ASSESSMENT OF THE ACCURACY OF APPROXIMATE METHODS IN THE ANALYSIS OF PLANE FRAME STRUCTURES

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Approximate methods of analysis are widely used in the analysis of frames for preliminary design due to their simplicity as compared to exact analysis methods although they are not as accurate. In this paper, the methods of hinges and portal frame are described and have been used to analyze a statically indeterminate frame by taking into account the bending stiffness of the frame members. The obtained results are comparable to the magnitudes of forces determined by exact methods of analysis.

Keywords: *Analysis, accuracy, approximate and exact methods, frame, hinges, stiffness, inflection points, statically determinate and indeterminate structures, superimpose*

INTRODUCTION

In the design of building frames, three dimensional analysis is seldom used in practice except in the case of very important and unusual buildings. Instead the three dimensional actual building frame is broken up into a series of plane frames parallel to each other. The applied loads are divided suitably among the constituent plane frames and each plane is assumed to act independent of the other frame. In this manner a three dimensional problem is reduced to a much simpler two dimensional one. Analysis of plane frames is carried out using what are termed as rigorous or exact methods [Wang, 1986]. Strictly speaking, there is nothing like an exact method since all methods are based on certain simplifying assumptions which are not completely valid in actual practice. These assumptions have, however become so common and well established that the methods based on them are generally referred to as the exact methods. The so called exact methods require a prior knowledge of the relative flexural stiffness of the members. The relative flexural stiffness which cannot be determined unless the cross-sections of the members have been chosen which in turn cannot be chosen unless the analysis has been completed.

In this context, the approximate methods of analysis are very useful since they usually do not require a prior knowledge of the relative flexural stiffness of the members of the frame. They may be used for the determination of the approximate values of the internal forces and the selection of tentative cross-sectional properties as a preliminary design. The tentative cross-sections chosen in the preliminary design may then be revised on the basis of the so called exact method. In some standards, approximate analysis is permitted even for the final design. Compared to exact methods of analysis, approximate methods are simpler and do not demand much higher effort for calculations.

This paper looks into the possibility of improving the accuracy of approximate methods in the analysis of frames subjected to both vertical and horizontal loads. In this case the approximate methods used for analysis of the frames are the methods of hinges and the portal frame method. The obtained results are compared to the results of exact analysis methods.

APPROXIMATE METHODS

Introduction

For approximate analysis, assumptions aimed at transforming a statically indeterminate structure into statically determinate approximately without changing the behaviour of the structure so that the static equilibrium equations can be applied to analyze the structure are made. The numbers of assumptions required are equal to the degree of statical indeterminacy of the structure. Several approximate methods of analysis exists, but in this paper only method of hinges and portal frame method will be applied [Wang, 1986].

Method of Hinges

The method of hinges is normally used for the analysis of statically indeterminate beams and frames without side sway subjected to vertical/gravity loads. In the method of hinges, assumptions to transform a statically indeterminate frame to statically determinate are made [Wang, 1986].

From the fundamental relationship between bending moments and the deflected line, the bending moment is zero at the inflection points. Therefore, hinges are introduced at the inflection points hence reducing the degree of statical indeterminacy but without changing the behaviour of a statically indeterminate structure.

The analysis of a structure becomes approximate because the location of inflection points is estimated. It has to be noted that the position of inflection points changes with loading and stiffness/type of supports. The location of inflection points on a beam for different loading conditions and types of supports is shown in figure 1 [Glock, 1982]

The assumptions made in the method of hinges are equal to the degree of statical indeterminacy in order to render a statically indeterminate structure statically determinate.

