 = i_1 - i_3 = ej\omega C$$

A DOUBLE - MASS VIBRATING SYSTEM

A wide range of what may be considered as double-mass vibrating systems are given below:

(a) A machine with mass m_1 mounted on two sets of shock absorbers the first one having compliance λ_1 while the second one has a compliance equal to λ_2 and mass m_2. The machine is brought into a vibrating state by force P_o.

(b) A machine with mass m_1 coupled to a shock absorber whose compliance is λ_1 and an oscillation absorber with a mass m_2

and compliance λ_2. A force P_o acts on the machine.

(c) Iron laminations of mass m_1 (figure 1) and compliance λ_1 connected to a massive body of mass m_2 and compliance λ_2 by a spring whose compliance is λ_3. A radial magnetic force

P_o acts on the lamination.

Mechanical and electrical impedances for the systems described above may be given as follows:

(a) For the mechanical system

$$Z_{m_1} = j\omega m_1$$

$$Z_{m_2} = r_{M_1} + \frac{1}{j\omega\lambda_1}$$

$$Z_{m_3} = r_{\omega_2} + j\omega m_2 + \frac{1}{j\omega\lambda_2}$$

For the electrical equivalent system

$$Z_{E_1} = j\omega L_1$$

$$Z_{E_2} = r_{E_2} + \frac{1}{j\omega C_1}$$

$$Z_{E_3} = Z_{E_2} + j\omega L_2 + \frac{1}{j\omega C_2}$$

(b) For the mechanical system

$$Z_{M_1} = j\omega m_1 + \frac{1}{j\omega C_1} + r_{E_1}$$

$$Z_{m_2} = r_{m_2} + \frac{1}{j\omega\lambda_2}$$

$$Z_{m_3} = j\omega m_2$$

For the electrical equivalent system

$$Z_{E_1} = j\omega L_1 + \frac{1}{j\omega C_1 + r_{E_1}}$$

$$Z_{E_2} = r_{E_2} + \frac{1}{j\omega C_2}$$

$$Z_{E_3} = j\omega L_2$$

(c) For the mechanical system (figure 1)

$$Z_{M_1} = j\omega m_1 + \frac{1}{j\omega\lambda_1 + r_{m_1}}$$

$$Z_{M_2} = r_{M_3} + \frac{1}{j\omega\lambda_3}$$

$$Z_{M_3} = j\omega m_1 + \frac{1}{j\omega\lambda_2 + r_{M_2}}$$

For the electrical equivalent system (also figure 1)

$$Z_{E_1} = jL\omega_1 + \frac{1}{j\omega C_1 + r_{E_1}}$$

$$Z_{E_2} + R_{E_3} + \frac{1}{j\omega C_3}$$

$$Z_{E_3} = j\omega L_3 + \frac{1}{j\omega C_2 + r_{E_2}}$$

Current and vibration speed for all the three examples given above have equal and same solutions given by:

For the mechanical system

$$y_o = \frac{P_o(Z_{m_2} + Z_{m_3})}{H_M}$$

$$y_1 = \frac{P_o Z_{m_3}}{H_M}$$

$$y_1 = P_o \frac{Z_{m_2}}{H_M}$$

And for the electrical equivalent system

$$i_o = \frac{e(Z_{E_2} + Z_{E_3})}{H_E}$$

$$i_1 = \frac{eZ_{E_3}}{H_E}$$

$$i_2 = \frac{eZ_{E_2}}{H_E}$$

where

$$H_E = Z_{E_2} Z_{E_3} + Z_{E_2} Z_{E_1} + Z_{E_3} Z_{E_1}$$

$$H_M = Z_{M_2} Z_{M_3} + Z_{M_2} Z_{M_1} + Z_{M_3} Z_{M_1}$$

The values of these resistances for the particular case of the low quality steel transformer have been obtained experimentally.

APPLICATION OF THE ELECTRICAL EQUIVALENT METHOD TO VIBRATION CALCULATIONS IN TRANSFORMERS.

The double-mass vibrating system is used to develop an electrical equivalent circuit which is a model for the vibration and therefore noise pressure level calculations in transformers.

