SIMULATION OF THE COMMINUTION PROCESS IN A PIN MILL

Ambrose J. M. Itika
Chemical and Process Engineering Department, University of Dar es Salaam
P. O. Box 35131, Dar es Salaam, Tanzania

Prediction of the performance of comminution equipment by mathematical models and simulation poses a great challenge to the chemical engineer. This paper describes the simulation of grinding results obtained using an impact mill. Single particle results have been used to generate breakage and selection coefficients that were used for the evaluation of the model. Results have shown that simulation model is in good agreement with the experimental data obtained at a circumferential speed above 60 m/s. At lower speeds, the models shows only partial agreement in the middle particle size ranges. The application of the model is therefore limited for higher speeds of the rotor producing mono-modal particle size distributions.

Keywords: *Simulation, pin mill, comminution, grinding, selection function, breakage function.*

INTRODUCTION

An attempt has been made to build a model for describing the product coming from a pin mill. The approach adopted here, that of selection and breakage functions, has evolved from the early attempt by Epstein, (1948) to statistically describe the breakage of materials. He introduced two important functions namely the probability of breakage and the size distribution of the product of comminution. He found out that a product of a multiple breakage tended to log-normal size distribution. Sedlatchek and Bass, (1953) presented the possibility to use differential equations to describe batch grinding. They made assumptions that the probability of breakage (selection function) is independent of the proportions of any other size fractions present. A further step was made by Bass (1954) in which he formulated a mathematical theory for the milling process and derived an integro-differential equation for mass balance in batch grinding. There have been also efforts to use matrix formulation to represent the breakage of particles notable among them being those of Broadbent and Callcott, and Gaudin and Meloy as cited by Reid (1965). Other authors proposed models specific for pin mill. Huth (1976) proposed a mass balance model for pin mills which, however, is criticised for considering the mill as a black box. Further, multiple stressing and change in the stressing energy within the active chamber is not considered. Husemann et al. (1979) have proposed the use of a particular specific parameter, in their case, specific surface area to develop a model for pin mills. They stated that the intensity of stressing particles in conventional pin mills differs with the distance from the centre. They put forward the following equation:

$$\frac{\partial S_v}{\partial t} = -\frac{\partial (v.S_v)}{\partial r} + \frac{\partial^2 (D_t S_v)}{\partial r^2} + G \qquad (1)$$

where: S_v is the specific surface area of the product related to volume, t is time, r is radius, v is convective transport velocity, D_t is diffusion coefficient and G is the source generator.

Assumptions are made that plug flow in the radial direction is established and the parameter, v, in the macroscopic transport term is equal to the radial velocity. Heiskanen (1978) incorporated mill parameters in building the selection function for a ball mill. The selection function therefore consists basically of the probability of breakage usually obtained from single particle impact experiments and, in this case, ball parameters.

Of late, there has been more criticism of the assumptions on which the selection and breakage functions are built. Kupur and Fuerstenau (1995) argue that: "The widely invoked assumptions of time-invariant breakage rate function (selection function) and energy independent breakage function in the population balance phenomenological models can not be reconciled with the single particle impact grinding results". King and Bourgeois (1993) assert that selection and breakage functions are descriptive but not predictive since they are not related to fundamental material properties and fracture mechanics governing comminution of the particles. According to Shoenert, (1991) the breakage function cannot be expressed by one normalized master function. It usually depends on particle size and energy absorption. As such the functions become broader with increasing particle size and energy.

Despite all the criticism, this approach to mill simulation still finds wide acceptance and application. The approach has been applied for the case of the pin mill to evaluate how far it simulates the real life situation.

THE MODEL

Ideally, modelling of comminution starts with the general transport equation widely applicable in chemical engineering:

$$\frac{\partial\Gamma}{\partial t} = -div(\Gamma v) + div(\delta . grad\Gamma) + G \qquad (2)$$

where: Γ is the characteristic process parameter, v is the convective transport velocity, t is time and G represents the source.