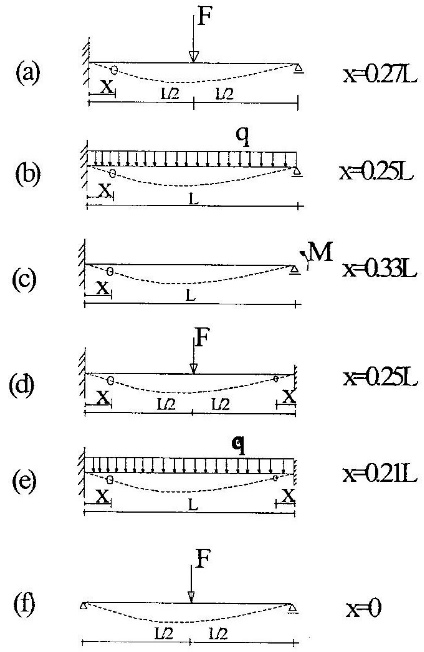

Figure. 1:Location of inflection points for beams

In frames with beams supported by columns, the location of inflection points also will vary in accordance to the type of joint (flexible or rigid/stiff joint) and the type of supports for the columns. In case of stiff/rigid joints, the location of the inflection points also depends on the stiffness of the members. When establishing the inflection points as indicated in figure 1, the bending stiffness of columns/supports are not normally considered. In this paper the stiffness of columns has been taken into account. This allows the determination of inflection points more accurately. A case of a beam connected to columns with stiff joints subjected to a point load is shown in table 1. In all cases shown in table 1, the stiffness of a beam is equal to EI while the stiffness of the columns varies i.e. nEI, where n = 1, 2, 3, ...

The following expression can be used to approximate the location of the inflection points in beams connected to columns with stiff joints subjected to a point/concentrated load.

$$x = \frac{3nl}{2(4+6n)} \qquad (1)$$

Table 1: Location of inflection point with stiffness of columns

Stiffness of Columns	E I	2EI	3EI	4EI
Location of inflection point, x	0.15l	0.1875l	0.2045l	0.214l

Where l is the span of the beam and n = *1, 2, 3* indicates the order of the stiffness, i. e. *nEI*. In table 2, variation of location of inflection points when the stiffness of beam varies while the stiffness of columns is constant and equal to EI is shown.

Table 2.: Location of inflection point as the stiffness of beam varies

Stiffness of beam	E I	2EI	3EI	4EI
Location of inflection point, x	0.15l	0.1l	0.0833l	0.068l

In case of a beam connected to simply supported columns with stiff joints subjected to distributed loads, the following expression can be used to estimate the location of inflection points in a beam.

$$x = 0.5l(1-\sqrt{\frac{n+2}{3n+2}}) \qquad (2)$$

When a simple frame with stiff joints for beams and columns is subjected to point vertical load only, the location of inflection points in the beam is normally determined using the average value of cases indicated in fig. 1(d) and (f). This approximation is based on the assumption of two extreme cases regarding the connections of the beam and columns; that is in one case the beam is considered to be simply supported by the columns (see figure 1 (f)), and in the second case the beam is considered to be fixed at the connections with columns (see figure 1 (d)) due to the fact that there exist bending moments at the joints. Therefore, the location of inflection points is approximated by taking the average value of the two cases.

The axial forces in the beam subjected to gravity loads are generally small and hence it is assumed that the axial force in the beam is zero. The column is considered to be loaded with a moment caused by the applied load on the beam and the connection of the beam and column is considered to be simple. Therefore, it is assumed that the column behaviour is as of the member shown in figure 1 (c). A hinge is, therefore, introduced at the location of inflection point.

It has to be noted that if the supports of the frame columns are fixed, then the values determined by expressions (1) and (2) can not be directly used as the type of supports has an influence on the stiffness of the frame. Since fixed supports increases the stiffness of the frame, then the distance for inflection points in beams from the joints will slightly increase. For example, in case when the bending stiffness of beam and columns is the same, the location of the inflection point can be taken to be 0.16l, and when the bending stiffness of a beam is twice greater than that of columns the location of the inflection point is approximately 0.12l.

Portal frame method

The portal frame method is used for frames with side sway loaded horizontally.

The assumption in this method is based on the different stiffness of the members making up a frame and results in the assumed distribution of the horizontal forces in the members [Wang, 1986].

Exact analysis for frames subjected to horizontal forces in the joints shows that inflection points in columns for all storeys are located approximately near the mid height except for the ground floor where they are located at distance two thirds from the base/supports [Wang, 1986]. Therefore, hinges are introduced in the location of inflection points hence reducing the degree of statical indeterminacy.