Noise is caused by vibrating lamination in the core of the transformer[4], the vibrations being by electromagnetic waves formed as a result of the flow of current through the windings engulfing the core. If consider the physical structure of the winding and the way it is normally put in relation to the core it is easy to see that larger electromagnetic forces will be produced along axes x and y (see figure 2). However, the laminations may be considered as not being affected by these forces because they are homogenous along these axes and therefore firmly fixed. A smaller force will act along the z - axis causing excessive vibrations. This is because the laminations along this axis are free to move and are therefore affected by the slightest force the line of action of which will affect in one go all the different laminations in the transformer. This argument is not true for the other axes since the forces line of action only affects in one go one firmly fixed lamination. The vibrations along the z - axis will therefore supercede those along axes x and y.

The calculations will hence consider only those vibrations along the Z - axis. It is assumed in the first instant, that the outer lamination labelled A and B in fig.3 (b) are coupled to massive bodies.

The real mechanical diagram, considering the x-z plane only, is shown in fig.3(a). Force P_0 which acts along the z - axis causing the system to vibrate is also shown. The equivalent mechanical diagram for the system can then be obtained and is similar to the examples for a double-mass vibrating system. In this case n is the total number of laminations across the transformer core along the z - axis. The mechanical system may be modelled electrically to obtain the circuit shown in Fig.3(c). It is realized from the diagrams shown that the laminations are usually lumped in steps. Since the dimensions of laminations within each step are equal they have the same values of capacitance, inductance and resistance. With the exception of the outer laminations, all the laminations within the same step and having the same parameters may be reduced to a single branch to obtain the circuit shown in figure 3(d). In this case s is the number of steps; the circuit having been reduced and therefore very simple to calculate for current i_0 which may then be converted to vibration and noiselevels. Hence the final circuit (fig.3(d)) is the one convenient for use in vibration calculations to the transformer. Usually n is several times larger as compared to s.

In reality, though, the outer laminations are not fixed as the middle part along the y - axis is free to vibrate. If this is taken into account several positions have to be considered each time changing the values of instantaneous inductances L_1 and L_2 alongside with source e. In this particular case, however, a position which has been assumed as emmitting the most serious vibrations was taken. This is midway between the most outward and most inward positions for either of them.

EXPERIMENTAL RESULTS

A laboratory prototype of the transformer, has been built and tested using the extended to multimass electric equivalent method. Plotted results obtained from the prototype are shown in figures 4 and 5.

Experimental and computed results appear side by side for easy comparison.

CONCLUSION

In general there was a satisfactory agreement between the computed and experimental results. The extension to a multimass system of the electrical equivalent method can therefore be used for noiselevel calculations. The method can hence prove useful to multimass systems still under design but before the production of a prototype to check noiselevels emitted by the object.

REFERENCES

1. Noise and vibration in electrical machines. Shubof, I.G. Energia Publication, 1973.

2. Electrical variable - speed drives. B.L. Jones and J.E. Brown, IEE Proceedings, Vol.131, PtA, No.7 September, 1984.

3. Power electronics, control of the electromechanical energy conversion process and some applications. J.D. Van Wyk, H.C. Skudelny and Muller - Hellmann. IEE Proceedings, Vol 133, PtB, No.6, November, 1984.

