LOAD FLOW SOLUTION FOR THREE-PHASE MV RADIAL DISTRIBUTION $\Delta - \Delta$ CONNECTED NETWORK SUPPLYING DIFFERENT LINE TOPOLOGIES

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In this paper, a load flow solution for the Medium Voltage (MV) three-phase radial distribution in delta-delta ($\Delta - \Delta$) connected network supplying different line topologies such as three phase, phase to phase and single-wire earth return (SWER) networks is presented. The general voltage related network equations are developed using Kirchhoff's voltage and current laws. Development of a solution to the load flow analysis applying the developed general voltage related network equations is proposed in order to evaluate the network voltage profile. Various numbers of arrays are developed for the identification of the network under study. The proposed method is a new concept and was tested using balanced and unbalanced data and the results are compatible to the basic theory of ohm's law.

***Keywords:** Load flow; Unbalanced three-phase radial distribution networks; computer programming*

INTRODUCTION

Distribution power networks in rural areas employ different topologies to supply electricity to consumers. Utility companies or distribution power companies in different countries adopt different topologies in order to reduce the cost of radial distribution networks. One of such systems uses three-phase three-line system as a backbone feeder connected between the secondary of substation transformer that is delta connected and the primary of the distribution transformer that is also delta connected as shown in Figure 1. The use of two phase and SWER networks are facilitated by tapping from the backbone feeder (Shiel, 1994). The choice of an appropriate system depends on the type of load to be served and the actual distance from the backbone feeder. In general, for any system, criteria for the choice of the appropriate line topology to be applied are based on the following (Herman et al, 1998):

- the length of the MV distribution line
- type of the load to be supplied i.e. low or high load density
- the magnitude of the load i.e. low load or high load
- prospects for load growth i.e. growth of population, increase in commercial activities, development of industries, etc.

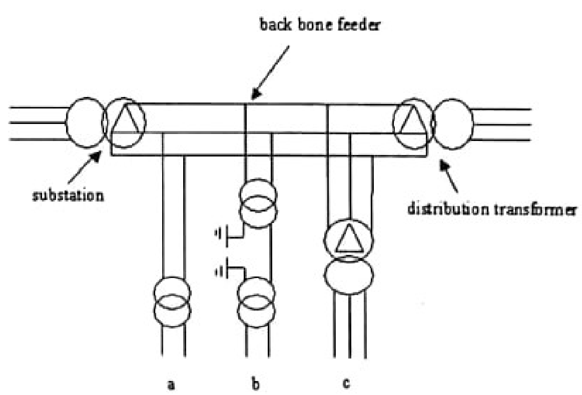

Figure 1: The simplified three-phase three wire delta connected backbone feeder feeding: (a) Single-phase topology. (b) SWER topology. (c) Three-phase topology

Existing solutions that express individual phases to ground such as those in (Teng, 2000; Kersting et al, 1992; Baran et al, 1997) cannot be able to model the network depicted in Figure1. Modelling a three-phase power networks as star-connected systems enable the application of Y-bus matrix for load flow analysis (Caramia et al, 1999). In (Kundy, 2003), a development of a formulation for radial distribution network is presented. It shows that, the structure of radial power distribution network allows each load point to be traced to the source. Having this characteristic, it is possible to develop a formulation that can relate each node with the source. The idea is to express the relationship in terms of the system currents, which means that, the individual load current is traced to the source. This is facilitated by the formulation of four main arrays as follows:

- path-array describing the path from any node to the supply node, this path contains all the branches connected between the node point in question and the supply node
- b-array describing the individual load currents through any branch of the network
- x2-array containing the total number of paths (i.e. total number of connected branches) from any node of the network to the supply node
- wx-array containing the total number of load currents through any branch of the network

The entire distribution network can be described by an array called "D-array" in which the total number of columns represents the total number of the branches and their identification. In this paper, the concepts outlined in (Kundy, 2003) are used to perform a load flow on three-phase radial distribution network of the nature depicted in Figure1 in conjunction with appropriate voltage related network equations that satisfy the network configuration. The three-phase radial distribution network is configured in four different categories that enables the individual phases to be defined. To perform computations on the network under study, a single line diagram is proposed to express all the system nodes. By referring to the single line diagram, four main arrays referred as path-array, b-array, x2-array and wx-array are developed. In the analysis only positive sequence voltages are being considered.

DEVELOPMENT OF THE VOLTAGE-RELATED NETWORK EQUATIONS

The power load flow for the distribution system described in this paper can be performed if system voltage-related equations are known. To develop the system voltage-related equations, it will be appropriate to consider only one load point with phase loads connected in delta as shown in Figure 2.

