DERIVATION OF AN EXPRESSION THAT DEFINES THE PULLING FORCE IN RELAYS WITH SMALL OPERATING GAPS

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This paper derives an expression that defines magnitude of the pulling force developed in relays with small operating gaps, that is, when the length of the operating gap in between the yoke and the armature is small as compared to the area of the poles at the yoke. To obtain the expression, saturation of the core material is not considered. Results from the expression are then compared with those obtained experimentally. The comparison is found to deliver acceptable accuracy.

Keywords: *Pulling force, small gap, operating gap, reluctance.*

INTRODUCTION

Operating gaps in relays are defined as large or small by considering the ratio of the effective operating airgap to the area of the poles. Because of the magnetic properties of the materials in the airgaps and the poles, a majority of available electromagnetic relays have large operating gaps. An expression that defines the pulling force developed in such relays is known [Atabekov, 1960, Van Warrington, 1968]. This paper develops an expression for pulling forces in relays with small operating gaps.

DEVELOPMENT OF THE EXPRESSION

When the length of the operating gap is small as compared to the area of the poles of the yoke and when the core material is not saturated it is possible to take the following expression [Russel, 1980]:

$$G_\delta = \frac{S}{\delta}\mu_o; \frac{dG_\delta}{d\delta} = -\frac{S}{\delta^2}\mu_o = \frac{G_\delta^2}{\mu_o S} \text{ [H]} \qquad (1)$$

where

G_δ Permeance of operating airgap dependent on displacement angle of armature
S Cross-sectional area of each plane in m^2
δ Airgap length in metres
μ_o Permeability of air

In this case for pulling force the following expression is obtained

$$F = \frac{(IN)^2}{2G_\delta^2}\frac{G_M^2 S}{\delta^2}\mu_o = \frac{(IN)^2 G_M^2}{2\mu_o S} \text{ [N]} \qquad (2)$$

where
F Pulling force
G_M Permeance of relay magnet
IN Magneto-motive force, mmf, (or ampere-turns)

The common permeance, for negligible effect of leakage fluxes, is:

$$G_M = \frac{1}{R_M} = \frac{1}{R_\pi + R_\delta} \text{ [H]} \qquad (3)$$

where

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R_m Reluctance of relay

R_δ Reluctance of the relay operating airgap dependent on displacement angle of armature

R_π Reluctance of the remaining parts of the active material (core, casing, base, armature, joints, pins) not dependent on displacement angle

Substituting in expression (2) instead of G_M its value results into:

$$F = \frac{(IN)^2}{2\mu_o S(R_\pi + R_\delta)^2} = \frac{B^2 S}{2\mu_o} \text{ [N]} \qquad (4)$$

where

B Magnetic induction

When the armature is fully attracted ($R_\delta = 0$) the pulling force becomes:

$$F_{\delta o} = \frac{(IN)^2}{2\mu_o S R_\pi^2} = \frac{\Phi_{\delta o}^2}{2\mu_o S} \text{ [N]} \qquad (5)$$

where

$F_{\delta o}$ Pulling force when the armature is fully attracted ($R_\delta = 0$)

$\Phi_{\delta o}$ Magnetic flux when the armature is fully attracted ($R_\delta = 0$)

The relationship of magnetic fluxes at unattracted and fully attracted positions of the armature is given by:

$$\frac{\Phi}{\Phi_{\delta o}} = \frac{R_\pi}{R_\pi + R_\delta} = \frac{\mu_o S R_\pi}{\mu_o S R_\pi + \delta} \qquad (6)$$

where

Φ Magnetic flux

from where

$$\Phi = \Phi_{\delta o} \frac{\mu_o S R_\pi}{\mu_o S R_\pi + \delta} \text{ [Wb]} \qquad (7)$$

Substituting in equation (4) the value of Φ results into:

$$F = \frac{\Phi_{\delta o}^2 (\mu_o S R_\pi)^2}{2\mu_o S(\mu_o S R_\pi + \delta)^2} = F_{\delta o} \frac{(\mu_o S R_\pi)^2}{(\mu_o S R_\pi + \delta)^2} \text{ [N]} \qquad (8)$$