4. The magnetic noise of polyphase induction motors. P.L. Alger, TAIEE, 1954, pg. 75 - 82 with ill.

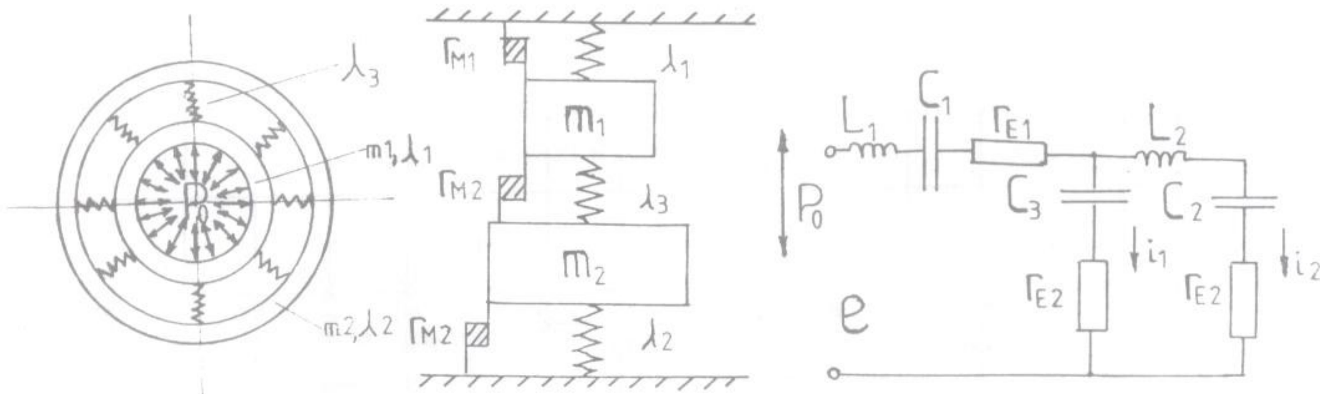

Figure 1 - Laminations connected to a massive body and its electrical equivalent system.

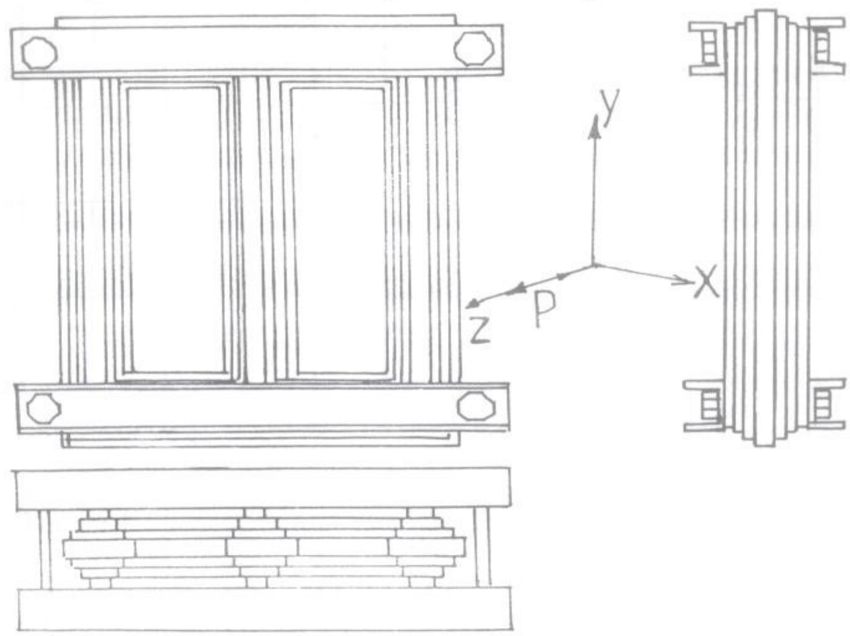

Figure 2 - General layout of a core of a transformer.