At node 1 in Figure2, the three phases are identified as phase ba, cb and ac with designated phase voltage $\overline{V}_{1,ba}$, $\overline{V}_{1,cb}$ and $\overline{V}_{1,ac}$ respectively. The phase voltage notation can be interpreted as follows:

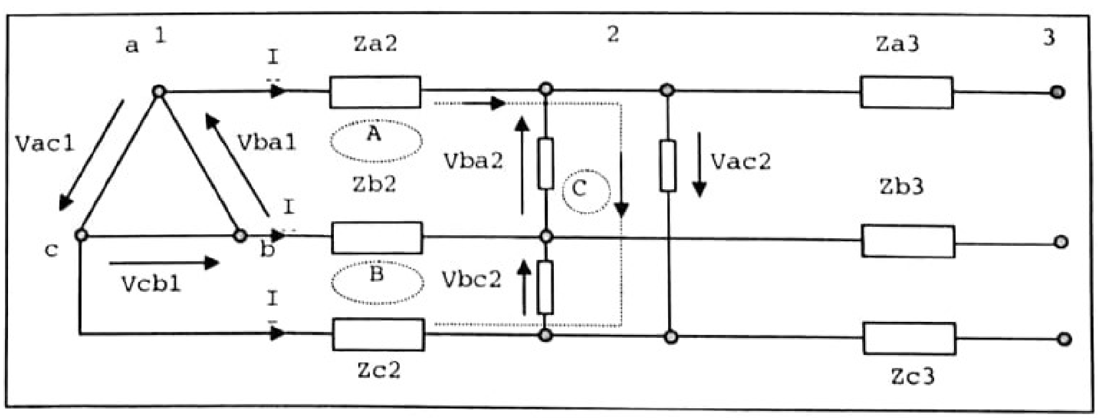

Figure 2: Transformed three single-phase loads connected at node 2

- point a is at higher potential than point b
- point b is at higher potential than point c
- point c is at higher potential than point a
- the direction of the system line to line voltages is from the lower potential to higher potential
- the direction of the phase currents are from the higher potential to lower potential
- the first letter denotes the point with lower potential

According to the above, the phase currents at node 2 are designated as $\bar{I}_{2,ab}$, $\bar{I}_{2,bc}$ and $\bar{I}_{2,ca}$ for phase ba, cb and ac respectively. By treating the supply phase voltage $\overline{V}_{1,ba}$ to be the reference phasor and adopting the ABC sequence, it follows that:

$$\begin{aligned} \overline{V}_{1,ba} &= |V_{1,ba}|\angle 0^0 \\ \overline{V}_{1,cb} &= V_{1,cb}\angle 240^0 \\ \overline{V}_{1,ac} &= V_{1,ac}\angle 120^0 \end{aligned} \tag{1}$$

Assume that currents $\bar{I}_x$, $\bar{I}_y$ and $\bar{I}_z$ are flowing in the line conductors as shown in Figure 2. By applying Kirchhoff's voltage law around loop A,

$$-\overline{V}_{1,ba} + \bar{I}_x.\overline{Z}_{a2} + \overline{V}_{2,ba} - \bar{I}_y.\overline{Z}_{b2} = 0 \tag{2}$$

According to (2),

$$\overline{V}_{ba2} = \overline{V}_{ba1} - \bar{I}_x.\overline{Z}_{a2} + \bar{I}_y.\overline{Z}_{b2} \tag{3}$$

According to Kirchhoff's current law:

$$\bar{I}_x = \bar{I}_{2,ab} - \bar{I}_{2,ca} \text{ and}$$

$$\bar{I}_y = \bar{I}_{2,bc} - \bar{I}_{2,ab} \tag{4}$$

Therefore,

$$\begin{aligned} \overline{V}_{2,ba} &= \overline{V}_{1,ba} + \overline{Z}_{b2}(\bar{I}_{2,bc} - \bar{I}_{2,ab}) - \overline{Z}_{a2}(\bar{I}_{2,ab} - \bar{I}_{2,ca}) \\ &= \overline{V}_{1,ba} - \bar{I}_{2,ab}(\overline{Z}_{a2} + \overline{Z}_{b2}) + \bar{I}_{2,bc}\overline{Z}_{b2} + \bar{I}_{2,ca}\overline{Z}_{a2} \end{aligned} \tag{5}$$