The equation states that:

local change = sum of (macroscopic transport, diffusion, and source)

The fundamental mass balance equation which was originally formulated for the batch grinding in ball mills (Reid, 1965) is in principle a variation of the above equation:

$$\frac{\partial^2}{\partial x \partial t} m(x,t) = -s(x)\frac{\partial}{\partial x} m(x,t) + \int_x^{x_0} \frac{\partial}{\partial x} B(x,\alpha)s(\alpha)\frac{\partial}{\partial x} m(\alpha,t)d\alpha \qquad (3)$$

where m(x,t) gives the mass fraction finer than x after grinding for time t. $B(x,\alpha)$ represents the fraction finer than x obtained by primary breakage of material of size α (i.e. without re-grinding of the fragments), also called the breakage function. x_o is the maximum particle size in the size distribution and $s(x)$ stands for the fraction of material of size x that breaks per unit time.

The equation as presented above, is of little practical use and cannot be solved without further simplification. In order to simplify the solution to the equation and extend its application to the continuous milling operation, it is normally discretized and applied to narrow particle size classes (Reid, 1965; Schoenert, 1983).

$$\frac{dm_i}{dt} = -s_i m_i + \sum_{j=1}^{i-1} s_j b_{ij} m_j \qquad (4)$$

where m_i is the mass fraction in class i, s_i is the grinding rate or the mass fraction that leaves class i in a unit time (also known as selection function), b_{ij} is the mass fraction that during the breakage falls from size class j to size class i. The first index gives the target class and the second one represents the originating class. The coefficient b_{ij} is dimension less.

The size classes must in this case obey the geometric similarity precondition. That means a constant factor, in this particular case $x_i = \sqrt{2}x_{i+1}, x_1/x_2 = \sqrt{2}$, is maintained in the selection of the size classes. Furthermore the following conditions are also fulfilled:

$$\sum_{i=1}^{n} m_i = 1 \qquad \text{with} \qquad m_i(t=0) = m_{io} \qquad (5)$$

$$\sum_{i=j}^{n} b_{ij} = 1 \text{ and } b_{jj} = 0 \qquad (6)$$

For the case whereby s and b are constant, the solution proposed by Reid (1965) and further expounded by Schoenert (1983) is valid i.e. m_i the mass fraction of class is given by:

$$m_i(t) = \sum_{\mu=1}^{i} A_{i\mu} \exp(-(s_\mu, t)) \qquad (7)$$

in which the coefficient $A_{i\mu}$ is calculated as:

$$A_{i\mu} = \sum_{j=\mu}^{i-1} \frac{b_{ij} s_j A_{j\mu}}{s_i - s_\mu} \quad (8)$$

where s_μ is the grinding speed for mean particle size and further,

$$A_{ii} = m_{io} - \sum_{\mu=1}^{i-1} A_{i\mu} \quad (9)$$

Furthermore, it is assumed that back mixing does not take place and therefore flow in the annular gap of the mill approximate plug flow. In this respect, convection and diffusion influences can be ignored. With this in mind, the transport equation above can be solved as already discussed. However, for the case of continuous milling, the time factor posses a hindrance and is of no physical significance. Rather, a concept of a model factor which is proportional to the number of stressing in the axial direction is introduced. The rate of change of mass in the particle size class i is correspondingly expressed as:

$$\frac{dm_i(\xi)}{d\xi} = -s_i m_i(\xi) + \sum_{j=1}^{i-1} s_j b_{ij} m_j(\xi) \quad (10)$$

where ξ is the new factor defining the number of stressing in the mill. The other symbols carry the same meaning as before. Using this modified approach, the solution for the mass fraction for class i to the model equation can henceforth be written as follows:

$$m_i(\xi) = \sum_{\mu=1}^{i} A_{i\mu} \exp(-s_\mu \xi) \quad (11)$$

and likewise

$$A_{i\mu} = \sum_{j=\mu}^{i-1} \frac{b_{ij} s_j A_{j\mu}}{s_i - s_j} \text{ and} \quad (12)$$

$$a_{ii} = m_{io} - \sum_{\mu=1}^{i-1} a_{i\mu}$$

Experimental data is required to find the variables b_{ij} and s_i from which the coefficients $a_{i\mu}$, can be calculated.