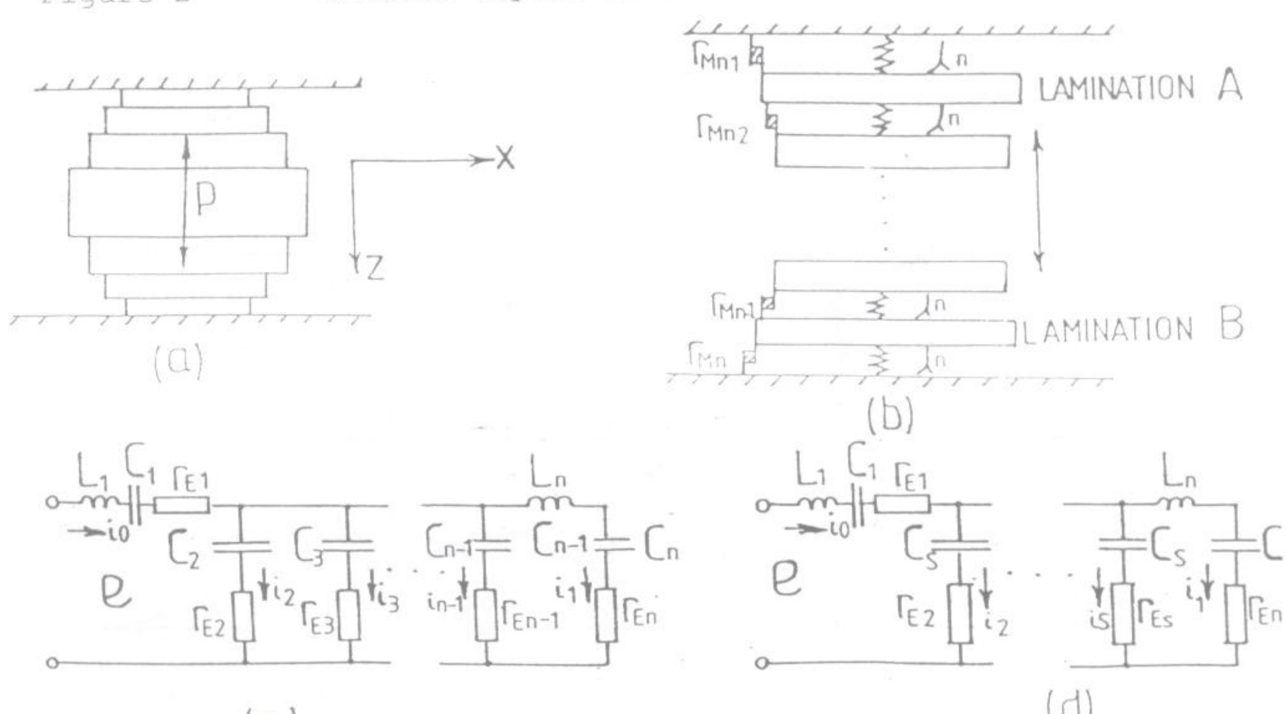

Figure 3 - The transformer mechanical vibrating system and its equivalent electrical system.