By applying Kirchhoff's voltage law around loop B,

$$-\overline{V}_{1,cb} + \bar{I}_y.\overline{Z}_{b2} + \overline{V}_{2,bc} - \bar{I}_z.\overline{Z}_{c2} = 0 \tag{6}$$

$$\overline{V}_{2,bc} = \overline{V}_{1,cb} + \bar{I}_y.\overline{Z}_{b2} - \bar{I}_z.\overline{Z}_{c2} \tag{7}$$

$$\text{But } \bar{I}_z = \bar{I}_{2,ca} - \bar{I}_{2,bc} \tag{8}$$

Therefore,

$$\begin{aligned} \overline{V}_{2,bc} &= \overline{V}_{1,cb} + \overline{Z}_{b2}(\bar{I}_{2,bc} - \bar{I}_{2,ab}) - \overline{Z}_{c2}(\bar{I}_{2,ca} - \bar{I}_{2,bc}) \\ &= \overline{V}_{1,cb} + \bar{I}_{2,ab}.\overline{Z}_{b2} - \bar{I}_{2,bc}(\overline{Z}_{b2} + \overline{Z}_{c2}) + \bar{I}_{2,ca}.\overline{Z}_{c2} \end{aligned} \tag{9}$$

By applying Kirchhoff's voltage law around loop C,

$$-\overline{V}_{1,ac} + \bar{I}_z.\overline{Z}_{c2} + \overline{V}_{2,ac} - \bar{I}_x.\overline{Z}_{a2} = 0 \tag{10}$$

$$\overline{V}_{2,ac} = \overline{V}_{1,ac} - \bar{I}_z.\overline{Z}_{c2} + \bar{I}_x.\overline{Z}_{a2} \tag{11}$$

Therefore,

$$\begin{aligned} \overline{V}_{2,ac} &= \overline{V}_{1,ac} - \overline{Z}_{c2}(\bar{I}_{2,ca} - \bar{I}_{2,bc}) - \overline{Z}_{a2}(\bar{I}_{2,ab} - \bar{I}_{2,ca}) \\ &= \overline{V}_{1,ac} + \bar{I}_{2,ab}\overline{Z}_{a2} + \bar{I}_{2,bc}\overline{Z}_{c2} - \bar{I}_{2,ca}(\overline{Z}_{a2} + \overline{Z}_{c2}) \end{aligned} \tag{12}$$

According to expressions (5), (9), and (12), line voltages $\overline{V}_{2,ba}$, $\overline{V}_{2,cb}$ and $\overline{V}_{2,ac}$ at node 2 can be expressed in a matrix form as:

$$\begin{bmatrix} \overline{V}_{2,ba} \\ \overline{V}_{2,cb} \\ \overline{V}_{2,ac} \end{bmatrix} = \begin{bmatrix} \overline{V}_{1,ba} \\ \overline{V}_{1,cb} \\ \overline{V}_{1,ac} \end{bmatrix} - \begin{bmatrix} \overline{Z}_{a2} + \overline{Z}_{b2} & -\overline{Z}_{b2} & -\overline{Z}_{a2} \\ -\overline{Z}_{b2} & \overline{Z}_{b2} + Z_{c2} & -\overline{Z}_{c2} \\ -\overline{Z}_{a2} & -\overline{Z}_{c2} & \overline{Z}_{a2} + \overline{Z}_{c2} \end{bmatrix} \begin{bmatrix} \bar{I}_{2,ab} \\ \bar{I}_{2,bc} \\ \bar{I}_{2,ca} \end{bmatrix} \tag{13}$$

In general, at any load k of the network, line to line voltages $\overline{V}_{k,ba}$, $\overline{V}_{k,cb}$ and $\overline{V}_{k,ac}$ can be expressed in matrix form as:

$$\begin{bmatrix} \overline{V}_{k,ba} \\ \overline{V}_{k,cb} \\ \overline{V}_{k,ac} \end{bmatrix} = \begin{bmatrix} \overline{V}_{k-1,ba} \\ \overline{V}_{k-1,cb} \\ \overline{V}_{k-1,ac} \end{bmatrix} - \begin{bmatrix} \overline{Z}_{ak} + \overline{Z}_{bk} & -\overline{Z}_{bk} & -\overline{Z}_{ak} \\ -\overline{Z}_{bk} & \overline{Z}_{bk} + Z_{ck} & -\overline{Z}_{ck} \\ -\overline{Z}_{ak} & -\overline{Z}_{ck} & \overline{Z}_{ak} + \overline{Z}_{ck} \end{bmatrix} \begin{bmatrix} \bar{I}_{k,ab} \\ \bar{I}_{k,bc} \\ \bar{I}_{k,ca} \end{bmatrix} \tag{14}$$

It should be kept in mind that, equation (14) expresses line-to-line voltages, currents through respective phases that are connected between phases and individual conductor impedance.