Breakage Function

The breakage function is in principle a cumulative function of the fragments from a breakage event. It describes, with the help of various functions, the quantitative variation of the products of breakage as a function of their size. For a discretized size distribution the following equation is true:

$$B_{ij} = B(y_i, x_j) = \frac{Q_3(y_i)}{s_j} = \frac{Q_3(y_i)}{Q_3(y_{j+1})}$$

$$(y_i \le x_{j+1}) \quad (13)$$

where B_{ij} is the breakage function

Single particle experiment data originally obtained by Droegemeier (1998) has been used for simulation purposes. He conducted investigations on different size classes that were geometrically divided. As already explained above, b_{ij} is the weight fraction of material that by simple breakage of a particle (non repetitive breakage) goes from class j to class i where i > j. For the case where measurement is made using sieves, this implies simply the fraction of material retained on the sieve. If b_{ij} is determined from cumulative size distributions then one has first to calculate the residue, R_i as:

$$R_i = 1 - Q_3 \quad (14)$$

The weight fraction b_{ij} is therefore given by:

$$b_{ij} = R_i - R_j \quad (15)$$

From primary breakage experiments for all feed classes one obtains a triangular matrix of b_{ij} values.

Selection Function

From experimental investigations of primary breakage of particles the probability of breakage found as the cumulative value for the fragments corresponding to the lower limit of the feed class. If this is determined for the whole range of feed size classes then a selection function is obtained. In this work, the selection function has been determined by combining experimental data with the theory of the distribution of cracks on the surface of the particles developed by Weichert (1992). The theory is based on experimental

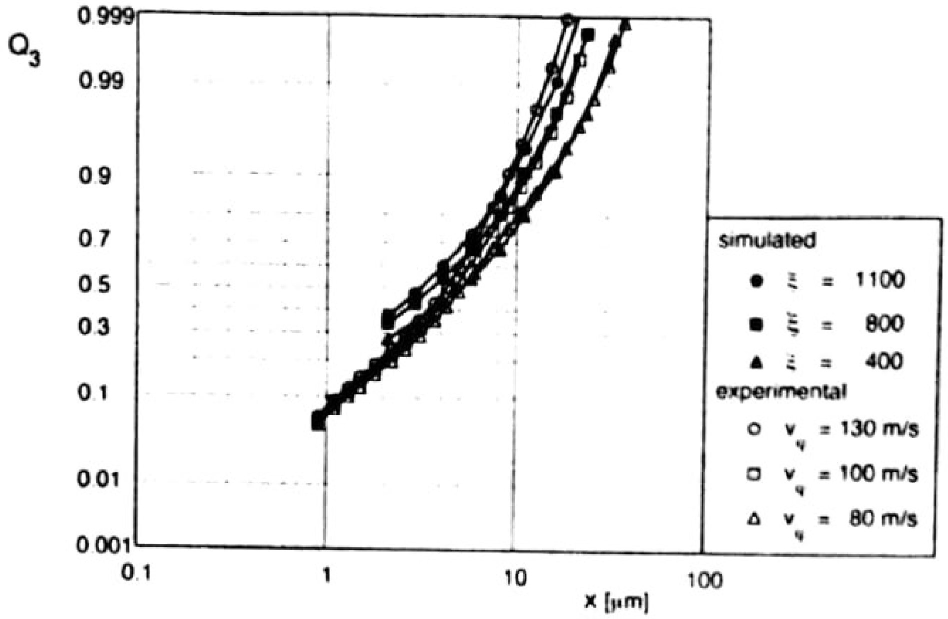

Figure 1: Comparison of the simulated and experimental results at between 80 and 130 m/s for pill mill

investigations by Weibull on the tensile strength of rods. The basic assumption here is that cracks on the surface of particles are distributed according to Weibull statistic. Bringing in energy and breakage mechanism considerations, an equation for the selection, s(x), function is represented by:

$$s(x) = 1 - ept(-cx^2 W_v^{\,z} k^{1-z}) \qquad (16)$$

where x is particle size, c and z are material constants, W_v is volume specific energy and k is number of contact points between particle and impact surface