If the effect of leakage fluxes is taken into account and neglect the reluctance of the soft iron of the core ($R_{si} = 0$) then in agreement with the equivalent circuit of the magnetic system of the relay [Kadete et al, 1991] it is possible to obtain the following expression:

$$\Phi_\delta (R_\delta + R_\pi) = \Phi_o \frac{R_g (R_\delta + R_\pi)}{R_g + R_\delta + R_\pi} \qquad (9)$$

where

R_g Reluctance of airgap

Φ_o Magnetic flux at distance

from where

$$\Phi_\delta = \frac{\Phi_o R_g}{R_g + R_\delta + R_\pi} = \frac{\Phi_o R_g}{R_g + R_\pi + \dfrac{\delta}{\mu_o S}} \text{ [Wb]} \qquad (10)$$

Substituting in equation (4) instead of $\Phi = \Phi_\delta$ the following expression is obtained:

$$F = \frac{\Phi_o^2 (\mu_o S R_g)^2}{2\mu_o S\left[\mu_o S(R_\pi + R_g) + \delta\right]^2} \text{ [N]} \qquad (11)$$

Hence in case of an optimum relationship of reluctances of individual parts of the core, the expression for pulling force will be of the following form:

$$F_{opt} = \frac{(IN)}{8\mu_o S R_\pi^2} = \frac{(IN)^2 S}{8\delta^2} \mu_o \text{ [N]} \qquad (12)$$

where

F_{opt} Optimum pulling force

RESULTS

The following results were obtained and considered in order to establish the validity of the expression.

Load characteristic

Load characteristic is the relationship between pulling force of the relay and magnetomotive force, mmf, at constant value of the operating airgap. If saturation of the core and the effect of leakage fluxes are neglected the pulling force is given, in agreement with equation (12), by:

$$F = \frac{(IN)^2 S}{8\delta^2}\mu_o [\text{N}] \tag{13}$$

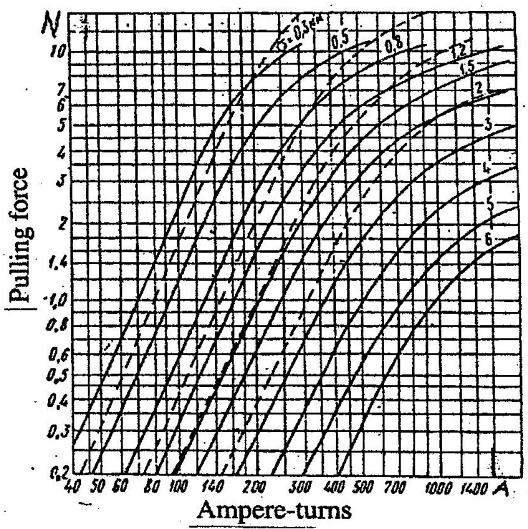

Figure 1 Load characteristics of (a) a drop-type relay of medium power – continuous line and (b) a solenoid type relay of medium power – dotted line

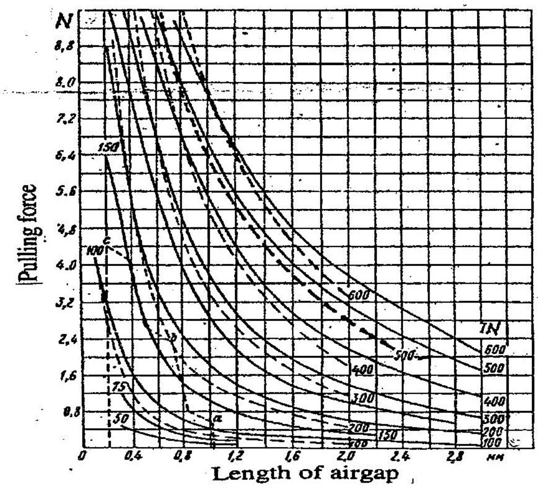

Figure 2 Electromechanical characteristic of a drop type relay of medium power (a) experimental – continuous line (b) computed (using the expression) – dotted line.