DEVELOPMENT OF A SOLUTION TO THE LOAD FLOW ANALYSIS

When dealing with a simplified power distribution network as depicted in Figure A1 shown in appendix-A, the individual phases should be clearly defined. This is very important so that, the consumer voltage values can be evaluated. Single-phase networks and SWER networks through isolating transformers are connected between two phases of the three-phase distribution feeder. In this case, SWER networks are regarded as single-phase networks (phase to phase connection) by nature of their connection to the three-phase backbone feeder. In this proposed method, the SWER networks are modeled as single-phase networks and therefore the impedance per conductor is treated to be half of the total line impedance between node points. As the result of such connection, the system line-to-line voltage becomes the phase voltage for single-phase loads. The line impedance data to be specified should distinguish the different phases. If the three-phase system is symmetrical, meaning that all loads are three phase loads fed from a three-phase backbone feeder, the D-array that is defined as $[2,q]$ matrix is sufficient for the load flow analysis and can be developed by the flow chart shown in appendix-B. Its elements in the first row show the sending end nodes in ascending order and the elements in the second row shows the receiving end nodes for each branch respectively. But as long as single-phase loads are present, the D-array will no longer be adequate. Therefore, each phase should be defined by separate arrays. In this paper, three arrays are employed called P21-array, P32-array and P13-array to describe the system network for the phase ba, cb and ac respectively. It should be understood that, to solve equation (14) the D-array is used. The P21-array, P32-array and P13-array are applied to get the voltage profile for the corresponding phase. The data input required for the network shown in Figure A1 for the load flow should be provided in five categories as follows:

Category 1: data to define the common D-array. According to Figure A1, the following data should be specified where Z is a variable that indicates the existence of a network connection between nodes m *and* n.

$$Z(m,n)=p \tag{15}$$

$$nt=21 \tag{16}$$

Category 2: data to define the branch impedance. For line 1, the branch impedance between the nodes is given a variable $Z1$. For line 2, the branch impedance is given a variable $Z2$, while for line 3 is given a variable $Z3$. According to Figure A1, the following data should be specified.

$$Z1(m,n)=k \tag{17}$$

$$Z2(m,n)=k \tag{18}$$

$$Z3(m,n)=k \tag{19}$$

Category 3: data to define the P21-array

$$Z21(m,n)=p \tag{20}$$

$$nt1=21 \tag{21}$$

Category 4: data to define the P32-array

$$Z32(m,n)=p \tag{22}$$

$$nt2=21 \tag{23}$$

Category 5: data to define the P13-array

$$Z13(m,n)=p \tag{24}$$

$$nt3=21 \tag{25}$$

where m is the sending end node, n is the receiving end node, p is any integer value, nt is the largest node number, k is the conductor impedance value between node m and n of the respective line. The variables $nt1$, $nt2$ and $nt3$ are the largest node number for phase ba, cb and ac respectively while variables $Z21$, $Z32$ *and* $Z13$ indicates the existence of a network connection between nodes m *and* n for phase ba, cb and ac respectively.

Table 1: The D-array

1	2	2	3	3	3	3	4	4	4	4	5	6	8	10	12	14	16	18	20
2	3	6	4	8	10	12	5	14	16	18	20	7	9	11	13	15	17	19	21

Table 2: The P21-array

1	2	2	3	3	4	4	5	6	10	18	20
2	3	6	4	10	5	18	20	7	11	19	21

Table 3: The P32-array

1	2	2	3	3	4	4	5	6	8	14	20
2	3	6	4	8	5	14	20	7	9	15	21

Table 4: The P13-array

1	2	2	3	3	4	4	5	6	12	16	20
2	3	6	4	12	5	16	20	7	13	7	21

After specifying the above input data, the arrays are evaluated automatically using the computer flow charts shown in appendix B, C, D and E will appear as shown in Table 1-4. The Tables are defined as $[2,q]$ matrix, where elements in the first row shows the sending end nodes in ascending order while elements in the second row shows the receiving end nodes for each branch respectively.

The path-array, b-array, wx-array and x2-array are formulated by the use of the D-array as discussed in (Kundy, 2003). The column impedance matrix designated Z11, Z22 and Z33 are for the line 1, 2, and 3 respectively. These are developed from the data in category 2 by using the flow chart shown in appendix-B. These column matrices represent the line impedance data to be applied in equation (14). The numbering of the nodes in three-phase radial distribution systems should adhere to the following protocol:

- number 1 is assigned to the supply node
- jumping of the numeric sequence order is not allowed
- numbering of the nodes should follow the direction of the current flow, that means, the current must always flow from the smaller number towards the bigger number
- The individual phase network should be numbered in such a way that the biggest node number for each phase network is connected to the node that is the biggest in relation to other nodes of the same phase network that are connected, as can clearly be seen in Figure A1.