For impact stressing, the value of k = 1 since the particle is stressed from only one side. The energy term, W_v is proportional to the square of the circumferential speed of the rotor. According to Droegemeier (1998), for a particular circumferential speed the energy term can be condensed together with the material constants to a parameter $c^*(v_\varphi)$ resulting into:

$$s(x, v_\varphi) = 1 - \exp[-c^*(v_\varphi)x^2] \qquad (17)$$

where v_φ is the tangential speed of the mill

The equation was used with values of s(x) obtained for a number of classes at the respective circumferential speeds to evaluate the values of $c^*(v_\varphi)$. It turned out that the values of $c^*(v_\varphi)$ increased with particle size and with the rotor circumferential speed. The decrease with particle size was smaller compared to increase with circumferential speed. Through testing it was established that changes in selection function could fully be compensated with the variation of the parameter ξ (Eqn. 11) representing the number of impacts. It was therefore decided to take the average value for the tested classes. It must be pointed out here that this approach is very approximate and the results obtained from it are purely for estimation purposes.

RESULTS OF SIMULATION

Based on the above description of the model, calculations were made to simulate the results obtained in experiments with a pin mill. Fig. 1 shows results obtained from simulation as compared to results of comminution in the pin mill at circumferential speeds of between 80 and 130 m/s. The number of impact events, ξ, was varied to obtain the best curve approximating the experimental particle size distribution. For particle sizes above 6 μm, the model simulated very well the experimental data in all the three cases of

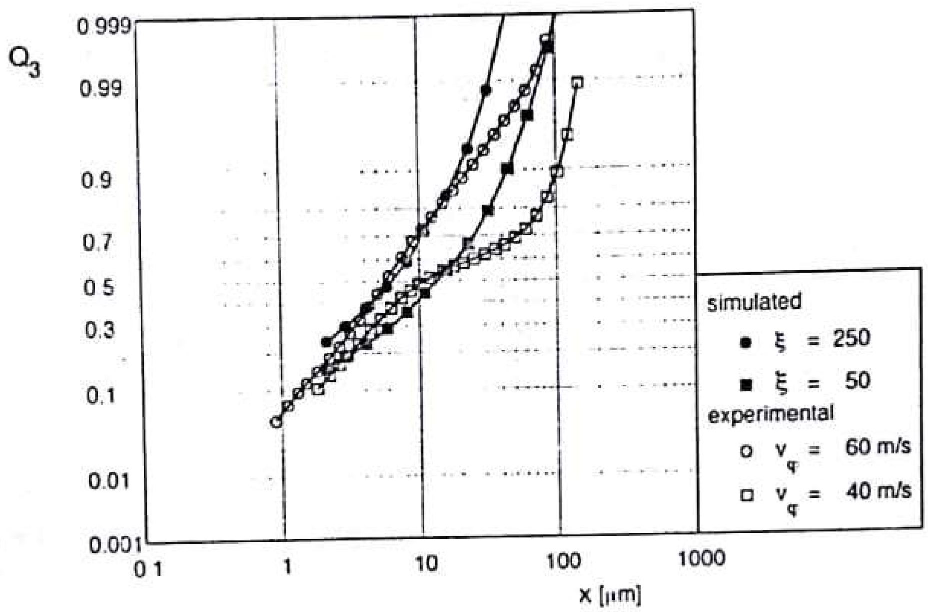

Figure 2: Simulation of the comminution results at circumferential speeds of 60 and 40 m/s

varying circumferential speed. However, below this size limit, there is marked deviation of the simulated data from the experimental ones. The model gives higher amounts for the fine material than what was determined experimentally. This is probably due to the fact that the model used the breakage function of the coarser size classes for the whole size spectrum as no data was available for the finer ones.

The number of impacts, ξ, of 400, 800 and 1100 for circumferential speeds of 80, 100, and 130 m/s as determined in this evaluation are higher by up to 61 % than those reported in Droegemeier (1998) for circumferential speeds of 83 and 125 m/s. This is indicative of higher number of impacts taking place in pin mill.