Transverse forces in columns for every storey are approximately proportional to I/h^3, where I is the moment of inertia and h is the storey height/height of column [Wang, 1986]. The transverse force/shear force in the column can be determined from the following equilibrium condition.

With the shear forces known in the columns, then the bending moments of columns in the joints can be determined. Considering the equilibrium of moments at the joints the moments in beams proportional to the stiffness i = EI/l can be determined. Exact analysis shows that the moments at mid span of all beams is zero.

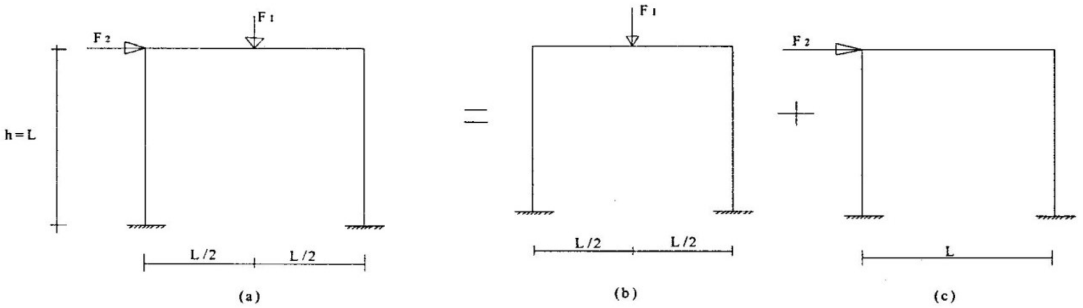

Figure. 2 Single storey frame ($F_1 = F_2 = F$)

$$V_k = \frac{I_k}{h_k^3} \sum F \Big/ \sum_{i=1}^{n} \frac{I_i}{h_i^3} \qquad (3)$$

Where, $\sum F$ is the sum of horizontal forces acting on a frame above the section in consideration, $\frac{I_i}{h_i^3}$ ratio for every column in the floor, and $\frac{I_k}{h_k^3}$ ratio for column in which the shear force is being determined.

For frames with several bays, the shear forces in all interior columns are equal and that the shear forces in the two exterior columns are equal to one half of the shear forces in the interior columns. The foregoing assumptions in respect of the distribution of the shear forces in the columns of the same storey are reasonable when all bays are approximately equal.

ANALYSIS OF FRAMES

Simple frame structure

Analysis by approximate methods

A single storey frame shown in figure 2a with stiffness for beams and columns equal to EI is analysed by applying the method of hinges and portal frame method and the principle of superposition. For fig. 2b where only a vertical load is applied, the method of hinges is used while for a frame in fig. 2(c) the portal frame method is used.

For the frame subjected to a vertical load, the inflection points in the beam are assumed to be at a distance equal to 0.1251 from the joints. This is an average value for the inflection points for cases (d) and (f) in fig. 1. For the columns, the inflection points are assumed to be at a distance equal to 0.33h from the supports. This is similar to case (c) in fig.1 Since the frame in fig. 2 is statically indeterminate to a degree of 3, then three hinges are introduced at 3 inflection points for fig. 2(b) hence transforming a frame into a statically determinate one.

Applying the static equilibrium equations, the reactions and internal forces are determined and the bending moment diagram shown in fig. 3(a) constructed.

For the case of loading shown in fig. 2 (c), the portal frame method is applied hence three hinges introduced at the inflection points at the mid span of the beam and mid height of the columns. Using expression (3) it is found that the horizontal forces at mid height of the columns is F/2. Applying the equilibrium equations, the reactions and internal forces are determined. The bending moment in fig. 3(b) is then constructed. Applying the principle of superposition of forces to the bending moment diagrams in 3(a) and 3(b), the bending moment diagram in fig. 3(c) is obtained which corresponds to the loading condition in fig. 2a.