CURRENTS DUE TO CONSTANT P-Q LOADS

The load current phasor $\bar{I}_{k-nm}$ for the phase nm at node k due to the constant $P-Q$ loads on phase nm can be expressed as:

$$\bar{I}_{k-nm} = (P_{k-nm} - jQ_{k-nm}) / \bar{V}_{k-nm}^{*} \qquad (26)$$

where

P_{k-nm} is the real load power for phase nm at node k

Q_{k-nm} is the reactive load power for phase nm at node k

$\bar{V}_{k-nm}^{*}$ is the conjugate of the voltage for phase nm at node k

COMPUTATION RESULTS

The computer program was written in MATLAB for load flow analysis using a forward and backward sweep algorithm as reported in (Baran et al, 1997). Figure A1 in the appendix-A, shows the three-phase distribution network used to test the proposed method. The line and load data applied

Table 1: Voltage profiles for phases ba, cb and ac using the load data in table A1

Node no.	$\lvert V_{ba}\rvert$ p.u.	$\lvert V_{cb}\rvert$ p.u.	$\lvert V_{ac}\rvert$ p.u.
1	1.0000	1.0000	1.0000
2	0.9702	0.9702	0.9702
3	0.9481	0.9481	0.9481
4	0.9335	0.9335	0.9335
5	0.9254	0.9254	0.9254
6	0.9625	0.9625	0.9625
7	0.9592	0.9592	0.9592
8		0.9432	
9		0.9405	
10	0.9432		
11	0.9405		
12			0.9432
13			0.9405
14		0.9292	
15		0.9269	
16			0.9292
17			0.9269
18	0.9292		
19	0.9269		
20	0.9173	0.9173	0.9173
21	0.9126	0.9126	0.9126

Table 2: Voltage profiles for phases ba, cb and ac using the load data in table A2

Node no.	$\lvert V_{ba}\rvert$ p.u.	$\lvert V_{cb}\rvert$ p.u.	$\lvert V_{ac}\rvert$ p.u.
1	1.0000	1.0000	1.0000
2	0.9702	0.9737	0.9749
3	0.9482	0.9551	0.9576
4	0.9331	0.9417	0.9448
5	0.9250	0.9337	0.9368
6	0.9625	0.9660	0.9672
7	0.9592	0.9627	0.9639
8		0.9515	
9		0.9497	
10	0.9428		
11	0.9401		
12			0.9555
13			0.9543
14		0.9381	

are given in Appendix-A. The convergence criteria are taken as 0.00001 and 0.0001 (the difference in p.u. nodal voltage magnitudes and phase angles per each iteration respectively). The voltage profiles evaluated for each phase are tabulated in Table 1 and Table 2 for balanced and unbalanced loading respectively.

DISCUSSION OF THE RESULTS

The results obtained for balanced loading in Table 1 shows clearly that the corresponding node points for each phase have the same value. This is the expected result taking into consideration that the network configuration is balanced. For the case of unbalanced loading, phase ba, which is more loaded, experiences more voltage drop as compared to other phases. This result to its voltage profile to have lower voltage magnitude compared to other phases as clearly shown in Table 2 as according to ohm's law. The same applies to phase cb, which is more loaded than phase ac. The results confirm the validity of the proposed method.

CONCLUSION

The method presented in this paper has demonstrated its capability of performing load flow analysis on three-phase delta-delta connected radial distribution networks feeding different topologies. Adoption of a single line diagram that expresses all the network nodes enables the development of various arrays that can be utilised to perform computations on the network under study. The development of arrays that are used to identify individual phases gives the proposed method the ability to recognise the system configuration. The results obtained for a balanced typology using balanced and unbalanced load data confirm the suitability of the proposed method.