Hence, at relatively small values of the effective airgap and optimum ratio of reluctances R_π and R_δ, pulling force at constant value δ is proportional to the square of mmf.

Depending on the value and form of the surface of poles and armature, the law that governs the change in permeance of the effective airgap during displacement of the armature may change its character. Hence the load characteristic of the relay may be of different forms.

Figure 1 shows load characteristics of both drop-type and solenoid type relays of medium power. From the characteristics it follows that for producing a relatively sizeable pulling force of 0.2N at an operating gap of 0.5mm, mmf of about 46A-turns is needed.

Electromechanical characteristic

Electromechanical characteristic of the relay defines the relationship between pulling force of the armature and the length of airgap at constant mmf. From equation (12) it follows that at no saturation of the steel and a relatively small operating gap the value of pulling force at constant mmf may be considered inversely proportional to the square of the length of operating gap. If there is a change of the value and form of pole endings and armature, it is possible to change the electromechanical characteristic. Figure 2 shows a series of electromechanical characteristic of medium size drop type relay obtained at different values of mmf. If there is an increase of the mmf then at some value, (IN), the pulling force remains equal to the opposing mechanical force (at point c) while when further increasing the mmf the armature of the relay starts to move. The value of the mmf $(IN)_{op}$ at this point is the operating mmf of the relay.

DISCUSSION OF THE RESULTS

The relay fully operates only in the case when the pulling force of the armature along its entire path of displacement is greater than the opposing force of the mechanical load. Hence the electromechanical characteristic corresponding to the operating mmf of the relay exists above the mechanical characteristic of this relay. From figure 2 it follows that operating mmf (or pulling mmf) of the relay is obtained not by initial or ultimate loads of the armature but by largely the loading interval representing the mechanical characteristics. The point of intersection, labelled b, of mechanical and electromechanical characteristics of the relay is the critical mmf or operating mmf. In some cases the critical point coincides with point a, which forms the mechanical characteristic.

In order to decrease the operating mmf and weaken the armature's impulsive force to the core it is desirable to match the mechanical to the electromechanical characteristics of the relay, that is, the angles of inclination of the characteristics and their coordinates should be as near to each other as is possible.

To initiate movement of the armature at release it is necessary to decrease the mmf to a value $(IN)_{opr}$ (operating mmf at release) at which the electromechanical (pulling) characteristic passes through the last point, which is also part of the mechanical characteristic.

CONCLUSION

The paper has developed an expression that defines the pulling force in relays with small gaps. The expression was tested through experimentation with relays at different conditions and the results obtained coincide with those obtained through computations using the derived expression. The expression can hence be used to predict the pulling force in such relays and may prove useful to both the designers and users of the relays.

NOMENCLATURE:

B Magnetic induction
F Pulling force
$F_{\delta o}$ Pulling force when the armature is fully attracted ($R_\delta = 0$)
F_{opt} Optimum pulling force
G_δ Permeance of operating airgap dependent on displacement angle of armature
G_M Permeance of relay magnet
IN Magneto-motive force, mmf, (or ampere-turns)
$(IN)_{op}$ Operating mmf
$(IN)_{opr}$ Operating mmf at release
mmf Magneto-motive force
R_δ Reluctance of the relay operating airgap dependent on displacement angle of armature
R_π Reluctance of the remaining parts of the active material (core, casing, base, armature, joints, pins) not dependent on displacement angle
R_B Reluctance in between the soft iron space
R_g Reluctance of airgap.
R_m Reluctance of relay
R_{si} Reluctance of the gap in between base and the armature along the motion axis of the latter
S Cross-sectional area of each plane in m^2

Greek Symbols

δ Airgap length in metres
μ_o Permeability of air
Φ Magnetic flux
$\Phi_{\delta o}$ Magnetic flux when the armature is fully attracted ($R_\delta = 0$)
Φ_o Magnetic flux at distance

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