Analysis by Exact Method

When exact analysis for the same framed structure shown in figure 2 is carried out using the force method, the bending moment diagrams shown in fig. 4 below are obtained. The bending moment diagram in figure 4(a) corresponds to the vertical loading condition in figure 2(b), the bending moment diagram in figure 4(b) corresponds to the loading condition in figure 2(c) while the bending moment diagram in figure 4(c) corresponds to the loading condition in figure 2(a).

Improving the accuracy of approximate methods

If for the same frame in fig. 2 the approximate method of hinges is again applied for the vertical loading condition shown in fig. 2(b) where the inflection points are assumed to be located at a distance equal to 0.15l from the joints as shown in table 1(the bending stiffness for beams and columns are equal), the bending moment diagram obtained after the analysis of this case is as shown in fig. 5(a).

If the portal frame method is applied for the frame with loading condition shown in fig. 2(c) and the inflection points in columns assumed to be at the height of 0.57h from the support/base(not at 0.5h as normally assumed) while for the beam the inflection point is at mid-span, the bending moment diagram obtained is as shown in fig. 5(b).

By superimposing the forces obtained from the

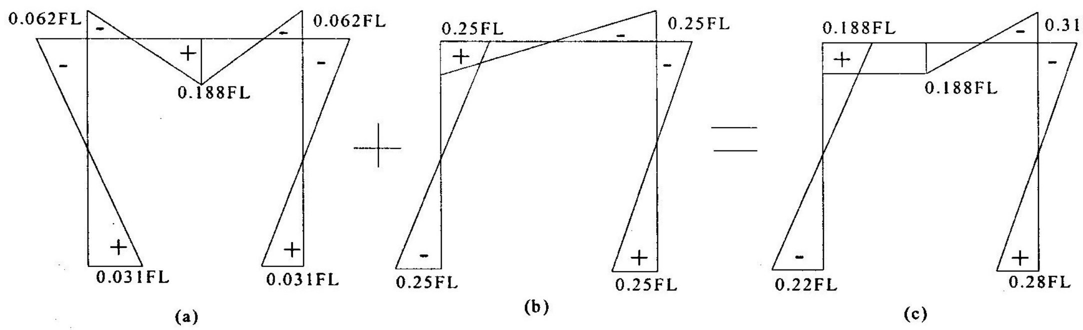

Figure. 3: Bending moment diagrams

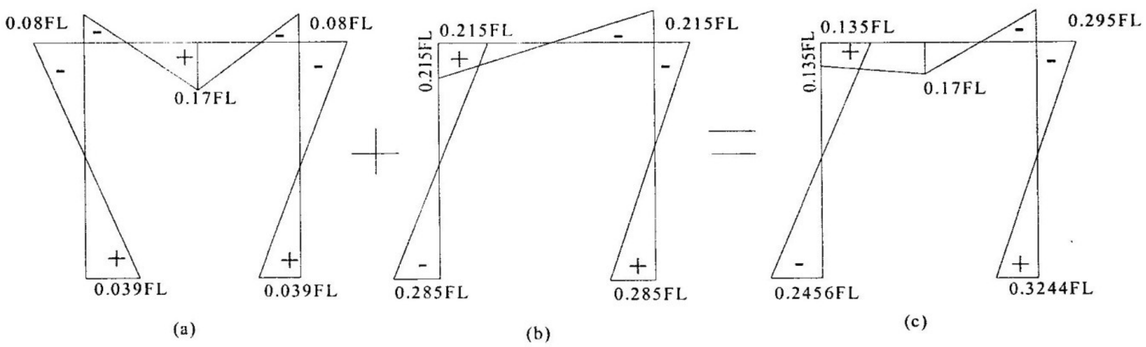

Figure. 4: Bending moment diagrams

two loading conditions described above, then the bending moment diagram for the loading shown in fig. 2(a) is as shown in fig. 5(c).

Two – bay frame structure

If a frame shown in fig. 6 is analyzed using the force method, the internal forces are determined and the bending moments diagrams shown in figure 7 are obtained.

Applying the improved approximate analysis approach described in section 3.3 above to the frame in figure 6, the approximate analysis is carried out as follows.