NOMENCLATURE

MV	Medium Voltage
SWER	Single- Wire Earth Return
Y-bus	admittance bus
$\Delta - \Delta$	delta – delta connected network

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APPENDIX-A

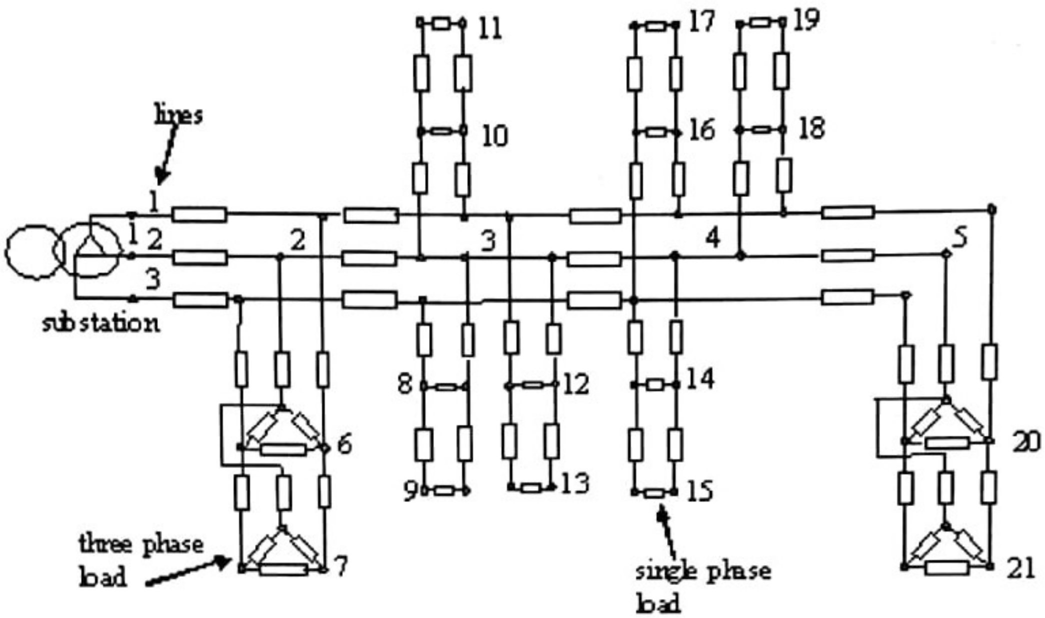


Figure A1: Three-phase backbone feeder tapped by single-phase, SWER and three-phase topologies used to test the proposed method.

Table A1: Balanced load data

node no.	phase ba kVA	phase cb kVA	phase ac kVA
6	50	50	50
7	50	50	50
8		50	
9		60	
10	50		
11	60		50
12			60
13		45	
14		50	
15			45
16			50
17			
18	45		
19	50		
20	50	50	50
21	50	50	50