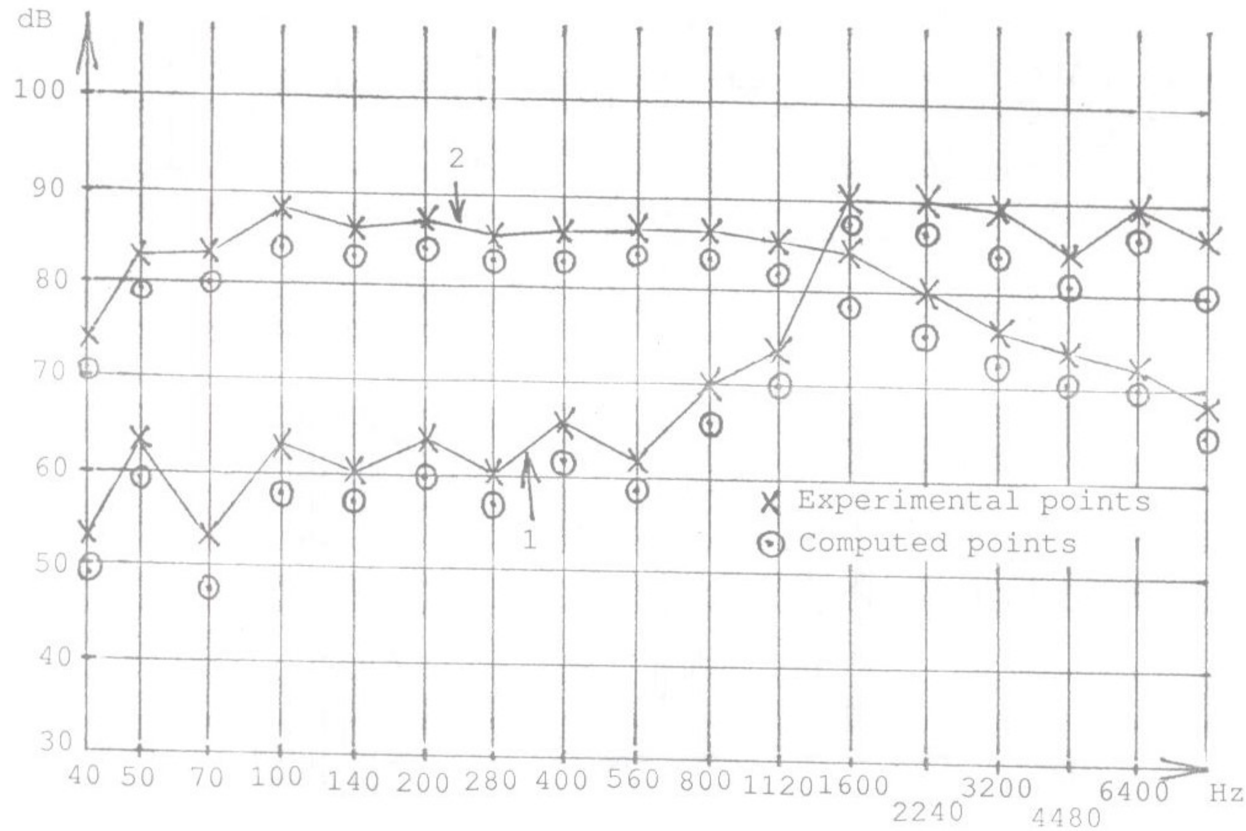

Figure 4 - Spectrogram showing vibrations (line 1) and noiselevels (line 2) for the test transformer (5kVA).

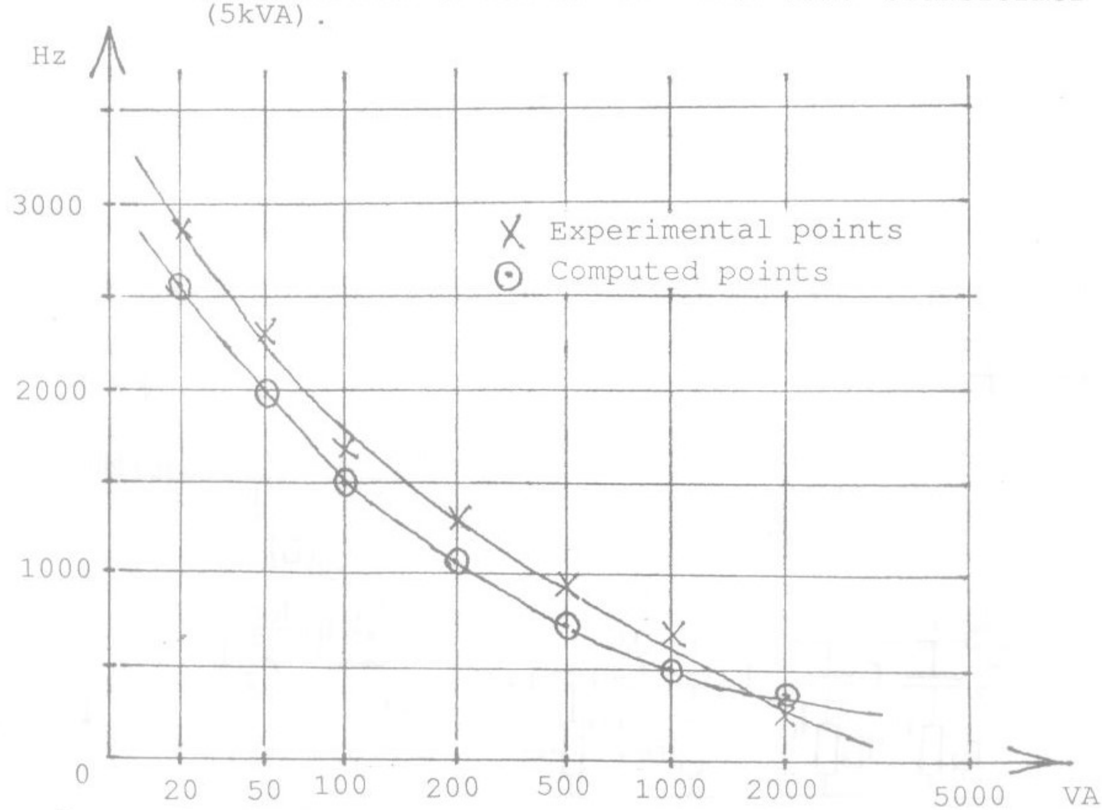

Figure 5 - Relationship between natural frequency of the core and power of the transformer.