When the approximate method of hinges is applied to a frame loaded as shown in fig. 6(b), the inflection points are introduced in the columns at a height of 0.57h from the supports. For beams the inflection points are introduced at a distance equal to 0.151 from joints D and F. Since the loading of members DE and FE can be considered to be the two extreme cases shown in fig. 1(a) and fig.(d), then the inflection points from E are approximated to be at a distance equal to 0.261. Since the frame under consideration is statically indeterminate to a degree of six, then six hinges are introduced at any six inflection points to transform the frame into a statically determinate one. Applying the equilibrium equations to this statically determinate frame then the internal forces are determined and a bending moment diagram

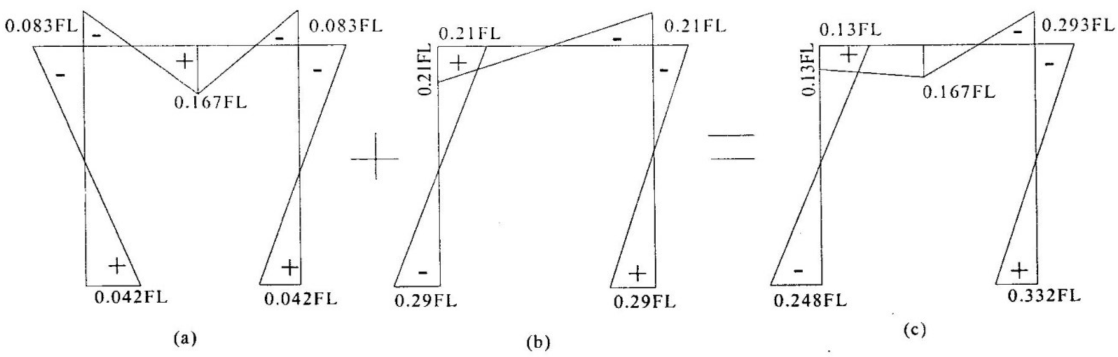

Figure. 5: Bending moment diagrams

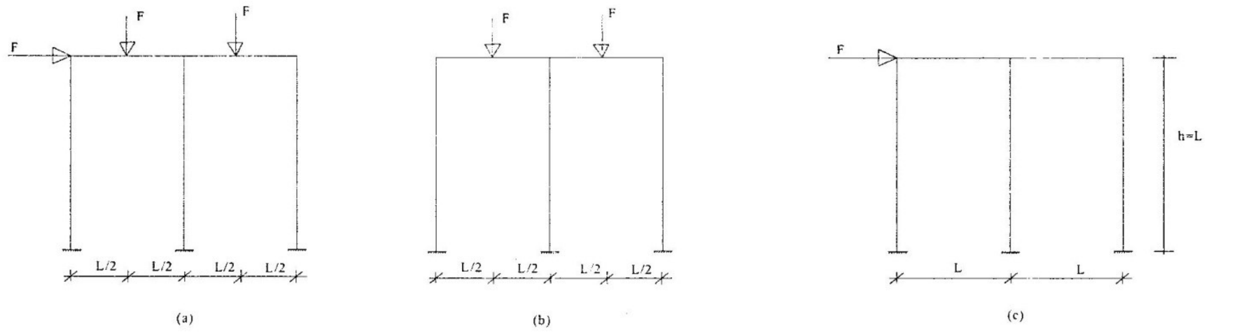

Figure. 6: A two – bay frame structure

shown in fig. 8(a) is constructed.

Applying the portal frame method of analysis to the frame loaded as shown in fig. 6(c), the internal forces are determined and the bending moment diagram shown in fig. 8(b) drawn. Superimposing the forces in fig. 8(a) and (b), the bending moment diagram in fig. 8(c) is obtained.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

From the analysis carried out above, the following conclusions are made.

(i) While for approximate analysis of frames subjected to lateral loads the inflection points in columns are assumed to be located at mid heights, exact analysis indicates that these points are located slightly above mid height, i.e. about 0.571 above the supports when the stiffness of members is the same. But if the stiffness of the beam is greater than that of columns, the inflection points are slightly below the mid heights of columns.