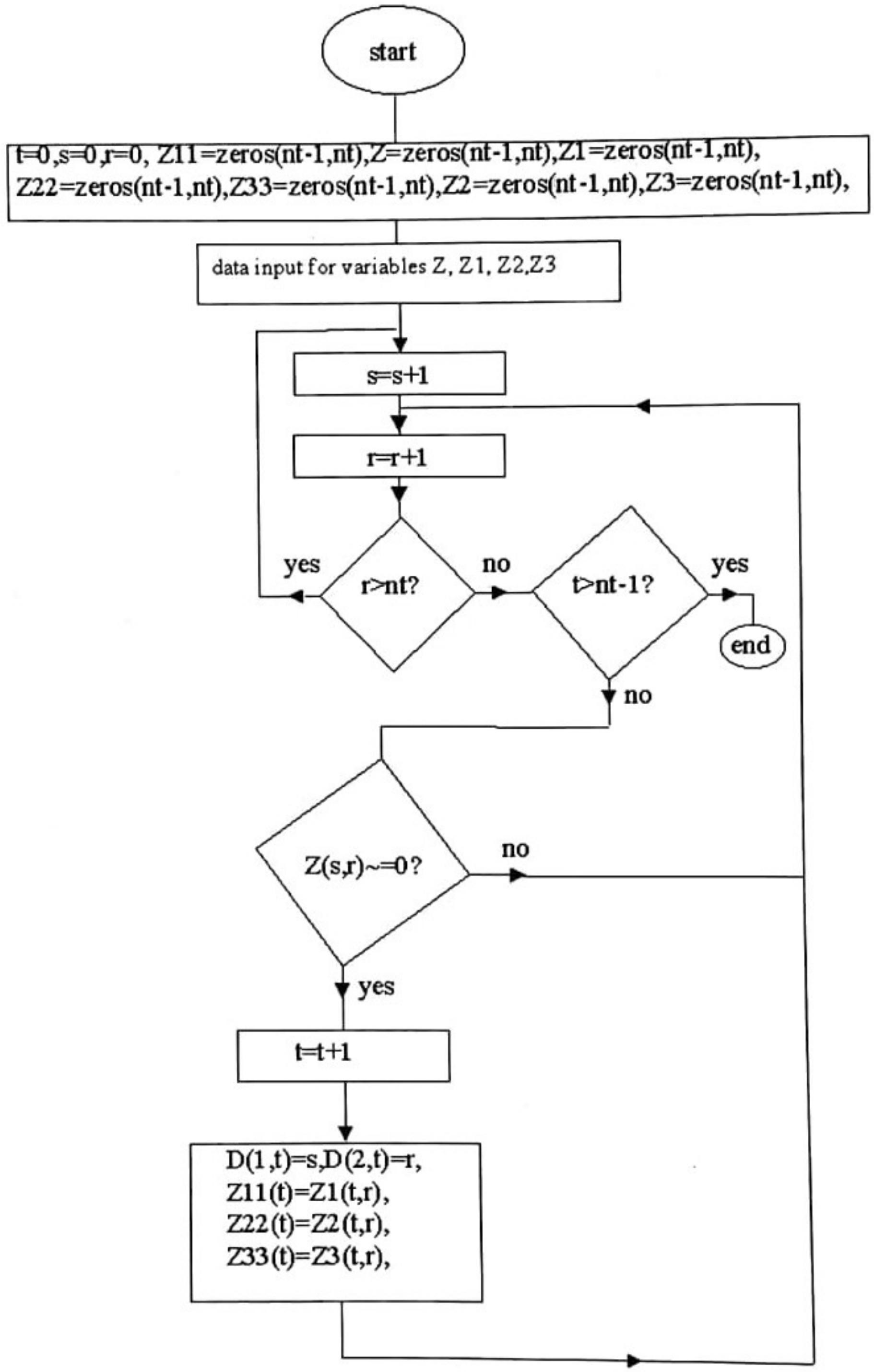
Table A2: Unbalanced load data

node no.	phase ba kVA	phase cb kVA	phase ac kVA
6	50	50	50
7	50	50	50
8		40	
9		40	
10	60		
11	60		
12			25
13			25
14		40	
15		40	
16			25
17			25
18	60		
19	60		
20	50	50	50
21	50	50	50

For testing purposes, the impedance per conductor between node points is assumed to be $2.32 + j1.19\Omega$ and the loads are all operating at a 0.8 power factor lagging. The operating voltage is $11kV$.