For lower circumferential speeds, the model does not simulate the results so well. Fig. 2 shows the simulated results compared to experimental ones. At a circumferential speed of 60 m/s the product is still mono-modal and can be reasonably well simulated in the middle ranges of particle sizes between 4 and 15 µm. For particle sizes outside this range the simulation gives higher amounts of materials for a particular size. At a circumferential speed of 40 m/s there is generally no good agreement between simulated results and experimental ones. The product is multi-modal with size distribution pattern that defies a single mathematical description. Observation of the curve reveals three regions each of which could have an independent mathematical definition. The simulation curve intersects the experimental one at two points with x = 2 µm and x = 15 µm.

CONCLUSION

The selection and breakage functions have been used to simulate results obtained from experiments with pin mill. It has been shown that the model has limited range of application in this case. For the higher circumferential speeds between 80 and 130 m/s, the model could be used to simulate the particles size distribution of the product reasonably well. But even in this case better agreement with comminution results is obtained for particle sizes above 6 µm. Below this size range the model made higher predictions than the actual product. At a circumferential speed of 60 m/s the model only partly described the middle ranges of the product while at a circumferential speed of 40 m/s it failed almost completely. The model can therefore only be used for a limited range of operation of the mill.

ACKNOWLEDGEMENT

I wish to acknowledge the financial support of the DAAD who funded my stay in Germany at the time this research was conducted. I also thank sincerely a research colleague Dr.-Ing. Droegemeier who assisted with the setting up of the research.

NOMENCLATURE

Symbol	Units	Description
a	-	constant
A_{ii}	-	coefficient
b	-	constant
b_{ij}	-	mass fraction going from class j to class i
B_{ij}	-	breakage function
c	-	material constants
k	-	collision number
k	-	number of contact points between particle and impact surface
m	kg	mass
m_i		mass fraction in class i
Q		cumulative volume fraction
R_i		residue
s_i		selection function(grinding rate)
S_v	m^2/m^3	volume related surface area of particles
t	s	time
W_m	J/g	mass specific comminution energy
W_z	J/m^3	is specific energy
x	m	particle size
z		material constants
ξ		number of impacts

REFERENCES

1. B. Epstein *Logarithmic-normal distribution in breakage of solids*, Industr. Eng. Chem., Vol. 40, No. 12, pp. 2289-2291, 1948
2. K. Sedlatschek, L. Bass : *Contribution to the theory of milling process*, Powder Metal, Vol. 6 No. 5, pp. 148-153, 1953
3. L. Bass: *Zur Theory der Mahlvorgaenge*, Zeitschrift Angewendet Mathematik und Physik, Vol. 5 No. 4, pp. 283-292, 1954
4. K. J. Reid: *A solution to a batch grinding equation*, Chemical Engineering Science, Vol. 20, pp. 953-963, 1965
5. W. Huth: *Zur maschinellen Prallzerkleinerung von Zuckerkristallen in einer Stiftmuehle*, Dissertation, Technical University Dresden, 1976
6. K. Husemann, E. Toepfer, W. Scheibe: *Beanspruchungsvorgaenge in Stiftmuehlen*, Aufbereitungstechnik No. 10, pp. 551-558, 1979
7. K. Heiskanen: *On the estimation of system parameters in mathematical simulation of batch grinding*, Dissertation, Helsinki University of Technology, 1978
8. P.C. Kapur, D. W. Fuerstenau: *A model analysis of the impact grinding of single particles*, Proceedings of the XIX International Mineral Processing Congress, Colorado, pp. 127-130, USA 1995
9. R. P. King, F Bourgeois: *A new conceptual model for ball milling*, XVIII International Mineral Processing Congress, Sydney, 23-28, pp. 8185, May 1993
10. K. Schoenert: Advances in comminution fundamental, and impact on Technology", XVII International Mineral Processing Congress, Dresden, 1991, p. 1-21
11. K. Schoenert: *Zerkleinern,* Skriptum zur Vorlesung an der Universitaet Changsha, VR China, March 1983

12. R. Droegemeier: *Feinstzerkleinerung von Kalkstein in einer neuartigen Rotorprallmuehle*, Dissertation, TU-Clausthal, 1998

13. R. Weichert: Anwendung von Fehlerstellenstatistik und Bruchmechanik zur Beschreibung von Zerkleinerungsvorgaengen, Cement.Lime.Gypsum, January 1992