(ii) For frames subjected to vertical loads, exact analysis indicates that the inflection points in beams are located at a distance of about 0.1661

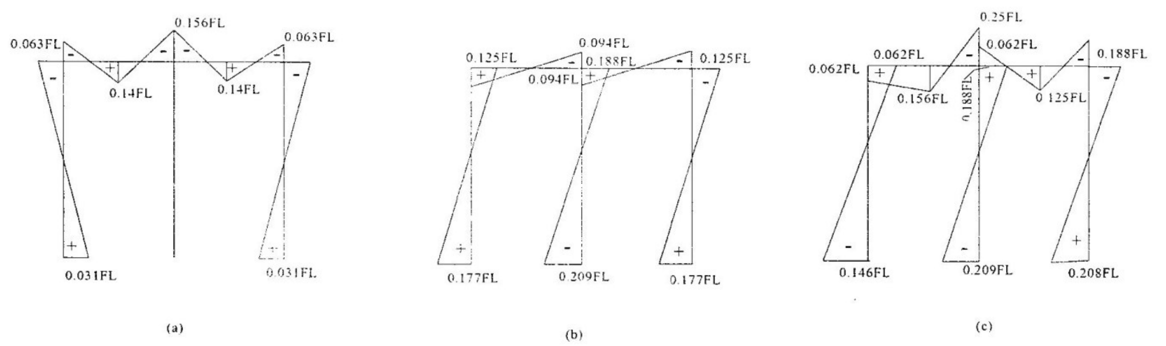

Figure. 7: Bending moment diagrams (exact analysis)

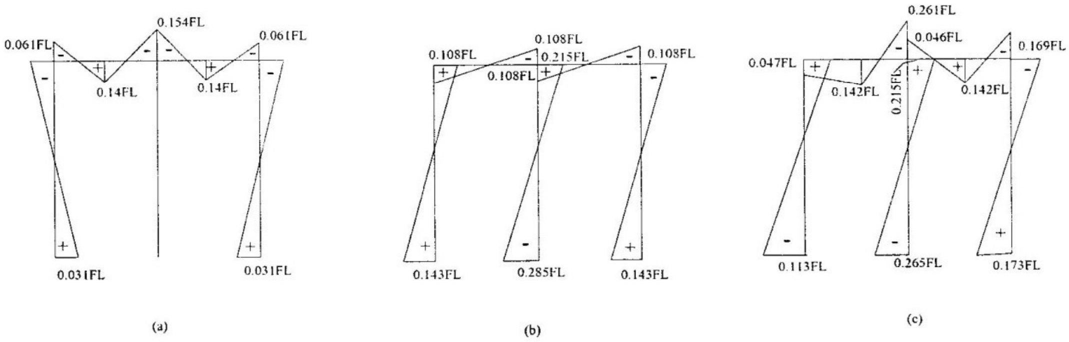

Figure 8: Bending moment diagrams (improved approximate analysis)

from the joints, while in the approximate analysis, the inflection points in beams are assumed to be located at a distance of 0.125l from the joints.

(iii) The magnitudes of bending moments obtained using approximate analysis

and exact analysis have a difference of up to 40% when the location of the inflection points for the beam is taken to be 0.125l (average of the value in fig. 1d and fig. 1f and without taking into consideration the bending stiffness of members). When the bending stiffness of members is taken into account in approximating the location of the inflection points (x = 0.15l), the difference of the results is about 20 %.

(iv) The results obtained in the approximate analysis when the inflection points in beams are estimated taking into account the bending stiffness of members are comparable to the results by the exact analysis methods.

Recommendations

(i) When analyzing framed structures subjected to lateral loads, inflection points in columns can be assumed to be located approximately at a distance of about 0.57l from the fixed supports.

(ii) For vertically loaded frames, the location of inflection points in beams can be estimated by using expression (1) or table 1 depending on the stiffness of the members and the type of columns' supports.

(iii) Approximate analysis of frames subjected to both lateral and vertical loads can be carried out by analyzing the two loadings separately and then using the principle of superposition of forces.

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