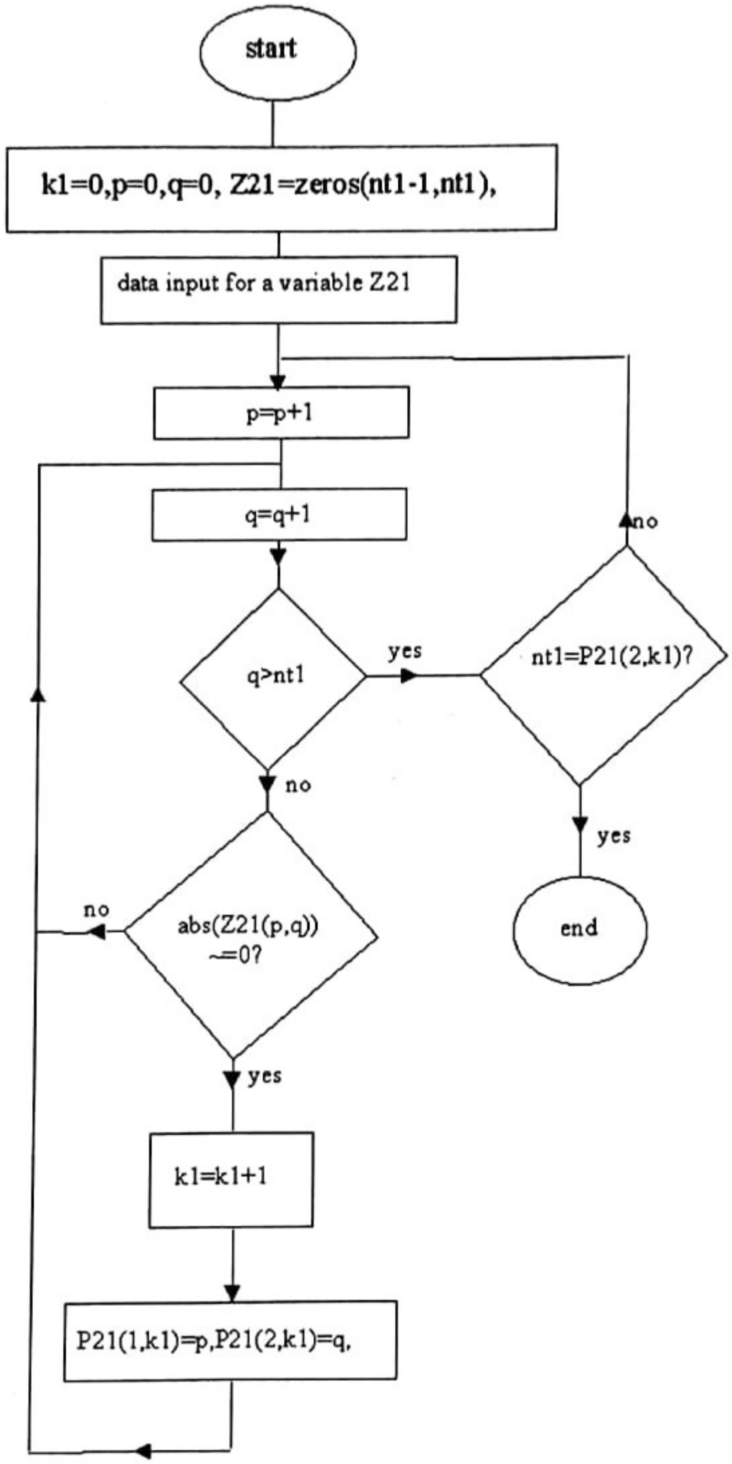
APPENDIX-B

The computer flow chart for creating the D-array, Z11-array, Z22-array and Z33-array



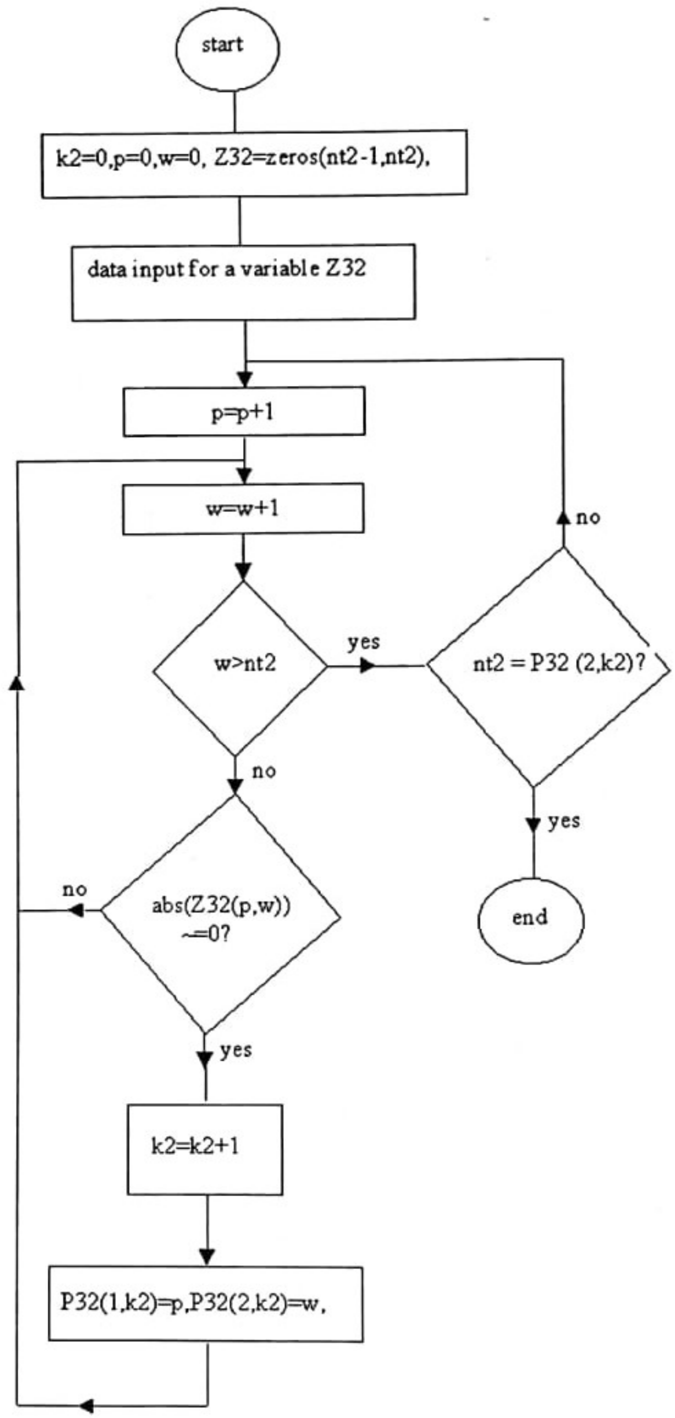
APPENDIX-C

The computer program flow chart for creating the P21-array



APPENDIX-D

The computer program flow chart for creating the P32-array



APPENDIX-E

The computer program flow chart for creating the P13- array